5.1 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered

VOCABULARY: Fill in the blanks.

1. Polynomial and rational functions are examples of ________ functions.
2. Exponential and logarithmic functions are examples of nonalgebraic functions, also called ________ functions.
3. You can use the ________ Property to solve simple exponential equations.
4. The exponential function given by $f(x) = e^x$ is called the ________ ________ function, and the base e is called the ________ base.
5. To find the amount A in an account after t years with principal P and an annual interest rate r compounded n times per year, you can use the formula ________.
6. To find the amount A in an account after t years with principal P and an annual interest rate r compounded continuously, you can use the formula ________.

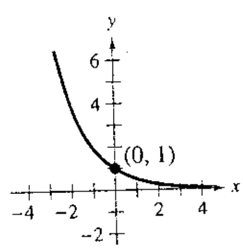
SKILLS AND APPLICATIONS

In Exercises 7–12, evaluate the function at the indicated value of x. Round your result to three decimal places.

Function	Value
7. $f(x) = 0.9^x$	$x = 1.4$
8. $f(x) = 2.3^x$	$x = \frac{3}{2}$
9. $f(x) = 5^x$	$x = -\pi$
10. $f(x) = \left(\frac{2}{3}\right)^{5x}$	$x = \frac{3}{10}$
11. $g(x) = 5000(2^x)$	$x = -1.5$
12. $f(x) = 200(1.2)^{12x}$	$x = 24$

In Exercises 13–16, match the exponential function with its graph. [The graphs are labeled (a), (b), (c), and (d).]

(a)



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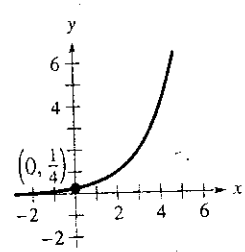

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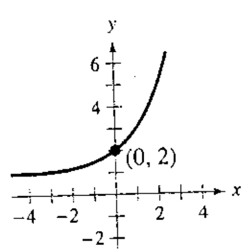
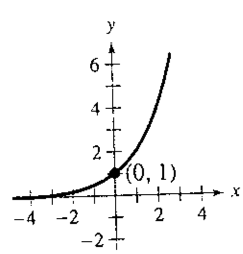


(d)



13. $f(x) = 2^x$
14. $f(x) = 2^x + 1$
15. $f(x) = 2^{-x}$
16. $f(x) = 2^{x-2}$

In Exercises 17–22, use a graphing utility to construc of values for the function. Then sketch the grapl function.

17. $f(x) = \left(\frac{1}{2}\right)^x$
18. $f(x) = \left(\frac{1}{2}\right)^{-x}$
19. $f(x) = 6^{-x}$
20. $f(x) = 6^x$
21. $f(x) = 2^{x-1}$
22. $f(x) = 4^{x-3} +$:

In Exercises 23–28, use the graph of f to des transformation that yields the graph of g.

23. $f(x) = 3^x$, $g(x) = 3^x + 1$
24. $f(x) = 4^x$, $g(x) = 4^{x-3}$
25. $f(x) = 2^x$, $g(x) = 3 - 2^x$
26. $f(x) = 10^x$, $g(x) = 10^{-x+3}$
27. $f(x) = \left(\frac{7}{2}\right)^x$, $g(x) = -\left(\frac{7}{2}\right)^{-x}$
28. $f(x) = 0.3^x$, $g(x) = -0.3^x + 5$

In Exercises 29–32, use a graphing utility to : exponential function.

29. $y = 2^{-x^2}$
30. $y = 3^{-|x|}$
31. $y = 3^{x-2} + 1$
32. $y = 4^{x+1} - 2$

In Exercises 33–38, evaluate the function at the value of x. Round your result to three decimal pla

Function	Value
33. $h(x) = e^{-x}$	$x = \frac{3}{4}$
34. $f(x) = e^x$	$x = 3.2$
35. $f(x) = 2e^{-5x}$	$x = 10$
36. $f(x) = 1.5e^{x/2}$	$x = 240$
37. $f(x) = 5000e^{0.06x}$	$x = 6$
38. $f(x) = 250e^{0.05x}$	$x = 20$

In Exercises 39–44, use a graphing utility to construct a table of values for the function. Then sketch the graph of the function.

39. $f(x) = e^x$
40. $f(x) = e^{-x}$
41. $f(x) = 3e^{x+4}$
42. $f(x) = 2e^{-0.5x}$
43. $f(x) = 2e^{x-2} + 4$
44. $f(x) = 2 + e^{x-5}$

In Exercises 45–50, use a graphing utility to graph the exponential function.

45. $y = 1.08^{-5x}$
46. $y = 1.08^{5x}$
47. $s(t) = 2e^{0.12t}$
48. $s(t) = 3e^{-0.2t}$
49. $g(x) = 1 + e^{-x}$
50. $h(x) = e^{x-2}$

In Exercises 51–58, use the One-to-One Property to solve the equation for x.

51. $3^{x+1} = 27$
52. $2^{x-3} = 16$
53. $\left(\frac{1}{2}\right)^x = 32$
54. $5^{x-2} = \frac{1}{125}$
55. $e^{3x+2} = e^3$
56. $e^{2x-1} = e^4$
57. $e^{x^2-3} = e^{2x}$
58. $e^{x^2+6} = e^{5x}$

COMPOUND INTEREST In Exercises 59–62, complete the table to determine the balance A for P dollars invested at rate r for t years and compounded n times per year.

n	1	2	4	12	365	Continuous
A						

59. $P = \$1500$, $r = 2\%$, $t = 10$ years
60. $P = \$2500$, $r = 3.5\%$, $t = 10$ years
61. $P = \$2500$, $r = 4\%$, $t = 20$ years
62. $P = \$1000$, $r = 6\%$, $t = 40$ years

COMPOUND INTEREST In Exercises 63–66, complete the table to determine the balance A for \$12,000 invested at rate r for t years, compounded continuously.

t	10	20	30	40	50
A					

63. $r = 4\%$
64. $r = 6\%$
65. $r = 6.5\%$
66. $r = 3.5\%$

67. **TRUST FUND** On the day of a child's birth, a deposit of \$30,000 is made in a trust fund that pays 5% interest, compounded continuously. Determine the balance in this account on the child's 25th birthday.

68. **TRUST FUND** A deposit of \$5000 is made in a trust fund that pays 7.5% interest, compounded continuously. It is specified that the balance will be given to the college from which the donor graduated after the money has earned interest for 50 years. How much will the college receive?

69. **INFLATION** If the annual rate of inflation averages 4% over the next 10 years, the approximate costs C of goods or services during any year in that decade will be modeled by $C(t) = P(1.04)^t$, where t is the time in years and P is the present cost. The price of an oil change for your car is presently \$23.95. Estimate the price 10 years from now.

70. **COMPUTER VIRUS** The number V of computers infected by a computer virus increases according to the model $V(t) = 100e^{4.6052t}$, where t is the time in hours. Find the number of computers infected after (a) 1 hour, (b) 1.5 hours, and (c) 2 hours.

71. **POPULATION GROWTH** The projected populations of California for the years 2015 through 2030 can be modeled by $P = 34.696e^{0.0098t}$, where P is the population (in millions) and t is the time (in years), with $t = 15$ corresponding to 2015.

(a) Use a graphing utility to graph the function for the years 2015 through 2030.
(b) Use the *table* feature of a graphing utility to create a table of values for the same time period as in part (a).
(c) According to the model, when will the population of California exceed 50 million?

72. **POPULATION** The populations P (in millions) of Italy from 1990 through 2008 can be approximated by the model $P = 56.8e^{0.0015t}$, where t represents the year, with $t = 0$ corresponding to 1990.

(a) According to the model, is the population of Italy increasing or decreasing? Explain.
(b) Find the populations of Italy in 2000 and 2008.
(c) Use the model to predict the populations of Italy in 2015 and 2020.

73. **RADIOACTIVE DECAY** Let Q represent a mass of radioactive plutonium (^{239}Pu) (in grams), whose half-life is 24,100 years. The quantity of plutonium present after t years is $Q = 16\left(\frac{1}{2}\right)^{t/24,100}$.

(a) Determine the initial quantity (when $t = 0$).
(b) Determine the quantity present after 75,000 years.
(c) Use a graphing utility to graph the function over the interval $t = 0$ to $t = 150,000$.

74. **RADIOACTIVE DECAY** Let Q represent a mass of carbon 14 (^{14}C) (in grams), whose half-life is 5715 years. The quantity of carbon 14 present after t years is $Q = 10\left(\frac{1}{2}\right)^{t/5715}$.
   (a) Determine the initial quantity (when $t = 0$).
   (b) Determine the quantity present after 2000 years.
   (c) Sketch the graph of this function over the interval $t = 0$ to $t = 10{,}000$.

75. **DEPRECIATION** After t years, the value of a wheelchair conversion van that originally cost \$30,500 depreciates so that each year it is worth $\frac{7}{8}$ of its value for the previous year.
   (a) Find a model for $V(t)$, the value of the van after t years.
   (b) Determine the value of the van 4 years after it was purchased.

76. **DRUG CONCENTRATION** Immediately following an injection, the concentration of a drug in the bloodstream is 300 milligrams per milliliter. After t hours, the concentration is 75% of the level of the previous hour.
   (a) Find a model for $C(t)$, the concentration of the drug after t hours.
   (b) Determine the concentration of the drug after 8 hours.

EXPLORATION

TRUE OR FALSE? In Exercises 77 and 78, determine whether the statement is true or false. Justify your answer.

77. The line $y = -2$ is an asymptote for the graph of $f(x) = 10^x - 2$.

78. $e = \dfrac{271{,}801}{99{,}990}$

THINK ABOUT IT In Exercises 79–82, use properties of exponents to determine which functions (if any) are the same.

79. $f(x) = 3^{x-2}$
$g(x) = 3^x - 9$
$h(x) = \frac{1}{9}(3^x)$

80. $f(x) = 4^x + 12$
$g(x) = 2^{2x+6}$
$h(x) = 64(4^x)$

81. $f(x) = 16(4^{-x})$
$g(x) = \left(\frac{1}{4}\right)^{x-2}$
$h(x) = 16(2^{-2x})$

82. $f(x) = e^{-x} + 3$
$g(x) = e^{3-x}$
$h(x) = -e^{x-3}$

83. Graph the functions given by $y = 3^x$ and $y = 4^x$ and use the graphs to solve each inequality.
   (a) $4^x < 3^x$ (b) $4^x > 3^x$

84. Use a graphing utility to graph each function. Us graph to find where the function is increasing decreasing, and approximate any relative maximu minimum values.
   (a) $f(x) = x^2e^{-x}$ (b) $g(x) = x2^{3-x}$

85. **GRAPHICAL ANALYSIS** Use a graphing utili graph $y_1 = (1 + 1/x)^x$ and $y_2 = e$ in the same vie window. Using the *trace* feature, explain what ha to the graph of y_1 as x increases.

86. **GRAPHICAL ANALYSIS** Use a graphing utili graph

$$f(x) = \left(1 + \frac{0.5}{x}\right)^x \quad \text{and} \quad g(x) = e^{0.5}$$

in the same viewing window. What is the relatic between f and g as x increases and decreases w bound?

87. **GRAPHICAL ANALYSIS** Use a graphing uti graph each pair of functions in the same viewing w Describe any similarities and differences in the gra
   (a) $y_1 = 2^x, y_2 = x^2$ (b) $y_1 = 3^x, y_2 = x^3$

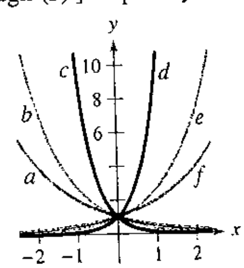
88. **THINK ABOUT IT** Which functions are expon
   (a) $3x$ (b) $3x^2$ (c) 3^x (d) 2^{-x}

89. **COMPOUND INTEREST** Use the formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

to calculate the balance of an account when $P =$ $r = 6\%$, and $t = 10$ years, and compounding (a) by the day, (b) by the hour, (c) by the min (d) by the second. Does increasing the nun compoundings per year result in unlimited gr the balance of the account? Explain.

90. **CAPSTONE** The figure shows the graphs of y $y = e^x$, $y = 10^x$, $y = 2^{-x}$, $y = e^{-x}$, and $y =$ Match each function with its graph. [The gra labeled (a) through (f).] Explain your reasonin



PROJECT: POPULATION PER SQUARE MILE an extended application analyzing the population p mile of the United States, visit this text's w *academic.cengage.com*. (Data Source: U.S. Ce

5.2 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered

VOCABULARY: Fill in the blanks.

1. The inverse function of the exponential function given by $f(x) = a^x$ is called the ________ function with base a
2. The common logarithmic function has base ________ .
3. The logarithmic function given by $f(x) = \ln x$ is called the ________ logarithmic function and has base ______
4. The Inverse Properties of logarithms and exponentials state that $\log_a a^x = x$ and ________.
5. The One-to-One Property of natural logarithms states that if $\ln x = \ln y$, then ________.
6. The domain of the natural logarithmic function is the set of ________ ________ ________ .

SKILLS AND APPLICATIONS

In Exercises 7–14, write the logarithmic equation in exponential form. For example, the exponential form of $\log_5 25 = 2$ is $5^2 = 25$.

7. $\log_4 16 = 2$
8. $\log_7 343 = 3$
9. $\log_9 \frac{1}{81} = -2$
10. $\log \frac{1}{1000} = -3$
11. $\log_{32} 4 = \frac{2}{5}$
12. $\log_{16} 8 = \frac{3}{4}$
13. $\log_{64} 8 = \frac{1}{2}$
14. $\log_8 4 = \frac{2}{3}$

In Exercises 15–22, write the exponential equation in logarithmic form. For example, the logarithmic form of $2^3 = 8$ is $\log_2 8 = 3$.

15. $5^3 = 125$
16. $13^2 = 169$
17. $81^{1/4} = 3$
18. $9^{3/2} = 27$
19. $6^{-2} = \frac{1}{36}$
20. $4^{-3} = \frac{1}{64}$
21. $24^0 = 1$
22. $10^{-3} = 0.001$

In Exercises 23–28, evaluate the function at the indicated value of x without using a calculator.

Function	Value
23. $f(x) = \log_2 x$	$x = 64$
24. $f(x) = \log_{25} x$	$x = 5$
25. $f(x) = \log_8 x$	$x = 1$
26. $f(x) = \log x$	$x = 10$
27. $g(x) = \log_a x$	$x = a^2$
28. $g(x) = \log_b x$	$x = b^{-3}$

In Exercises 29–32, use a calculator to evaluate $f(x) = \log x$ at the indicated value of x. Round your result to three decimal places.

29. $x = \frac{7}{8}$
30. $x = \frac{1}{500}$
31. $x = 12.5$
32. $x = 96.75$

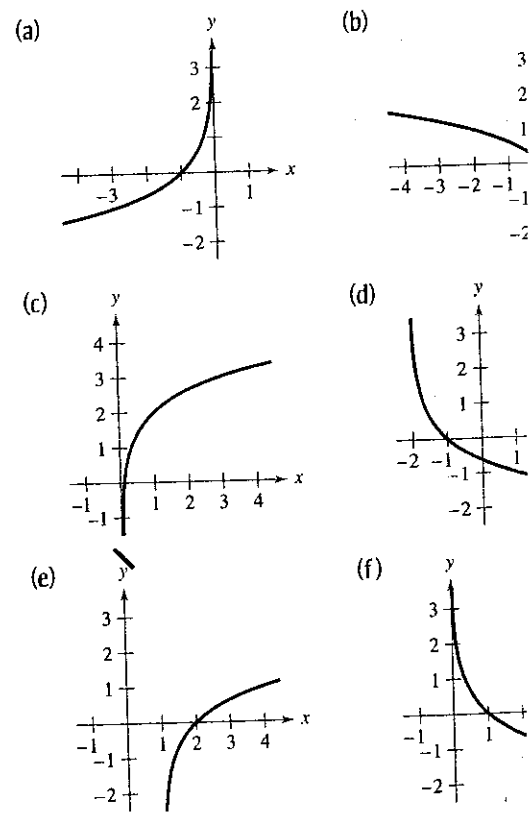
In Exercises 33–36, use the properties of logarithms to simplify the expression.

33. $\log_{11} 11^7$
34. $\log_{3.2} 1$
35. $\log_\pi \pi$
36. $9^{\log_9 15}$

In Exercises 37–44, find the domain, x-intercept, a
asymptote of the logarithmic function and sketch i

37. $f(x) = \log_4 x$
38. $g(x) = \log_6 x$
39. $y = -\log_3 x + 2$
40. $h(x) = \log_4(x$
41. $f(x) = -\log_6(x + 2)$
42. $y = \log_5(x -$
43. $y = \log\left(\frac{x}{7}\right)$
44. $y = \log(-x)$

In Exercises 45–50, use the graph of $g(x) = \log_3$ the given function with its graph. Then describe t ship between the graphs of f and g. [The graphs (a), (b), (c), (d), (e), and (f).]



45. $f(x) = \log_3 x - 2$

46. $f(x) = -\log_3 x$

47. $f(x) = -\log_3(x - 2)$

48. $f(x) = \log_3(x - 1)$

49. $f(x) = \log_3(1 - x)$

50. $f(x) = -\log_3(-x)$

In Exercises 51–58, write the logarithmic equation in exponential form.

51. $\ln \frac{1}{2} = -0.693\ldots$

52. $\ln \frac{2}{5} = -0.916\ldots$

53. $\ln 7 = 1.945\ldots$

54. $\ln 10 = 2.302\ldots$

55. $\ln 250 = 5.521\ldots$

56. $\ln 1084 = 6.988\ldots$

57. $\ln 1 = 0$

58. $\ln e = 1$

In Exercises 59–66, write the exponential equation in logarithmic form.

59. $e^4 = 54.598\ldots$

60. $e^2 = 7.3890\ldots$

61. $e^{1/2} = 1.6487\ldots$

62. $e^{1/3} = 1.3956\ldots$

63. $e^{-0.9} = 0.406\ldots$

64. $e^{-4.1} = 0.0165\ldots$

65. $e^x = 4$

66. $e^{2x} = 3$

In Exercises 67–70, use a calculator to evaluate the function at the indicated value of x. Round your result to three decimal places.

Function	*Value*
67. $f(x) = \ln x$	$x = 18.42$
68. $f(x) = 3 \ln x$	$x = 0.74$
69. $g(x) = 8 \ln x$	$x = 0.05$
70. $g(x) = -\ln x$	$x = \frac{1}{2}$

In Exercises 71–74, evaluate $g(x) = \ln x$ at the indicated value of x without using a calculator.

71. $x = e^5$

72. $x = e^{-4}$

73. $x = e^{-5/6}$

74. $x = e^{-5/2}$

In Exercises 75–78, find the domain, x-intercept, and vertical asymptote of the logarithmic function and sketch its graph.

75. $f(x) = \ln(x - 4)$

76. $h(x) = \ln(x + 5)$

77. $g(x) = \ln(-x)$

78. $f(x) = \ln(3 - x)$

In Exercises 79–84, use a graphing utility to graph the function. Be sure to use an appropriate viewing window.

79. $f(x) = \log(x + 9)$

80. $f(x) = \log(x - 6)$

81. $f(x) = \ln(x - 1)$

82. $f(x) = \ln(x + 2)$

83. $f(x) = \ln x + 8$

84. $f(x) = 3 \ln x - 1$

In Exercises 85–92, use the One-to-One Property to solve the equation for x.

85. $\log_5(x + 1) = \log_5 6$

86. $\log_2(x - 3) = \log_2 9$

87. $\log(2x - 1) = \log 15$

88. $\log(5x - 3) = \log 12$

89. $\ln(x - 4) = \ln 12$

90. $\ln(x - 7) = \ln 7$

91. $\ln(x^2 - 2) = \ln 23$

92. $\ln(x^2 - x) = \ln 6$

93. **MONTHLY PAYMENT** The model

$$t = 16.625 \ln\left(\frac{x}{x - 750}\right), \quad x > 750$$

approximates the length of a home mortgage of \$150,000 at 6% in terms of the monthly payment. In the model, t is the length of the mortgage in years and x is the monthly payment in dollars.

(a) Use the model to approximate the lengths of a \$150,000 mortgage at 6% when the monthly payment is \$897.72 and when the monthly payment is \$1659.24.

(b) Approximate the total amounts paid over the term of the mortgage with a monthly payment of \$897.72 and with a monthly payment of \$1659.24.

(c) Approximate the total interest charges for a monthly payment of \$897.72 and for a monthly payment of \$1659.24.

(d) What is the vertical asymptote for the model? Interpret its meaning in the context of the problem.

94. **COMPOUND INTEREST** A principal P, invested at $5\frac{1}{2}\%$ and compounded continuously, increases to an amount K times the original principal after t years, where t is given by $t = (\ln K)/0.055$.

(a) Complete the table and interpret your results.

K	1	2	4	6	8	10	12
t							

(b) Sketch a graph of the function.

95. **CABLE TELEVISION** The numbers of cable television systems C (in thousands) in the United States from 2001 through 2006 can be approximated by the model

$$C = 10.355 - 0.298t \ln t, \quad 1 \le t \le 6$$

where t represents the year, with $t = 1$ corresponding to 2001.

(a) Complete the table.

t	1	2	3	4	5	6
C						

(b) Use a graphing utility to graph the function.

(c) Can the model be used to predict the numbers of cable television systems beyond 2006? Explain.

96. **POPULATION** The time t in years for the world population to double if it is increasing at a continuous rate of r is given by $t = (\ln 2)/r$.

(a) Complete the table and interpret your results.

r	0.005	0.010	0.015	0.020	0.025	0.030
t						

(b) Use a graphing utility to graph the function.

97. **HUMAN MEMORY MODEL** Students in a mathematics class were given an exam and then retested monthly with an equivalent exam. The average scores for the class are given by the human memory model $f(t) = 80 - 17\log(t + 1), 0 \le t \le 12$, where t is the time in months.

(a) Use a graphing utility to graph the model over the specified domain.

(b) What was the average score on the original exam $(t = 0)$?

(c) What was the average score after 4 months?

(d) What was the average score after 10 months?

98. **SOUND INTENSITY** The relationship between the number of decibels β and the intensity of a sound I in watts per square meter is

$$\beta = 10\log\left(\frac{I}{10^{-12}}\right).$$

(a) Determine the number of decibels of a sound with an intensity of 1 watt per square meter.

(b) Determine the number of decibels of a sound with an intensity of 10^{-2} watt per square meter.

(c) The intensity of the sound in part (a) is 100 times as great as that in part (b). Is the number of decibels 100 times as great? Explain.

EXPLORATION

TRUE OR FALSE? In Exercises 99 and 100, determine whether the statement is true or false. Justify your answer.

99. You can determine the graph of $f(x) = \log_6 x$ by graphing $g(x) = 6^x$ and reflecting it about the x-axis.

100. The graph of $f(x) = \log_3 x$ contains the point $(27, 3)$.

In Exercises 101–104, sketch the graphs of f and g and describe the relationship between the graphs of f and g. What is the relationship between the functions f and g?

101. $f(x) = 3^x, \quad g(x) = \log_3 x$

102. $f(x) = 5^x, \quad g(x) = \log_5 x$

103. $f(x) = e^x, \quad g(x) = \ln x$

104. $f(x) = 8^x, \quad g(x) = \log_8 x$

105. **THINK ABOUT IT** Complete the table for $f(x) = 10^x$.

x	-2	-1	0	1	2
$f(x)$					

Complete the table for $f(x) = \log x$.

x	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100
$f(x)$					

Compare the two tables. What is the relationship between $f(x) = 10^x$ and $f(x) = \log x$?

106. **GRAPHICAL ANALYSIS** Use a graphing utility to graph f and g in the same viewing window and determine which is increasing at the greater rate as x approaches $+\infty$. What can you conclude about the rate of growth of the natural logarithmic function?

(a) $f(x) = \ln x, \quad g(x) = \sqrt{x}$

(b) $f(x) = \ln x, \quad g(x) = \sqrt[4]{x}$

107. (a) Complete the table for the function given by $f(x) = (\ln x)/x$.

x	1	5	10	10^2	10^4	10^6
$f(x)$						

(b) Use the table in part (a) to determine what value $f(x)$ approaches as x increases without bound.

(c) Use a graphing utility to confirm the result of part (b).

108. **CAPSTONE** The table of values was obtained by evaluating a function. Determine which of the statements may be true and which must be false.

x	y
1	0
2	1
8	3

(a) y is an exponential function of x.

(b) y is a logarithmic function of x.

(c) x is an exponential function of y.

(d) y is a linear function of x.

109. **WRITING** Explain why $\log_a x$ is defined only for $0 < a < 1$ and $a > 1$.

In Exercises 110 and 111, (a) use a graphing utility to graph the function, (b) use the graph to determine the intervals in which the function is increasing and decreasing, and (c) approximate any relative maximum or minimum values of the function.

110. $f(x) = |\ln x|$

111. $h(x) = \ln(x^2 + 1)$

5.3 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY

In Exercises 1–3, fill in the blanks.

1. To evaluate a logarithm to any base, you can use the ________ formula.
2. The change-of-base formula for base e is given by $\log_a x =$ ________.
3. You can consider $\log_a x$ to be a constant multiple of $\log_b x$; the constant multiplier is ________.

In Exercises 4–6, match the property of logarithms with its name.

4. $\log_a(uv) = \log_a u + \log_a v$ (a) Power Property
5. $\ln u^n = n \ln u$ (b) Quotient Property
6. $\log_a \frac{u}{v} = \log_a u - \log_a v$ (c) Product Property

SKILLS AND APPLICATIONS

In Exercises 7–14, rewrite the logarithm as a ratio of (a) common logarithms and (b) natural logarithms.

7. $\log_5 16$
8. $\log_3 47$
9. $\log_{1/5} x$
10. $\log_{1/3} x$
11. $\log_x \frac{3}{10}$
12. $\log_x \frac{3}{4}$
13. $\log_{2.6} x$
14. $\log_{7.1} x$

In Exercises 15–22, evaluate the logarithm using the change-of-base formula. Round your result to three decimal places.

15. $\log_3 7$
16. $\log_7 4$
17. $\log_{1/2} 4$
18. $\log_{1/4} 5$
19. $\log_9 0.1$
20. $\log_{20} 0.25$
21. $\log_{15} 1250$
22. $\log_3 0.015$

In Exercises 23–28, use the properties of logarithms to rewrite and simplify the logarithmic expression.

23. $\log_4 8$
24. $\log_2(4^2 \cdot 3^4)$
25. $\log_5 \frac{1}{250}$
26. $\log \frac{9}{300}$
27. $\ln(5e^6)$
28. $\ln \frac{6}{e^2}$

In Exercises 29–44, find the exact value of the logarithmic expression without using a calculator. (If this is not possible, state the reason.)

29. $\log_3 9$
30. $\log_5 \frac{1}{125}$
31. $\log_2 \sqrt[4]{8}$
32. $\log_6 \sqrt[3]{6}$
33. $\log_4 16^2$
34. $\log_3 81^{-3}$
35. $\log_2(-2)$
36. $\log_3(-27)$
37. $\ln e^{4.5}$
38. $3 \ln e^4$
39. $\ln \frac{1}{\sqrt{e}}$
40. $\ln \sqrt[4]{e^3}$
41. $\ln e^2 + \ln e^5$
42. $2 \ln e^6 - \ln e^5$
43. $\log_5 75 - \log_5 3$
44. $\log_4 2 + \log_4 32$

In Exercises 45–66, use the properties of logarithms to expand the expression as a sum, difference, and/or constant multiple of logarithms. (Assume all variables are positive.)

45. $\ln 4x$
46. $\log_3 10z$
47. $\log_8 x^4$
48. $\log_{10} \frac{y}{2}$
49. $\log_5 \frac{5}{x}$
50. $\log_6 \frac{1}{z^3}$
51. $\ln \sqrt{z}$
52. $\ln \sqrt[3]{t}$
53. $\ln xyz^2$
54. $\log 4x^2y$
55. $\ln z(z-1)^2, \quad z > 1$
56. $\ln\left(\frac{x^2-1}{x^3}\right), \quad x > 1$
57. $\log_2 \frac{\sqrt{a-1}}{9}, \quad a > 1$
58. $\ln \frac{6}{\sqrt{x^2+1}}$
59. $\ln \sqrt[3]{\frac{x}{y}}$
60. $\ln \sqrt{\frac{x^2}{y^3}}$
61. $\ln x^2 \sqrt{\frac{y}{z}}$
62. $\log_2 x^4 \sqrt{\frac{y}{z^3}}$
63. $\log_5 \frac{x^2}{y^2 z^3}$
64. $\log_{10} \frac{xy^4}{z^5}$
65. $\ln \sqrt[4]{x^3(x^2+3)}$
66. $\ln \sqrt{x^2(x+2)}$

In Exercises 67–84, condense the expression to the logarithm of a single quantity.

67. $\ln 2 + \ln x$

68. $\ln y + \ln t$

69. $\log_4 z - \log_4 y$

70. $\log_5 8 - \log_5 t$

71. $2 \log_2 x + 4 \log_2 y$

72. $\frac{2}{3} \log_7(z - 2)$

73. $\frac{1}{4} \log_3 5x$

74. $-4 \log_6 2x$

75. $\log x - 2 \log(x + 1)$

76. $2 \ln 8 + 5 \ln(z - 4)$

77. $\log x - 2 \log y + 3 \log z$

78. $3 \log_3 x + 4 \log_3 y - 4 \log_3 z$

79. $\ln x - [\ln(x + 1) + \ln(x - 1)]$

80. $4[\ln z + \ln(z + 5)] - 2 \ln(z - 5)$

81. $\frac{1}{3}[2 \ln(x + 3) + \ln x - \ln(x^2 - 1)]$

82. $2[3 \ln x - \ln(x + 1) - \ln(x - 1)]$

83. $\frac{1}{3}[\log_8 y + 2 \log_8(y + 4)] - \log_8(y - 1)$

84. $\frac{1}{2}[\log_4(x + 1) + 2 \log_4(x - 1)] + 6 \log_4 x$

In Exercises 85 and 86, compare the logarithmic quantities. If two are equal, explain why.

85. $\dfrac{\log_2 32}{\log_2 4}$, $\log_2 \dfrac{32}{4}$, $\log_2 32 - \log_2 4$

86. $\log_7 \sqrt{70}$, $\log_7 35$, $\frac{1}{2} + \log_7 \sqrt{10}$

SOUND INTENSITY In Exercises 87–90, use the following information. The relationship between the number of decibels β and the intensity of a sound I in watts per square meter is given by

$$\beta = 10 \log\left(\frac{I}{10^{-12}}\right).$$

87. Use the properties of logarithms to write the formula in simpler form, and determine the number of decibels of a sound with an intensity of 10^{-6} watt per square meter.

88. Find the difference in loudness between an average office with an intensity of 1.26×10^{-7} watt per square meter and a broadcast studio with an intensity of 3.16×10^{-10} watt per square meter.

89. Find the difference in loudness between a vacuum cleaner with an intensity of 10^{-4} watt per square meter and rustling leaves with an intensity of 10^{-11} watt per square meter.

90. You and your roommate are playing your stereos at the same time and at the same intensity. How much louder is the music when both stereos are playing compared with just one stereo playing?

CURVE FITTING In Exercises 91–94, find a log equation that relates y and x. Explain the steps usec the equation.

91.

x	1	2	3	4	5
y	1	1.189	1.316	1.414	1.495

92.

x	1	2	3	4	5
y	1	1.587	2.080	2.520	2.924

93.

x	1	2	3	4	5
y	2.5	2.102	1.9	1.768	1.672

94.

x	1	2	3	4	5
y	0.5	2.828	7.794	16	27.951

95. **GALLOPING SPEEDS OF ANIMALS** Fo animals run with two different types of motior and galloping. An animal that is trotting has a foot on the ground at all times, whereas an ani galloping has all four feet off the ground at s in its stride. The number of strides per minute an animal breaks from a trot to a gallop depe weight of the animal. Use the table to find a lo equation that relates an animal's weight x (i and its lowest galloping speed y (in strides pe

Weight, x	Galloping speed
25	191.5
35	182.7
50	173.8
75	164.2
500	125.9
1000	114.2

96. **NAIL LENGTH** The approximate lengths an (in inches) of common nails are shown i Find a logarithmic equation that relates the of a common nail to its length x.

Length, x	Diameter, y
1	0.072
2	0.120
3	0.148

Length, x	D
4	
5	
6	

97. **COMPARING MODELS** A cup of water at an initial temperature of 78°C is placed in a room at a constant temperature of 21°C. The temperature of the water is measured every 5 minutes during a half-hour period. The results are recorded as ordered pairs of the form (t, T), where t is the time (in minutes) and T is the temperature (in degrees Celsius).

(0, 78.0°), (5, 66.0°), (10, 57.5°), (15, 51.2°), (20, 46.3°), (25, 42.4°), (30, 39.6°)

(a) The graph of the model for the data should be asymptotic with the graph of the temperature of the room. Subtract the room temperature from each of the temperatures in the ordered pairs. Use a graphing utility to plot the data points (t, T) and $(t, T - 21)$.

(b) An exponential model for the data $(t, T - 21)$ is given by $T - 21 = 54.4(0.964)^t$. Solve for T and graph the model. Compare the result with the plot of the original data.

(c) Take the natural logarithms of the revised temperatures. Use a graphing utility to plot the points $(t, \ln(T - 21))$ and observe that the points appear to be linear. Use the *regression* feature of the graphing utility to fit a line to these data. This resulting line has the form $\ln(T - 21) = at + b$. Solve for T, and verify that the result is equivalent to the model in part (b).

(d) Fit a rational model to the data. Take the reciprocals of the y-coordinates of the revised data points to generate the points

$$\left(t, \frac{1}{T - 21}\right).$$

Use a graphing utility to graph these points and observe that they appear to be linear. Use the *regression* feature of a graphing utility to fit a line to these data. The resulting line has the form

$$\frac{1}{T - 21} = at + b.$$

Solve for T, and use a graphing utility to graph the rational function and the original data points.

(e) Why did taking the logarithms of the temperatures lead to a linear scatter plot? Why did taking the reciprocals of the temperatures lead to a linear scatter plot?

EXPLORATION

98. **PROOF** Prove that $\log_b \dfrac{u}{v} = \log_b u - \log_b v$.

99. **PROOF** Prove that $\log_b u^n = n \log_b u$.

100. **CAPSTONE** A classmate claims that the following are true.

(a) $\ln(u + v) = \ln u - \ln v = \ln(uv)$

(b) $\ln(u - v) = \ln u - \ln v = \ln \dfrac{u}{v}$

(c) $(\ln u)^n = n(\ln u) = \ln u^n$

Discuss how you would demonstrate that these claims are not true.

TRUE OR FALSE? In Exercises 101–106, determine whether the statement is true or false given that $f(x) = \ln x$. Justify your answer.

101. $f(0) = 0$

102. $f(ax) = f(a) + f(x), \quad a > 0, x > 0$

103. $f(x - 2) = f(x) - f(2), \quad x > 2$

104. $\sqrt{f(x)} = \frac{1}{2} f(x)$

105. If $f(u) = 2f(v)$, then $v = u^2$.

106. If $f(x) < 0$, then $0 < x < 1$.

In Exercises 107–112, use the change-of-base formula to rewrite the logarithm as a ratio of logarithms. Then use a graphing utility to graph the ratio.

107. $f(x) = \log_2 x$

108. $f(x) = \log_4 x$

109. $f(x) = \log_{1/2} x$

110. $f(x) = \log_{1/4} x$

111. $f(x) = \log_{11.8} x$

112. $f(x) = \log_{12.4} x$

113. **THINK ABOUT IT** Consider the functions below.

$$f(x) = \ln \frac{x}{2}, \quad g(x) = \frac{\ln x}{\ln 2}, \quad h(x) = \ln x - \ln 2$$

Which two functions should have identical graphs? Verify your answer by sketching the graphs of all three functions on the same set of coordinate axes.

114. **GRAPHICAL ANALYSIS** Use a graphing utility to graph the functions given by $y_1 = \ln x - \ln(x - 3)$ and $y_2 = \ln \dfrac{x}{x - 3}$ in the same viewing window. Does the graphing utility show the functions with the same domain? If so, should it? Explain your reasoning.

115. **THINK ABOUT IT** For how many integers between 1 and 20 can the natural logarithms be approximated given the values $\ln 2 \approx 0.6931$, $\ln 3 \approx 1.0986$, and $\ln 5 \approx 1.6094$? Approximate these logarithms (do not use a calculator).

5.4 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. To ________ an equation in x means to find all values of x for which the equation is true.
2. To solve exponential and logarithmic equations, you can use the following One-to-One and Inverse Properties.
   (a) $a^x = a^y$ if and only if ________. (b) $\log_a x = \log_a y$ if and only if ________.
   (c) $a^{\log_a x} =$ ________ (d) $\log_a a^x =$ ________
3. To solve exponential and logarithmic equations, you can use the following strategies.
   (a) Rewrite the original equation in a form that allows the use of the ________ Properties of exponential or logarithmic functions.
   (b) Rewrite an exponential equation in ________ form and apply the Inverse Property of ________ functions.
   (c) Rewrite a logarithmic equation in ________ form and apply the Inverse Property of ________ functions.
4. An ________ solution does not satisfy the original equation.

SKILLS AND APPLICATIONS

In Exercises 5–12, determine whether each x-value is a solution (or an approximate solution) of the equation.

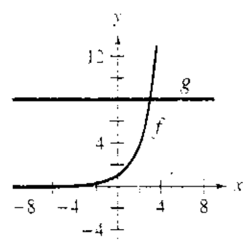
5. $4^{2x-7} = 64$
   (a) $x = 5$
   (b) $x = 2$
6. $2^{3x-1} = 32$
   (a) $x = -1$
   (b) $x = 2$
7. $3e^{x+2} = 75$
   (a) $x = -2 + e^{25}$
   (b) $x = -2 + \ln 25$
   (c) $x \approx 1.219$
8. $4e^{x-1} = 60$
   (a) $x = 1 + \ln 15$
   (b) $x \approx 3.7081$
   (c) $x = \ln 16$
9. $\log_4(3x) = 3$
   (a) $x \approx 21.333$
   (b) $x = -4$
   (c) $x = \frac{64}{3}$
10. $\log_2(x - 3) = 10$
   (a) $x = 1021$
   (b) $x = 17$
   (c) $x = 10^2 - 3$
11. $\ln(2x + 3) = 5.8$
   (a) $x = \frac{1}{2}(-3 + \ln 5.8)$
   (b) $x = \frac{1}{2}(-3 + e^{5.8})$
   (c) $x \approx 163.650$
12. $\ln(x - 1) = 3.8$
   (a) $x = 1 + e^{3.8}$
   (b) $x \approx 45.701$
   (c) $x = 1 + \ln 3.8$

In Exercises 13–24, solve for x.

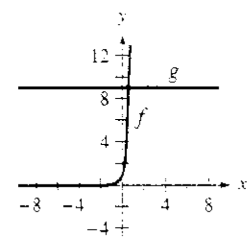
13. $4^x = 16$
14. $3^x = 243$
15. $(\frac{1}{2})^x = 32$
16. $(\frac{1}{4})^x = 64$
17. $\ln x - \ln 2 = 0$
18. $\ln x - \ln 5 = 0$
19. $e^x = 2$
20. $e^x = 4$
21. $\ln x = -1$
22. $\log x = -2$
23. $\log_4 x = 3$
24. $\log_5 x = \frac{1}{2}$

In Exercises 25–28, approximate the point of intersection of the graphs of f and g. Then solve the equation $f(x) = g(x)$ algebraically to verify your approximation.

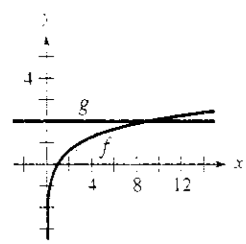
25. $f(x) = 2^x$
    $g(x) = 8$



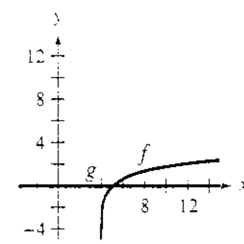
26. $f(x) = 27^x$
    $g(x) = 9$



27. $f(x) = \log_3 x$
    $g(x) = 2$



28. $f(x) = \ln(x - 4)$
    $g(x) = 0$



In Exercises 29–70, solve the exponential equation algebraically. Approximate the result to three decimal places.

29. $e^x = e^{x^2-2}$
30. $e^{2x} = e^{x^2-8}$
31. $e^{x^2-3} = e^{x-2}$
32. $e^{-x^2} = e^{x^2-2x}$
33. $4(3^x) = 20$
34. $2(5^x) = 32$
35. $2e^x = 10$
36. $4e^x = 91$
37. $e^x - 9 = 19$
38. $6^x + 10 = 47$
39. $3^{2x} = 80$
40. $6^{5x} = 3000$
41. $5^{-t/2} = 0.20$
42. $4^{-3t} = 0.10$
43. $3^{x-1} = 27$
44. $2^{x-3} = 32$
45. $2^{3-x} = 565$
46. $8^{-2-x} = 431$

47. $8(10^{3x}) = 12$
48. $5(10^{x-6}) = 7$
49. $3(5^{x-1}) = 21$
50. $8(3^{6-x}) = 40$
51. $e^{3x} = 12$
52. $e^{2x} = 50$
53. $500e^{-x} = 300$
54. $1000e^{-4x} = 75$
55. $7 - 2e^x = 5$
56. $-14 + 3e^x = 11$
57. $6(2^{3x-1}) - 7 = 9$
58. $8(4^{6-2x}) + 13 = 41$
59. $e^{2x} - 4e^x - 5 = 0$
60. $e^{2x} - 5e^x + 6 = 0$
61. $e^{2x} - 3e^x - 4 = 0$
62. $e^{2x} + 9e^x + 36 = 0$
63. $\dfrac{500}{100 - e^{x/2}} = 20$
64. $\dfrac{400}{1 + e^{-x}} = 350$
65. $\dfrac{3000}{2 + e^{2x}} = 2$
66. $\dfrac{119}{e^{6x} - 14} = 7$
67. $\left(1 + \dfrac{0.065}{365}\right)^{365t} = 4$
68. $\left(4 - \dfrac{2.471}{40}\right)^{9t} = 21$
69. $\left(1 + \dfrac{0.10}{12}\right)^{12t} = 2$
70. $\left(16 - \dfrac{0.878}{26}\right)^{3t} = 30$

In Exercises 71–80, use a graphing utility to graph and solve the equation. Approximate the result to three decimal places. Verify your result algebraically.

71. $7 = 2^x$
72. $5^x = 212$
73. $6e^{1-x} = 25$
74. $-4e^{-x-1} + 15 = 0$
75. $3e^{3x/2} = 962$
76. $8e^{-2x/3} = 11$
77. $e^{0.09t} = 3$
78. $-e^{1.8x} + 7 = 0$
79. $e^{0.125t} - 8 = 0$
80. $e^{2.724x} = 29$

In Exercises 81–112, solve the logarithmic equation algebraically. Approximate the result to three decimal places.

81. $\ln x = -3$
82. $\ln x = 1.6$
83. $\ln x - 7 = 0$
84. $\ln x + 1 = 0$
85. $\ln 2x = 2.4$
86. $2.1 = \ln 6x$
87. $\log x = 6$
88. $\log 3z = 2$
89. $3 \ln 5x = 10$
90. $2 \ln x = 7$
91. $\ln\sqrt{x + 2} = 1$
92. $\ln\sqrt{x - 8} = 5$
93. $7 + 3 \ln x = 5$
94. $2 - 6 \ln x = 10$
95. $-2 + 2 \ln 3x = 17$
96. $2 + 3 \ln x = 12$
97. $6 \log_3(0.5x) = 11$
98. $4 \log(x - 6) = 11$
99. $\ln x - \ln(x + 1) = 2$
100. $\ln x + \ln(x + 1) = 1$
101. $\ln x + \ln(x - 2) = 1$
102. $\ln x + \ln(x + 3) = 1$
103. $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$
104. $\ln(x + 1) - \ln(x - 2) = \ln x$
105. $\log_2(2x - 3) = \log_2(x + 4)$
106. $\log(3x + 4) = \log(x - 10)$
107. $\log(x + 4) - \log x = \log(x + 2)$
108. $\log_2 x + \log_2(x + 2) = \log_2(x + 6)$
109. $\log_4 x - \log_4(x - 1) = \frac{1}{2}$
110. $\log_3 x + \log_3(x - 8) = 2$
111. $\log 8x - \log(1 + \sqrt{x}) = 2$
112. $\log 4x - \log(12 + \sqrt{x}) = 2$

In Exercises 113–116, use a graphing utility to grapł solve the equation. Approximate the result to decimal places. Verify your result algebraically.

113. $3 - \ln x = 0$
114. $10 - 4 \ln(x - 2)$
115. $2 \ln(x + 3) = 3$
116. $\ln(x + 1) = 2 - $

COMPOUND INTEREST In Exercises 117–120, \$25 invested in an account at interest rate r, compou continuously. Find the time required for the amou (a) double and (b) triple.

117. $r = 0.05$
118. $r = 0.045$
119. $r = 0.025$
120. $r = 0.0375$

In Exercises 121–128, solve the equation algebraically. R the result to three decimal places. Verify your answer us graphing utility.

121. $2x^2e^{2x} + 2xe^{2x} = 0$
122. $-x^2e^{-x} + 2xe^{-x} =$
123. $-xe^{-x} + e^{-x} = 0$
124. $e^{-2x} - 2xe^{-2x} = ($
125. $2x \ln x + x = 0$
126. $\dfrac{1 - \ln x}{x^2} = 0$
127. $\dfrac{1 + \ln x}{2} = 0$
128. $2x \ln\left(\dfrac{1}{x}\right) - x = 0$

129. **DEMAND** The demand equation for a lin edition coin set is

$$p = 1000\left(1 - \frac{5}{5 + e^{-0.001x}}\right).$$

Find the demand x for a price of (a) $p = \$139.5($ (b) $p = \$99.99$.

130. **DEMAND** The demand equation for a hand- electronic organizer is

$$p = 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right).$$

Find the demand x for a price of (a) $p = \$600$ (b) $p = \$400$.

131. **FOREST YIELD** The yield V (in millions of cubic feet per acre) for a forest at age t years is given by $V = 6.7e^{-48.1/t}$.

(a) Use a graphing utility to graph the function.

(b) Determine the horizontal asymptote of the function. Interpret its meaning in the context of the problem.

(c) Find the time necessary to obtain a yield of 1.3 million cubic feet.

132. **TREES PER ACRE** The number N of trees of a given species per acre is approximated by the model $N = 68(10^{-0.04x})$, $5 \le x \le 40$, where x is the average diameter of the trees (in inches) 3 feet above the ground. Use the model to approximate the average diameter of the trees in a test plot when $N = 21$.

133. **U.S. CURRENCY** The values y (in billions of dollars) of U.S. currency in circulation in the years 2000 through 2007 can be modeled by $y = -451 + 444 \ln t$, $10 \le t \le 17$, where t represents the year, with $t = 10$ corresponding to 2000. During which year did the value of U.S. currency in circulation exceed \$690 billion? (Source: Board of Governors of the Federal Reserve System)

134. **MEDICINE** The numbers y of freestanding ambulatory care surgery centers in the United States from 2000 through 2007 can be modeled by

$$y = 2875 + \frac{2635.11}{1 + 14.215e^{-0.8038t}}, \quad 0 \le t \le 7$$

where t represents the year, with $t = 0$ corresponding to 2000. (Source: Verispan)

(a) Use a graphing utility to graph the model.

(b) Use the *trace* feature of the graphing utility to estimate the year in which the number of surgery centers exceeded 3600.

135. **AVERAGE HEIGHTS** The percent m of American males between the ages of 18 and 24 who are no more than x inches tall is modeled by

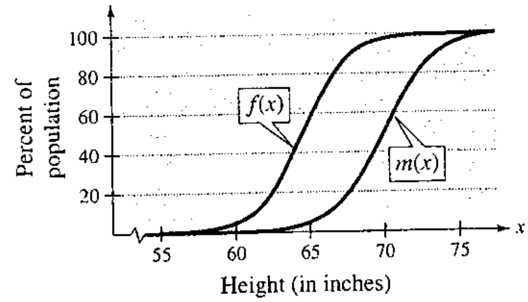
$$m(x) = \frac{100}{1 + e^{-0.6114(x-69.71)}}$$

and the percent f of American females between the ages of 18 and 24 who are no more than x inches tall is modeled by

$$f(x) = \frac{100}{1 + e^{-0.66607(x-64.51)}}.$$

(Source: U.S. National Center for Health Statistics)

(a) Use the graph to determine any horizontal asymptotes of the graphs of the functions. Interpret the meaning in the context of the problem.



(b) What is the average height of each sex?

136. **LEARNING CURVE** In a group project in learning theory, a mathematical model for the proportion P of correct responses after n trials was found to be $P = 0.83/(1 + e^{-0.2n})$.

(a) Use a graphing utility to graph the function.

(b) Use the graph to determine any horizontal asymptotes of the graph of the function. Interpret the meaning of the upper asymptote in the context of this problem.

(c) After how many trials will 60% of the responses be correct?

137. **AUTOMOBILES** Automobiles are designed with crumple zones that help protect their occupants in crashes. The crumple zones allow the occupants to move short distances when the automobiles come to abrupt stops. The greater the distance moved, the fewer g's the crash victims experience. (One g is equal to the acceleration due to gravity. For very short periods of time, humans have withstood as much as 40 g's.) In crash tests with vehicles moving at 90 kilometers per hour, analysts measured the numbers of g's experienced during deceleration by crash dummies that were permitted to move x meters during impact. The data are shown in the table. A model for the data is given by $y = -3.00 + 11.88 \ln x + (36.94/x)$, where y is the number of g's.

x	g's
0.2	158
0.4	80
0.6	53
0.8	40
1.0	32

(a) Complete the table using the model.

x	0.2	0.4	0.6	0.8	1.0
y					

(b) Use a graphing utility to graph the data points and the model in the same viewing window. How do they compare?

(c) Use the model to estimate the distance traveled during impact if the passenger deceleration must not exceed 30 g's.

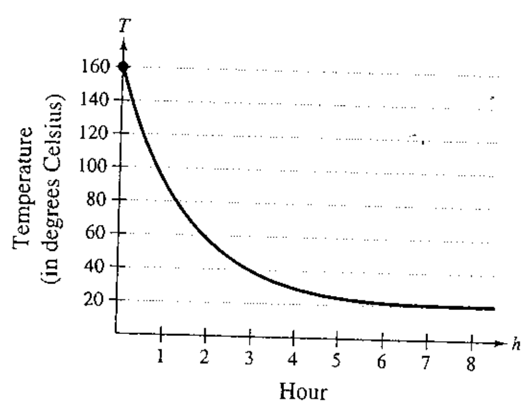
(d) Do you think it is practical to lower the number of g's experienced during impact to fewer than 23? Explain your reasoning.

138. DATA ANALYSIS An object at a temperature of 160°C was removed from a furnace and placed in a room at 20°C. The temperature T of the object was measured each hour h and recorded in the table. A model for the data is given by $T = 20[1 + 7(2^{-h})]$. The graph of this model is shown in the figure.

Hour, h	Temperature, T
0	160°
1	90°
2	56°
3	38°
4	29°
5	24°

(a) Use the graph to identify the horizontal asymptote of the model and interpret the asymptote in the context of the problem.

(b) Use the model to approximate the time when the temperature of the object was 100°C.



EXPLORATION

TRUE OR FALSE? In Exercises 139–142, rewrite each verbal statement as an equation. Then decide whether the statement is true or false. Justify your answer.

139. The logarithm of the product of two numbers is equal to the sum of the logarithms of the numbers.

140. The logarithm of the sum of two nur the product of the logarithms of the n

141. The logarithm of the difference of equal to the difference of the log numbers.

142. The logarithm of the quotient of two n to the difference of the logarithms of t

143. THINK ABOUT IT Is it possible fo equation to have more than one extra Explain.

144. FINANCE You are investing P dolla interest rate of r, compounded cont years. Which of the following would resu value of the investment? Explain your

(a) Double the amount you invest.

(b) Double your interest rate.

(c) Double the number of years.

145. THINK ABOUT IT Are the times re investments in Exercises 117–120 to q as long as the times for them to double' for your answer and verify your answe

146. The *effective yield* of a savings plan increase in the balance after 1 year. Fi yield for each savings plan when $1000 a savings account. Which savings plan l effective yield? Which savings plan highest balance after 5 years?

(a) 7% annual interest rate, compounde

(b) 7% annual interest rate, compounde

(c) 7% annual interest rate, compounde

(d) 7.25% annual interest rate, compou

147. GRAPHICAL ANALYSIS Let $f(x)$ $g(x) = a^x$, where $a > 1$.

(a) Let $a = 1.2$ and use a graphing utilit two functions in the same viewing v do you observe? Approximate ar intersection of the two graphs.

(b) Determine the value(s) of a for w graphs have one point of intersectior

(c) Determine the value(s) of a for w graphs have two points of intersectio

148. CAPSTONE Write two or three senter the general guidelines that you follow w (a) exponential equations and (b) l equations.

5.5 EXERCISES

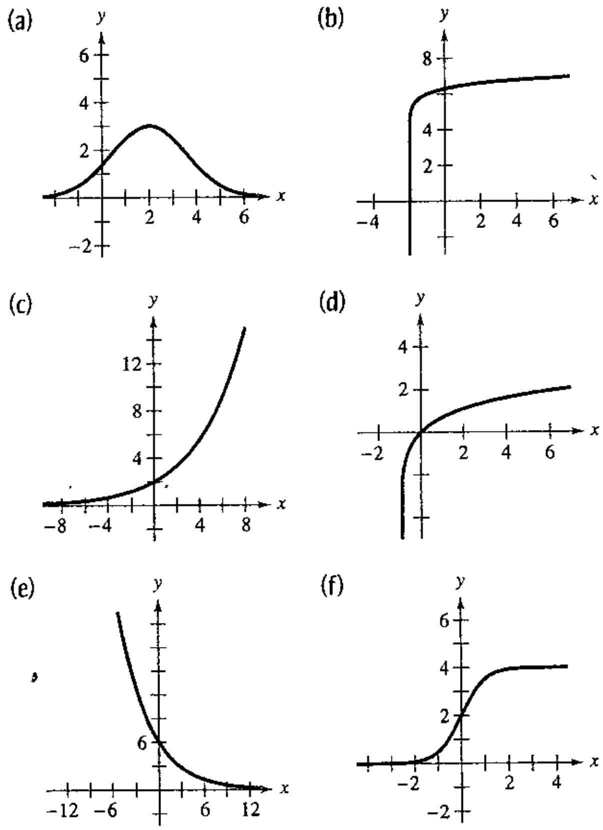
See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. An exponential growth model has the form ________ and an exponential decay model has the form ________.
2. A logarithmic model has the form ________ or ________.
3. Gaussian models are commonly used in probability and statistics to represent populations that are ________ ________.
4. The graph of a Gaussian model is ________ shaped, where the ________ ________ is the maximum y-value of the graph.
5. A logistic growth model has the form ________.
6. A logistic curve is also called a ________ curve.

SKILLS AND APPLICATIONS

In Exercises 7–12, match the function with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]



7. $y = 2e^{x/4}$
8. $y = 6e^{-x/4}$
9. $y = 6 + \log(x + 2)$
10. $y = 3e^{-(x-2)^2/5}$
11. $y = \ln(x + 1)$
12. $y = \dfrac{4}{1 + e^{-2x}}$

In Exercises 13 and 14, (a) solve for P and (b) solve for t.

13. $A = Pe^{rt}$
14. $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$

COMPOUND INTEREST In Exercises 15–22, complete the table for a savings account in which interest is compounded continuously.

	Initial Investment	*Annual % Rate*	*Time to Double*	*Amount After 10 Years*
15.	\$1000	3.5%		
16.	\$750	$10\frac{1}{2}$%		
17.	\$750		$7\frac{3}{4}$ yr	
18.	\$10,000		12 yr	
19.	\$500			\$1505.00
20.	\$600			\$19,205.00
21.		4.5%		\$10,000.00
22.		2%		\$2000.00

COMPOUND INTEREST In Exercises 23 and 24, determine the principal P that must be invested at rate r, compounded monthly, so that \$500,000 will be available for retirement in t years.

23. $r = 5\%, t = 10$
24. $r = 3\frac{1}{2}\%, t = 15$

COMPOUND INTEREST In Exercises 25 and 26, determine the time necessary for \$1000 to double if it is invested at interest rate r compounded (a) annually, (b) monthly, (c) daily, and (d) continuously.

25. $r = 10\%$
26. $r = 6.5\%$

27. **COMPOUND INTEREST** Complete the table for the time t (in years) necessary for P dollars to triple if interest is compounded continuously at rate r.

r	2%	4%	6%	8%	10%	12%
t						

28. **MODELING DATA** Draw a scatter plot of the data in Exercise 27. Use the *regression* feature of a graphing utility to find a model for the data.

29. **COMPOUND INTEREST** Complete the table for the time t (in years) necessary for P dollars to triple if interest is compounded annually at rate r.

r	2%	4%	6%	8%	10%	12%
t						

30. **MODELING DATA** Draw a scatter plot of the data in Exercise 29. Use the *regression* feature of a graphing utility to find a model for the data.

31. **COMPARING MODELS** If \$1 is invested in an account over a 10-year period, the amount in the account, where t represents the time in years, is given by $A = 1 + 0.075[\![t]\!]$ or $A = e^{0.07t}$ depending on whether the account pays simple interest at $7\frac{1}{2}\%$ or continuous compound interest at 7%. Graph each function on the same set of axes. Which grows at a higher rate? (Remember that $[\![t]\!]$ is the greatest integer function discussed in Section 2.4.)

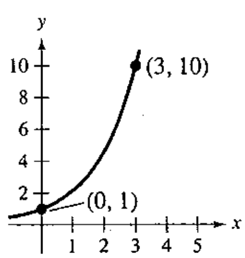
32. **COMPARING MODELS** If \$1 is invested in an account over a 10-year period, the amount in the account, where t represents the time in years, is given by $A = 1 + 0.06[\![t]\!]$ or $A = [1 + (0.055/365)]^{[\![365t]\!]}$ depending on whether the account pays simple interest at 6% or compound interest at $5\frac{1}{2}\%$ compounded daily. Use a graphing utility to graph each function in the same viewing window. Which grows at a higher rate?

RADIOACTIVE DECAY In Exercises 33–38, complete the table for the radioactive isotope.

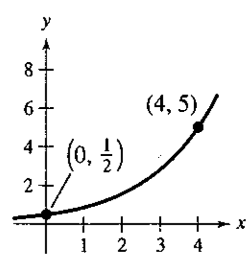
	Isotope	*Half-life (years)*	*Initial Quantity*	*Amount After 1000 Years*
33.	^{226}Ra	1599	10 g	
34.	^{14}C	5715	6.5 g	
35.	^{239}Pu	24,100	2.1 g	
36.	^{226}Ra	1599		2 g
37.	^{14}C	5715		2 g
38.	^{239}Pu	24,100		0.4 g

In Exercises 39–42, find the exponential model $y = ae^{bx}$ that fits the points shown in the graph or table.

39.



40.



41.

x	0	4
y	5	1

42.

x	0	3
y	1	$\frac{1}{4}$

43. **POPULATION** The populations P (in thousands) of Horry County, South Carolina from 1970 through 2007 can be modeled by

$$P = -18.5 + 92.2e^{0.0282t}$$

where t represents the year, with $t = 0$ corresponding to 1970. (Source: U.S. Census Bureau)

(a) Use the model to complete the table.

Year	1970	1980	1990	2000	2007
Population					

(b) According to the model, when will the population of Horry County reach 300,000?

(c) Do you think the model is valid for long-term predictions of the population? Explain.

44. **POPULATION** The table shows the populations (in millions) of five countries in 2000 and the projected populations (in millions) for the year 2015. (Source: U.S. Census Bureau)

Country	2000	2015
Bulgaria	7.8	6.9
Canada	31.1	35.1
China	1268.9	1393.4
United Kingdom	59.5	62.2
United States	282.2	325.5

(a) Find the exponential growth or decay model $y = ae^{bt}$ or $y = ae^{-bt}$ for the population of each country by letting $t = 0$ correspond to 2000. Use the model to predict the population of each country in 2030.

(b) You can see that the populations of the United States and the United Kingdom are growing at different rates. What constant in the equation $y = ae^{bt}$ is determined by these different growth rates? Discuss the relationship between the different growth rates and the magnitude of the constant.

(c) You can see that the population of China is increasing while the population of Bulgaria is decreasing. What constant in the equation $y = ae^{bt}$ reflects this difference? Explain.

45. **WEBSITE GROWTH** The number y of hits a new search-engine website receives each month can be modeled by $y = 4080e^{kt}$, where t represents the number of months the website has been operating. In the website's third month, there were 10,000 hits. Find the value of k, and use this value to predict the number of hits the website will receive after 24 months.

46. **VALUE OF A PAINTING** The value V (in millions of dollars) of a famous painting can be modeled by $V = 10e^{kt}$, where t represents the year, with $t = 0$ corresponding to 2000. In 2008, the same painting was sold for $65 million. Find the value of k, and use this value to predict the value of the painting in 2014.

47. **POPULATION** The populations P (in thousands) of Reno, Nevada from 2000 through 2007 can be modeled by $P = 346.8e^{kt}$, where t represents the year, with $t = 0$ corresponding to 2000. In 2005, the population of Reno was about 395,000. (Source: U.S. Census Bureau)
    (a) Find the value of k. Is the population increasing or decreasing? Explain.
    (b) Use the model to find the populations of Reno in 2010 and 2015. Are the results reasonable? Explain.
    (c) According to the model, during what year will the population reach 500,000?

48. **POPULATION** The populations P (in thousands) of Orlando, Florida from 2000 through 2007 can be modeled by $P = 1656.2e^{kt}$, where t represents the year, with $t = 0$ corresponding to 2000. In 2005, the population of Orlando was about 1,940,000. (Source: U.S. Census Bureau)
    (a) Find the value of k. Is the population increasing or decreasing? Explain.
    (b) Use the model to find the populations of Orlando in 2010 and 2015. Are the results reasonable? Explain.
    (c) According to the model, during what year will the population reach 2.2 million?

49. **BACTERIA GROWTH** The number of bacteria in a culture is increasing according to the law of exponential growth. After 3 hours, there are 100 bacteria, and after 5 hours, there are 400 bacteria. How many bacteria will there be after 6 hours?

50. **BACTERIA GROWTH** The number of bacteria in a culture is increasing according to the law of exponential growth. The initial population is 250 bacteria, and the population after 10 hours is double the population after 1 hour. How many bacteria will there be after 6 hours?

51. **CARBON DATING**
    (a) The ratio of carbon 14 to carbon 12 in a piece of wood discovered in a cave is $R = 1/8^{14}$. Estimate the age of the piece of wood.
    (b) The ratio of carbon 14 to carbon 12 in a piece of paper buried in a tomb is $R = 1/13^{11}$. Estimate the age of the piece of paper.

52. **RADIOACTIVE DECAY** Carbon 14 dating assumes that the carbon dioxide on Earth today has the same radioactive content as it did centuries ago. If this is true, the amount of ^{14}C absorbed by a tree that grew several centuries ago should be the same as the amount of ^{14}C absorbed by a tree growing today. A piece of ancient charcoal contains only 15% as much radioactive carbon as a piece of modern charcoal. How long ago was the tree burned to make the ancient charcoal if the half-life of ^{14}C is 5715 years?

53. **DEPRECIATION** A sport utility vehicle that costs $23,300 new has a book value of $12,500 after 2 years.
    (a) Find the linear model $V = mt + b$.
    (b) Find the exponential model $V = ae^{kt}$.
    (c) Use a graphing utility to graph the two models in the same viewing window. Which model depreciates faster in the first 2 years?
    (d) Find the book values of the vehicle after 1 year and after 3 years using each model.
    (e) Explain the advantages and disadvantages of using each model to a buyer and a seller.

54. **DEPRECIATION** A laptop computer that costs $1150 new has a book value of $550 after 2 years.
    (a) Find the linear model $V = mt + b$.
    (b) Find the exponential model $V = ae^{kt}$.
    (c) Use a graphing utility to graph the two models in the same viewing window. Which model depreciates faster in the first 2 years?
    (d) Find the book values of the computer after 1 year and after 3 years using each model.
    (e) Explain the advantages and disadvantages of using each model to a buyer and a seller.

55. **SALES** The sales S (in thousands of units) of a new CD burner after it has been on the market for t years are modeled by $S(t) = 100(1 - e^{kt})$. Fifteen thousand units of the new product were sold the first year.
    (a) Complete the model by solving for k.
    (b) Sketch the graph of the model.
    (c) Use the model to estimate the number of units sold after 5 years.

56. **LEARNING CURVE** The management at a plastics factory has found that the maximum number of units a worker can produce in a day is 30. The learning curve for the number N of units produced per day after a new employee has worked t days is modeled by $N = 30(1 - e^{kt})$. After 20 days on the job, a new employee produces 19 units.

(a) Find the learning curve for this employee (first, find the value of k).

(b) How many days should pass before this employee is producing 25 units per day?

57. **IQ SCORES** The IQ scores for a sample of a class of returning adult students at a small northeastern college roughly follow the normal distribution $y = 0.0266e^{-(x-100)^2/450}$, $70 \le x \le 115$, where x is the IQ score.

(a) Use a graphing utility to graph the function.

(b) From the graph in part (a), estimate the average IQ score of an adult student.

58. **EDUCATION** The amount of time (in hours per week) a student utilizes a math-tutoring center roughly follows the normal distribution $y = 0.7979e^{-(x-5.4)^2/0.5}$, $4 \le x \le 7$, where x is the number of hours.

(a) Use a graphing utility to graph the function.

(b) From the graph in part (a), estimate the average number of hours per week a student uses the tutoring center.

59. **CELL SITES** A cell site is a site where electronic communications equipment is placed in a cellular network for the use of mobile phones. The numbers y of cell sites from 1985 through 2008 can be modeled by

$$y = \frac{237{,}101}{1 + 1950e^{-0.355t}}$$

where t represents the year, with $t = 5$ corresponding to 1985. (Source: CTIA-The Wireless Association)

(a) Use the model to find the numbers of cell sites in the years 1985, 2000, and 2006.

(b) Use a graphing utility to graph the function.

(c) Use the graph to determine the year in which the number of cell sites will reach 235,000.

(d) Confirm your answer to part (c) algebraically.

60. **POPULATION** The populations P (in thousands) of Pittsburgh, Pennsylvania from 2000 through 2007 can be modeled by

$$P = \frac{2632}{1 + 0.083e^{0.0500t}}$$

where t represents the year, with $t = 0$ corresponding to 2000. (Source: U.S. Census Bureau)

(a) Use the model to find the populations of Pittsburgh in the years 2000, 2005, and 2007.

(b) Use a graphing utility to graph the function.

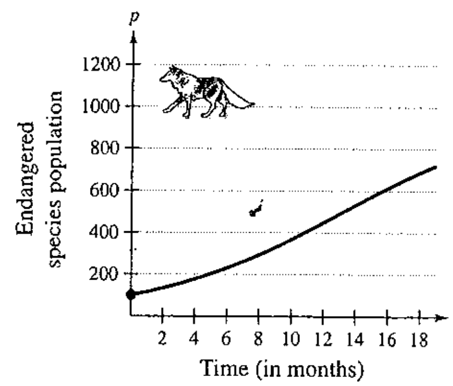
(c) Use the graph to determine the year in which the population will reach 2.2 million.

(d) Confirm your answer to part (c) algebraically.

61. **POPULATION GROWTH** A conservation organization releases 100 animals of an endangered species into a game preserve. The organization believes that the preserve has a carrying capacity of 1000 animals and that the growth of the pack will be modeled by the logistic curve

$$p(t) = \frac{1000}{1 + 9e^{-0.1656t}}$$

where t is measured in months (see figure).



(a) Estimate the population after 5 months.

(b) After how many months will the population be 500?

(c) Use a graphing utility to graph the function. Use the graph to determine the horizontal asymptotes, and interpret the meaning of the asymptotes in the context of the problem.

62. **SALES** After discontinuing all advertising for a tool kit in 2004, the manufacturer noted that sales began to drop according to the model

$$S = \frac{500{,}000}{1 + 0.4e^{kt}}$$

where S represents the number of units sold and $t = 4$ represents 2004. In 2008, the company sold 300,000 units.

(a) Complete the model by solving for k.

(b) Estimate sales in 2012.

GEOLOGY In Exercises 63 and 64, use the Richter scale

$$R = \log \frac{I}{I_0}$$

for measuring the magnitudes of earthquakes.

63. Find the intensity I of an earthquake measuring R on the Richter scale (let $I_0 = 1$).
 (a) Southern Sumatra, Indonesia in 2007, $R = 8.5$
 (b) Illinois in 2008, $R = 5.4$
 (c) Costa Rica in 2009, $R = 6.1$

64. Find the magnitude R of each earthquake of intensity I (let $I_0 = 1$).
 (a) $I = 199{,}500{,}000$ (b) $I = 48{,}275{,}000$
 (c) $I = 17{,}000$

INTENSITY OF SOUND In Exercises 65–68, use the following information for determining sound intensity. The level of sound β, in decibels, with an intensity of I, is given by $\beta = 10 \log(I/I_0)$, where I_0 is an intensity of 10^{-12} watt per square meter, corresponding roughly to the faintest sound that can be heard by the human ear. In Exercises 65 and 66, find the level of sound β.

65. (a) $I = 10^{-10}$ watt per m^2 (quiet room)
 (b) $I = 10^{-5}$ watt per m^2 (busy street corner)
 (c) $I = 10^{-8}$ watt per m^2 (quiet radio)
 (d) $I = 10^{0}$ watt per m^2 (threshold of pain)

66. (a) $I = 10^{-11}$ watt per m^2 (rustle of leaves)
 (b) $I = 10^{2}$ watt per m^2 (jet at 30 meters)
 (c) $I = 10^{-4}$ watt per m^2 (door slamming)
 (d) $I = 10^{-2}$ watt per m^2 (siren at 30 meters)

67. Due to the installation of noise suppression materials, the noise level in an auditorium was reduced from 93 to 80 decibels. Find the percent decrease in the intensity level of the noise as a result of the installation of these materials.

68. Due to the installation of a muffler, the noise level of an engine was reduced from 88 to 72 decibels. Find the percent decrease in the intensity level of the noise as a result of the installation of the muffler.

pH LEVELS In Exercises 69–74, use the acidity model given by $\text{pH} = -\log[H^+]$, where acidity (pH) is a measure of the hydrogen ion concentration $[H^+]$ (measured in moles of hydrogen per liter) of a solution.

69. Find the pH if $[H^+] = 2.3 \times 10^{-5}$.
70. Find the pH if $[H^+] = 1.13 \times 10^{-5}$.
71. Compute $[H^+]$ for a solution in which $\text{pH} = 5.8$.
72. Compute $[H^+]$ for a solution in which $\text{pH} = 3.2$.
73. Apple juice has a pH of 2.9 and drinking water ha of 8.0. The hydrogen ion concentration of the app is how many times the concentration of drinking
74. The pH of a solution is decreased by one un hydrogen ion concentration is increased by what

75. **FORENSICS** At 8:30 A.M., a coroner was calle home of a person who had died during the n order to estimate the time of death, the coroner t person's temperature twice. At 9:00 A. temperature was 85.7°F, and at 11:00 A. temperature was 82.8°F. From these two tempe the coroner was able to determine that the time since death and the body temperature were re the formula

$$t = -10 \ln \frac{T - 70}{98.6 - 70}$$

where t is the time in hours elapsed since th died and T is the temperature (in degrees Fahre the person's body. (This formula is derived general cooling principle called *Newton's Cooling*. It uses the assumptions that the pers normal body temperature of 98.6°F at death, the room temperature was a constant 70°F.) formula to estimate the time of death of the pe

76. **HOME MORTGAGE** A \$120,000 home mor 30 years at $7\frac{1}{2}\%$ has a monthly payment of \$83 of the monthly payment is paid toward th charge on the unpaid balance, and the remain payment is used to reduce the principal. Th that is paid toward the interest is

$$u = M - \left(M - \frac{Pr}{12}\right)\left(1 + \frac{r}{12}\right)^{12t}$$

and the amount that is paid toward the reduc principal is

$$v = \left(M - \frac{Pr}{12}\right)\left(1 + \frac{r}{12}\right)^{12t}.$$

In these formulas, P is the size of the mor the interest rate, M is the monthly payment, time (in years).

 (a) Use a graphing utility to graph each fun same viewing window. (The viewin should show all 30 years of mortgage pa
 (b) In the early years of the mortgage, is the of the monthly payment paid toward the the principal? Approximate the time monthly payment is evenly divide interest and principal reduction.
 (c) Repeat parts (a) and (b) for a repayme 20 years ($M = \$966.71$). What can you

77. **HOME MORTGAGE** The total interest u paid on a home mortgage of P dollars at interest rate r for t years is

$$u = P\left[\frac{rt}{1 - \left(\frac{1}{1 + r/12}\right)^{12t}} - 1\right].$$

Consider a \$120,000 home mortgage at $7\frac{1}{2}\%$.

(a) Use a graphing utility to graph the total interest function.

(b) Approximate the length of the mortgage for which the total interest paid is the same as the size of the mortgage. Is it possible that some people are paying twice as much in interest charges as the size of the mortgage?

78. **DATA ANALYSIS** The table shows the time t (in seconds) required for a car to attain a speed of s miles per hour from a standing start.

Speed, s	Time, t
30	3.4
40	5.0
50	7.0
60	9.3
70	12.0
80	15.8
90	20.0

Two models for these data are as follows.

$t_1 = 40.757 + 0.556s - 15.817 \ln s$

$t_2 = 1.2259 + 0.0023s^2$

(a) Use the *regression* feature of a graphing utility to find a linear model t_3 and an exponential model t_4 for the data.

(b) Use a graphing utility to graph the data and each model in the same viewing window.

(c) Create a table comparing the data with estimates obtained from each model.

(d) Use the results of part (c) to find the sum of the absolute values of the differences between the data and the estimated values given by each model. Based on the four sums, which model do you think best fits the data? Explain.

EXPLORATION

TRUE OR FALSE? In Exercises 79–82, determine whether the statement is true or false. Justify your answer.

79. The domain of a logistic growth function cannot be the set of real numbers.

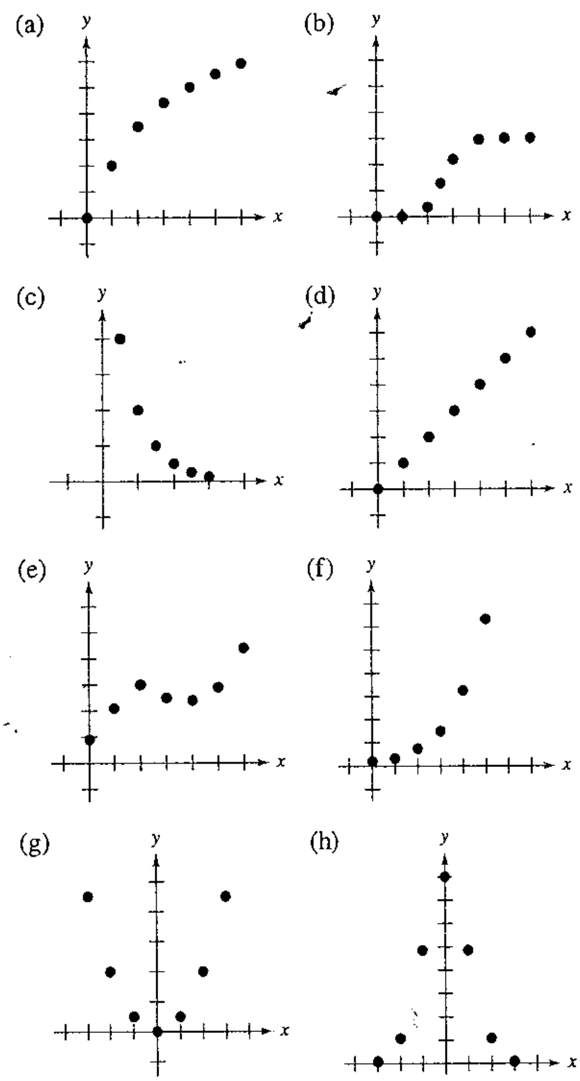
80. A logistic growth function will always have an x-intercept.

81. The graph of $f(x) = \dfrac{4}{1 + 6e^{-2x}} + 5$ is the graph of $g(x) = \dfrac{4}{1 + 6e^{-2x}}$ shifted to the right five units.

82. The graph of a Gaussian model will never have an x-intercept.

83. **WRITING** Use your school's library, the Internet, or some other reference source to write a paper describing John Napier's work with logarithms.

84. **CAPSTONE** Identify each model as exponential, Gaussian, linear, logarithmic, logistic, quadratic, or none of the above. Explain your reasoning.



PROJECT: SALES PER SHARE To work an extended application analyzing the sales per share for Kohl's Corporation from 1992 through 2007, visit this text's website at *academic.cengage.com*. (Data Source: Kohl's Corporation)