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PRECALCULUS I - MIXED PRACTICE

Distance, Midpoint, Circle Equations, Linear & Rational Equations, x-intercepts

Essential Formulas:

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Standard Form of Circle: $(x-h)^2 + (y-k)^2 = r^2$

x-intercepts: Set $y = 0$ and solve for x

Exercises

Problem 1: Distance Formula

Find the distance between the points $A(-3, 4)$ and $B(5, -2)$.

Problem 2: Midpoint Formula

Find the midpoint of the line segment connecting the points $P(-6, 8)$ and $Q(4, -2)$.

Problem 3: Distance and Midpoint Combined

Given points $A(2, -1)$ and $B(8, 7)$:

a) Find the distance between points A and B

b) Find the midpoint of segment AB

c) Find the distance from point A to the midpoint

Problem 4: Standard Form of Circle

Write the equation of a circle with center $(3, -4)$ and radius 5.

Problem 5: Circle from Center and Point

Find the equation of the circle with center $(-2, 3)$ that passes through the point $(1, 7)$.

Problem 6: Linear Equation

Solve for x: $3(2x-4)-5(x+2)=7x-18$

Problem 7: Rational Equation with Multiple Terms

Solve for x: $\frac{3}{x-2}+\frac{1}{x+1}=\frac{2}{x^2-x-2}$

Problem 8: x-intercepts of Linear Function

Find the x-intercept(s) of the function $f(x)=4x-12$.

Problem 9: x-intercepts of Quadratic Function

Find the x-intercepts of the function $g(x)=x^2-5x-14$.

Solutions

Solution 1: Distance Formula

Step 1: Identify the coordinates

Point $A(-3, 4)$: $x_1 = -3, y_1 = 4$

Point $B(5, -2)$: $x_2 = 5, y_2 = -2$

Step 2: Apply the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(5 - (-3))^2 + (-2 - 4)^2}$$

$$d = \sqrt{(8)^2 + (-6)^2}$$

$$d = \sqrt{64 + 36}$$

$$d = \sqrt{100} = 10$$

Answer:

The distance between points A and B is 10 units.

Solution 2: Midpoint Formula

Step 1: Identify the coordinates

Point $P(-6, 8)$: $x_1 = -6, y_1 = 8$

Point $Q(4, -2)$: $x_2 = 4, y_2 = -2$

Step 2: Apply the midpoint formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{-6 + 4}{2}, \frac{8 + (-2)}{2}\right)$$

$$M = \left(\frac{-2}{2}, \frac{6}{2}\right)$$

$$M = (-1, 3)$$

Answer:

The midpoint is $(-1, 3)$.

Solution 3: Distance and Midpoint Combined

Part (a): Distance between A and B

$A(2, -1)$ and $B(8, 7)$

$$d_{AB} = \sqrt{(8-2)^2 + (7-(-1))^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

Part (b): Midpoint of AB

$$M = \left(\frac{2+8}{2}, \frac{-1+7}{2}\right) = \left(\frac{10}{2}, \frac{6}{2}\right) = (5, 3)$$

Part (c): Distance from A to midpoint

Distance from $A(2, -1)$ to $M(5, 3)$:

$$d_{AM} = \sqrt{(5-2)^2 + (3-(-1))^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Answers:

a) Distance AB = 10 units

b) Midpoint = (5, 3)

c) Distance A to midpoint = 5 units

Solution 4: Standard Form of Circle

Step 1: Identify given information

Center: $(h, k) = (3, -4)$

Radius: $r = 5$

Step 2: Apply the standard form

Standard form: $(x - h)^2 + (y - k)^2 = r^2$

$$\text{Substituting: } (x - 3)^2 + (y - (-4))^2 = 5^2$$

$$(x - 3)^2 + (y + 4)^2 = 25$$

Answer:

$(x - 3)^2 + (y + 4)^2 = 25$

Solution 5: Circle from Center and Point

Step 1: Find the radius

Center: $(-2, 3)$, Point on circle: $(1, 7)$

Radius = distance from center to point:

$$r = \sqrt{(1-(-2))^2 + (7-3)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Step 2: Write the equation

Center: $(h, k) = (-2, 3)$, Radius: $r = 5$

$$(x - (-2))^2 + (y - 3)^2 = 5^2$$

$$(x + 2)^2 + (y - 3)^2 = 25$$

Answer:

$(x + 2)^2 + (y - 3)^2 = 25$

Solution 6: Linear Equation

Step 1: Expand both sides

$$3(2x-4)-5(x+2)=7x-18$$

$$6x-12-5x-10=7x-18$$

Step 2: Combine like terms

$$x-22=7x-18$$

Step 3: Solve for x

$$x-7x=-18+22$$

$$-6x=4$$

$$x=-\frac{4}{6}=-\frac{2}{3}$$

Verification:

Substitute $x=-\frac{2}{3}$ into original equation:

LHS:
$3(2(-\frac{2}{3})-4)-5(-\frac{2}{3}+2)=3(-\frac{4}{3}-4)-5(\frac{4}{3})=3(-\frac{16}{3})-\frac{20}{3}=-16-\frac{20}{3}=-\frac{68}{3}$

RHS: $7(-\frac{2}{3})-18=-\frac{14}{3}-18=-\frac{68}{3}$ ✓

Solution 7: Rational Equation with Multiple Terms

Step 1: Factor the denominator on the right

$$\frac{3}{x-2} + \frac{1}{x+1} = \frac{2}{x^2 - x - 2}$$

Note: $x^2 - x - 2 = (x-2)(x+1)$

Step 2: Multiply through by LCD: $(x-2)(x+1)$

$$3(x+1) + 1(x-2) = 2$$

$$3x + 3 + x - 2 = 2$$

$$4x + 1 = 2$$

Step 3: Solve for x

$$4x = 1$$

$$x = \frac{1}{4}$$

Check domain and solution:

Domain restrictions: $x \neq 2$ and $x \neq -1$

Since $x = \frac{1}{4}$ satisfies domain restrictions, it is valid.

Answer: $x = \frac{1}{4}$

Solution 8: x-intercepts of Linear Function

Step 1: Set function equal to zero

$$f(x) = 4x - 12$$

For x-intercept: $f(x) = 0$

$$4x - 12 = 0$$

Step 2: Solve for x

$$4x = 12$$

$$x = 3$$

Answer:

The x-intercept is $(3, 0)$ or simply $x = 3$.

Solution 9: x-intercepts of Quadratic Function

Step 1: Set function equal to zero

$$g(x) = x^2 - 5x - 14$$

$$\text{For x-intercepts: } g(x) = 0$$

$$x^2 - 5x - 14 = 0$$

Step 2: Factor the quadratic

Look for two numbers that multiply to -14 and add to -5:

Numbers: -7 and 2 (since $(-7) \times 2 = -14$ and $-7 + 2 = -5$)

$$x^2 - 5x - 14 = (x - 7)(x + 2) = 0$$

Step 3: Apply zero product property

$$x - 7 = 0 \quad \text{or} \quad x + 2 = 0$$

$$x = 7 \quad \text{or} \quad x = -2$$

Answer:

The x-intercepts are $(7, 0)$ and $(-2, 0)$, or simply $x = 7$ and $x = -2$.