3.1 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

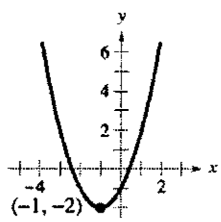
VOCABULARY: Fill in the blanks.

1. Linear, constant, and squaring functions are examples of ________ functions.
2. A polynomial function of degree n and leading coefficient a_n is a function of the form $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$ $(a_n \neq 0)$ where n is a ________ ________ and $a_n, a_{n-1}, \ldots, a_1, a_0$ are ________ numbers.
3. A ________ function is a second-degree polynomial function, and its graph is called a ________.
4. The graph of a quadratic function is symmetric about its ________.
5. If the graph of a quadratic function opens upward, then its leading coefficient is ________ and the vertex of the graph is a ________.
6. If the graph of a quadratic function opens downward, then its leading coefficient is ________ and the vertex of the graph is a ________.

SKILLS AND APPLICATIONS

In Exercises 7–12, match the quadratic function with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

(a)



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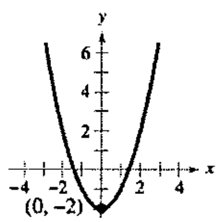

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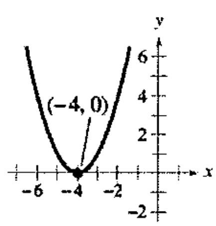

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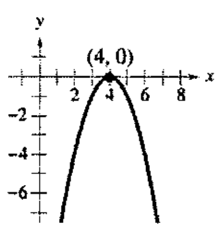

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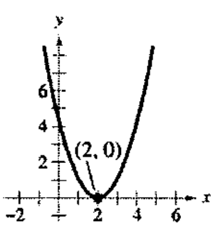

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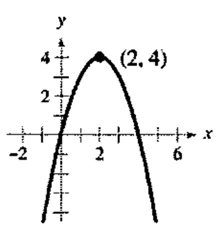


7. $f(x) = (x - 2)^2$
8. $f(x) = (x + 4)^2$
9. $f(x) = x^2 - 2$
10. $f(x) = (x + 1)^2 - 2$
11. $f(x) = 4 - (x - 2)^2$
12. $f(x) = -(x - 4)^2$

In Exercises 13–16, graph each function. Compare the graph of each function with the graph of $y = x^2$.

13. (a) $f(x) = \frac{1}{2}x^2$ (b) $g(x) = -\frac{1}{8}x^2$ (c) $h(x) = \frac{3}{2}x^2$ (d) $k(x) = -3x^2$
14. (a) $f(x) = x^2 + 1$ (b) $g(x) = x^2 - 1$ (c) $h(x) = x^2 + 3$ (d) $k(x) = x^2 - 3$
15. (a) $f(x) = (x - 1)^2$ (b) $g(x) = (3x)^2 + 1$ (c) $h(x) = \left(\frac{1}{3}x\right)^2 - 3$ (d) $k(x) = (x + 3)^2$
16. (a) $f(x) = -\frac{1}{2}(x - 2)^2 + 1$
    (b) $g(x) = \left[\frac{1}{2}(x - 1)\right]^2 - 3$
    (c) $h(x) = -\frac{1}{2}(x + 2)^2 - 1$
    (d) $k(x) = [2(x + 1)]^2 + 4$

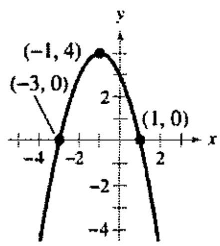
In Exercises 17–34, sketch the graph of the quadratic function without using a graphing utility. Identify the vertex, axis of symmetry, and x-intercept(s).

17. $f(x) = 1 - x^2$
18. $g(x) = x^2 - 8$
19. $f(x) = x^2 + 7$
20. $h(x) = 12 - x^2$
21. $f(x) = \frac{1}{2}x^2 - 4$
22. $f(x) = 16 - \frac{1}{4}x^2$
23. $f(x) = (x + 4)^2 - 3$
24. $f(x) = (x - 6)^2 + 8$
25. $h(x) = x^2 - 8x + 16$
26. $g(x) = x^2 + 2x + 1$
27. $f(x) = x^2 - x + \frac{5}{4}$
28. $f(x) = x^2 + 3x + \frac{1}{4}$
29. $f(x) = -x^2 + 2x + 5$
30. $f(x) = -x^2 - 4x + 1$
31. $h(x) = 4x^2 - 4x + 21$
32. $f(x) = 2x^2 - x + 1$
33. $f(x) = \frac{1}{4}x^2 - 2x - 12$
34. $f(x) = -\frac{1}{3}x^2 + 3x - 6$

In Exercises 35–42, use a graphing utility to graph the quadratic function. Identify the vertex, axis of symmetry, and x-intercepts. Then check your results algebraically by writing the quadratic function in standard form.

35. $f(x) = -(x^2 + 2x - 3)$
36. $f(x) = -(x^2 + x - 30)$
37. $g(x) = x^2 + 8x + 11$
38. $f(x) = x^2 + 10x + 14$
39. $f(x) = 2x^2 - 16x + 31$
40. $f(x) = -4x^2 + 24x - 41$
41. $g(x) = \frac{1}{2}(x^2 + 4x - 2)$
42. $f(x) = \frac{3}{5}(x^2 + 6x - 5)$

In Exercises 43–46, write an equation for the parabola in standard form.

43.


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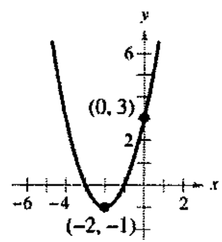

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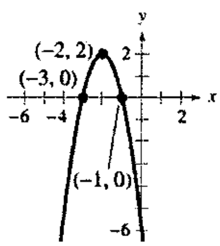

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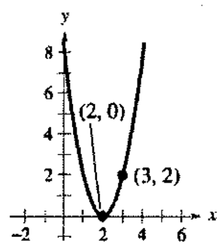


In Exercises 47–56, write the standard form of the equation of the parabola that has the indicated vertex and whose graph passes through the given point.

47. Vertex: $(-2, 5)$; point: $(0, 9)$

48. Vertex: $(4, -1)$; point: $(2, 3)$

49. Vertex: $(1, -2)$; point: $(-1, 14)$

50. Vertex: $(2, 3)$; point: $(0, 2)$

51. Vertex: $(5, 12)$; point: $(7, 15)$

52. Vertex: $(-2, -2)$; point: $(-1, 0)$

53. Vertex: $\left(-\frac{1}{4}, \frac{3}{2}\right)$; point: $(-2, 0)$

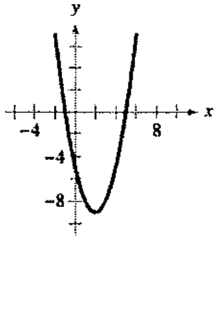
54. Vertex: $\left(\frac{5}{2}, -\frac{3}{4}\right)$; point: $(-2, 4)$

55. Vertex: $\left(-\frac{5}{2}, 0\right)$; point: $\left(-\frac{7}{2}, -\frac{16}{3}\right)$

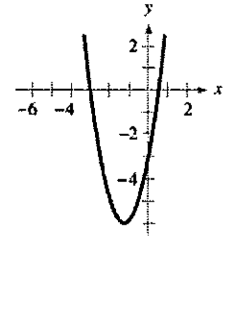
56. Vertex: $(6, 6)$; point: $\left(\frac{61}{10}, \frac{3}{2}\right)$

GRAPHICAL REASONING **In Exercises 57 and 58, determine the x-intercept(s) of the graph visually. Then find the x-intercept(s) algebraically to confirm your results.**

57. $y = x^2 - 4x - 5$



58. $y = 2x^2 + 5x - 3$



In Exercises 59–64, use a graphing utility to graph the quadratic function. Find the x-intercepts of the graph and compare them with the solutions of the corresponding quadratic equation when $f(x) = 0$.

59. $f(x) = x^2 - 4x$

60. $f(x) = -2x^2 + 10x$

61. $f(x) = x^2 - 9x + 18$

62. $f(x) = x^2 - 8x - 20$

63. $f(x) = 2x^2 - 7x - 30$

64. $f(x) = \frac{7}{10}(x^2 + 12x - 45)$

In Exercises 65–70, find two quadratic functions, one that opens upward and one that opens downward, whose graphs have the given x-intercepts. (There are many correct answers.)

65. $(-1, 0), (3, 0)$

66. $(-5, 0), (5, 0)$

67. $(0, 0), (10, 0)$

68. $(4, 0), (8, 0)$

69. $(-3, 0), \left(-\frac{1}{2}, 0\right)$

70. $\left(-\frac{5}{2}, 0\right), (2, 0)$

In Exercises 71–74, find two positive real numbers whose product is a maximum.

71. The sum is 110.

72. The sum is S.

73. The sum of the first and twice the second is 24.

74. The sum of the first and three times the second is 42.

75. PATH OF A DIVER The path of a diver is given by

$$y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$$

where y is the height (in feet) and x is the horizontal distance from the end of the diving board (in feet). What is the maximum height of the diver?

76. HEIGHT OF A BALL The height y (in feet) of a punted football is given by

$$y = -\frac{16}{2025}x^2 + \frac{9}{5}x + 1.5$$

where x is the horizontal distance (in feet) from the point at which the ball is punted.

(a) How high is the ball when it is punted?

(b) What is the maximum height of the punt?

(c) How long is the punt?

77. MINIMUM COST A manufacturer of lighting fixtures has daily production costs of $C = 800 - 10x + 0.25x^2$, where C is the total cost (in dollars) and x is the number of units produced. How many fixtures should be produced each day to yield a minimum cost?

78. MAXIMUM PROFIT The profit P (in hundreds of dollars) that a company makes depends on the amount x (in hundreds of dollars) the company spends on advertising according to the model $P = 230 + 20x - 0.5x^2$. What expenditure for advertising will yield a maximum profit?

79. MAXIMUM REVENUE The total revenue R earned (in thousands of dollars) from manufacturing handheld video games is given by

$R(p) = -25p^2 + 1200p$

where p is the price per unit (in dollars).

(a) Find the revenues when the price per unit is \$20, \$25, and \$30.

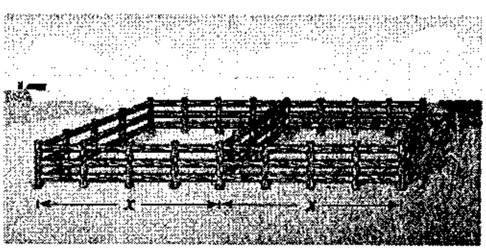
(b) Find the unit price that will yield a maximum revenue. What is the maximum revenue? Explain your results.

80. MAXIMUM REVENUE The total revenue R earned per day (in dollars) from a pet-sitting service is given by $R(p) = -12p^2 + 150p$, where p is the price charged per pet (in dollars).

(a) Find the revenues when the price per pet is \$4, \$6, and \$8.

(b) Find the price that will yield a maximum revenue. What is the maximum revenue? Explain your results.

81. NUMERICAL, GRAPHICAL, AND ANALYTICAL ANALYSIS A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals (see figure).



(a) Write the area A of the corrals as a function of x.

(b) Create a table showing possible values of x and the corresponding areas of the corral. Use the table to estimate the dimensions that will produce the maximum enclosed area.

(c) Use a graphing utility to graph the area function. Use the graph to approximate the dimensions that will produce the maximum enclosed area.

(d) Write the area function in standard form to find analytically the dimensions that will produce the maximum area.

(e) Compare your results from parts (b), (c), and (d).

82. GEOMETRY An indoor physical fitness room consists of a rectangular region with a semicircle on each end. The perimeter of the room is to be a 200-meter single-lane running track.

(a) Draw a diagram that illustrates the problem. Let x and y represent the length and width of the rectangular region, respectively.

(b) Determine the radius of each semicircular end of the room. Determine the distance, in terms of y, around the inside edge of each semicircular part of the track.

(c) Use the result of part (b) to write an equation, in terms of x and y, for the distance traveled in one lap around the track. Solve for y.

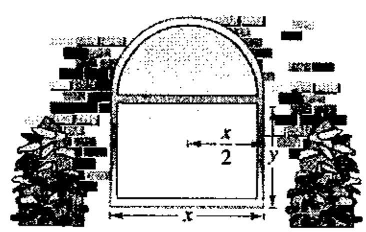
(d) Use the result of part (c) to write the area A of the rectangular region as a function of x. What dimensions will produce a rectangle of maximum area?

83. MAXIMUM REVENUE A small theater has a seating capacity of 2000. When the ticket price is \$20, attendance is 1500. For each \$1 decrease in price, attendance increases by 100.

(a) Write the revenue R of the theater as a function of ticket price x.

(b) What ticket price will yield a maximum revenue? What is the maximum revenue?

84. MAXIMUM AREA A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see figure). The perimeter of the window is 16 feet.



(a) Write the area A of the window as a function of x.

(b) What dimensions will produce a window of maximum area?

85. GRAPHICAL ANALYSIS From 1950 through 2005, the per capita consumption C of cigarettes by Americans (age 18 and older) can be modeled by $C = 3565.0 + 60.30t - 1.783t^2$, $0 \le t \le 55$, where t is the year, with $t = 0$ corresponding to 1950. (Source: *Tobacco Outlook Report*)

(a) Use a graphing utility to graph the model.

(b) Use the graph of the model to approximate the maximum average annual consumption. Beginning in 1966, all cigarette packages were required by law to carry a health warning. Do you think the warning had any effect? Explain.

(c) In 2005, the U.S. population (age 18 and over) was 296,329,000. Of those, about 59,858,458 were smokers. What was the average annual cigarette consumption *per smoker* in 2005? What was the average daily cigarette consumption *per smoker*?

86. DATA ANALYSIS: SALES The sales y (in billions of dollars) for Harley-Davidson from 2000 through 2007 are shown in the table. (Source: U.S. Harley-Davidson, Inc.)

Year	Sales, y
2000	2.91
2001	3.36
2002	4.09
2003	4.62
2004	5.02
2005	5.34
2006	5.80
2007	5.73

(a) Use a graphing utility to create a scatter plot of the data. Let x represent the year, with $x = 0$ corresponding to 2000.

(b) Use the *regression* feature of the graphing utility to find a quadratic model for the data.

(c) Use the graphing utility to graph the model in the same viewing window as the scatter plot. How well does the model fit the data?

(d) Use the *trace* feature of the graphing utility to approximate the year in which the sales for Harley-Davidson were the greatest.

(e) Verify your answer to part (d) algebraically.

(f) Use the model to predict the sales for Harley-Davidson in 2010.

EXPLORATION

TRUE OR FALSE? In Exercises 87–90, determine whether the statement is true or false. Justify your answer.

87. The function given by $f(x) = -12x^2 - 1$ has no x-intercepts.

88. The graphs of $f(x) = -4x^2 - 10x + 7$ and $g(x) = 12x^2 + 30x + 1$ have the same axis of symmetry.

89. The graph of a quadratic function with a negative leading coefficient will have a maximum value at its vertex.

90. The graph of a quadratic function with a positive leading coefficient will have a minimum value at its vertex.

THINK ABOUT IT In Exercises 91–94, find the values of b such that the function has the given maximum or minimum value.

91. $f(x) = -x^2 + bx - 75$; Maximum value: 25

92. $f(x) = -x^2 + bx - 16$; Maximum value: 48

93. $f(x) = x^2 + bx + 26$; Minimum value: 10

94. $f(x) = x^2 + bx - 25$; Minimum value: -50

95. Write the quadratic function

$$f(x) = ax^2 + bx + c$$

in standard form to verify that the vertex occurs at

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

96. CAPSTONE The profit P (in millions of dollars) for a recreational vehicle retailer is modeled by a quadratic function of the form

$$P = at^2 + bt + c$$

where t represents the year. If you were president of the company, which of the models below would you prefer? Explain your reasoning.

(a) a is positive and $-b/(2a) \le t$.

(b) a is positive and $t \le -b/(2a)$.

(c) a is negative and $-b/(2a) \le t$.

(d) a is negative and $t \le -b/(2a)$.

97. GRAPHICAL ANALYSIS

(a) Graph $y = ax^2$ for $a = -2, -1, -0.5, 0.5, 1$ and 2. How does changing the value of a affect the graph?

(b) Graph $y = (x - h)^2$ for $h = -4, -2, 2,$ and 4. How does changing the value of h affect the graph?

(c) Graph $y = x^2 + k$ for $k = -4, -2, 2,$ and 4. How does changing the value of k affect the graph?

98. Describe the sequence of transformation from f to g given that $f(x) = x^2$ and $g(x) = a(x - h)^2 + k$. (Assume a, h, and k are positive.)

99. Is it possible for a quadratic equation to have only one x-intercept? Explain.

100. Assume that the function given by

$$f(x) = ax^2 + bx + c, \quad a \neq 0$$

has two real zeros. Show that the x-coordinate of the vertex of the graph is the average of the zeros of f. (*Hint:* Use the Quadratic Formula.)

PROJECT: HEIGHT OF A BASKETBALL To work an extended application analyzing the height of a basketball after it has been dropped, visit this text's website at *academic.cengage.com*.

3.2 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

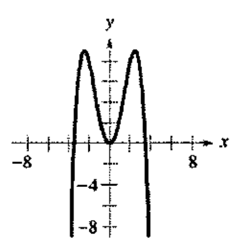
VOCABULARY: Fill in the blanks.

1. The graphs of all polynomial functions are ________, which means that the graphs have no breaks, holes, or gaps.
2. The ________ ________ ________ is used to determine the left-hand and right-hand behavior of the graph of a polynomial function.
3. Polynomial functions of the form $f(x) =$ ________ are often referred to as power functions.
4. A polynomial function of degree n has at most ________ real zeros and at most ________ turning points.
5. If $x = a$ is a zero of a polynomial function f, then the following three statements are true.
   (a) $x = a$ is a ________ of the polynomial equation $f(x) = 0$.
   (b) ________ is a factor of the polynomial $f(x)$.
   (c) $(a, 0)$ is an ________ of the graph of f.
6. If a real zero of a polynomial function is of even multiplicity, then the graph of f ________ the x-axis at $x = a$, and if it is of odd multiplicity, then the graph of f ________ the x-axis at $x = a$.
7. A polynomial function is written in ________ form if its terms are written in descending order of exponents from left to right.
8. The ________ ________ Theorem states that if f is a polynomial function such that $f(a) \neq f(b)$, then, in the interval $[a, b]$, f takes on every value between $f(a)$ and $f(b)$.

SKILLS AND APPLICATIONS

In Exercises 9–16, match the polynomial function with its graph. [The graphs are labeled (a), (b), (c), (d), (e), (f), (g), and (h).]

(a)



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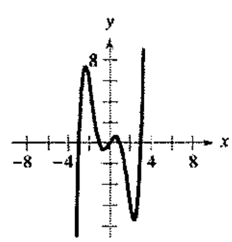

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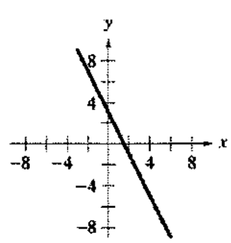

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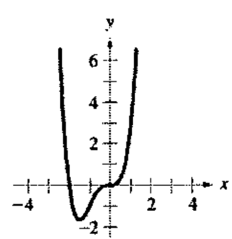

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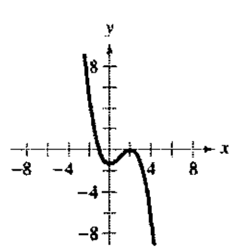

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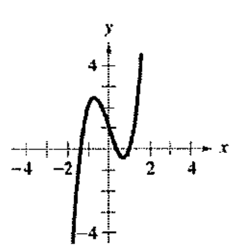

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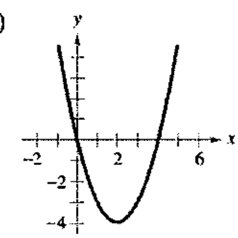

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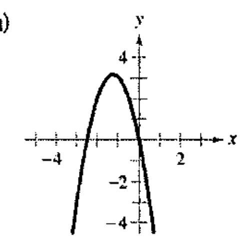


9. $f(x) = -2x + 3$
10. $f(x) = x^2 - 4x$
11. $f(x) = -2x^2 - 5x$
12. $f(x) = 2x^3 - 3x + 1$
13. $f(x) = -\frac{1}{4}x^4 + 3x^2$
14. $f(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$
15. $f(x) = x^4 + 2x^3$
16. $f(x) = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$

In Exercises 17–20, sketch the graph of $y = x^n$ and each transformation.

17. $y = x^3$
   (a) $f(x) = (x - 4)^3$ (b) $f(x) = x^3 - 4$
   (c) $f(x) = -\frac{1}{4}x^3$ (d) $f(x) = (x - 4)^3 - 4$
18. $y = x^5$
   (a) $f(x) = (x + 1)^5$ (b) $f(x) = x^5 + 1$
   (c) $f(x) = 1 - \frac{1}{2}x^5$ (d) $f(x) = -\frac{1}{2}(x + 1)^5$
19. $y = x^4$
   (a) $f(x) = (x + 3)^4$ (b) $f(x) = x^4 - 3$
   (c) $f(x) = 4 - x^4$ (d) $f(x) = \frac{1}{2}(x - 1)^4$
   (e) $f(x) = (2x)^4 + 1$ (f) $f(x) = \left(\frac{1}{2}x\right)^4 - 2$

20. $y = x^6$

(a) $f(x) = -\frac{1}{8}x^6$ (b) $f(x) = (x + 2)^6 - 4$

(c) $f(x) = x^6 - 5$ (d) $f(x) = -\frac{1}{4}x^6 + 1$

(e) $f(x) = \left(\frac{1}{4}x\right)^6 - 2$ (f) $f(x) = (2x)^6 - 1$

In Exercises 21–30, describe the right-hand and left-hand behavior of the graph of the polynomial function.

21. $f(x) = \frac{1}{5}x^3 + 4x$ **22.** $f(x) = 2x^2 - 3x + 1$

23. $g(x) = 5 - \frac{7}{2}x - 3x^2$ **24.** $h(x) = 1 - x^6$

25. $f(x) = -2.1x^5 + 4x^3 - 2$

26. $f(x) = 4x^5 - 7x + 6.5$

27. $f(x) = 6 - 2x + 4x^2 - 5x^3$

28. $f(x) = (3x^4 - 2x + 5)/4$

29. $h(t) = -\frac{3}{4}(t^2 - 3t + 6)$

30. $f(s) = -\frac{7}{8}(s^3 + 5s^2 - 7s + 1)$

GRAPHICAL ANALYSIS **In Exercises 31–34, use a graphing utility to graph the functions f and g in the same viewing window. Zoom out sufficiently far to show that the right-hand and left-hand behaviors of f and g appear identical.**

31. $f(x) = 3x^3 - 9x + 1,\quad g(x) = 3x^3$

32. $f(x) = -\frac{1}{3}(x^3 - 3x + 2),\quad g(x) = -\frac{1}{3}x^3$

33. $f(x) = -(x^4 - 4x^3 + 16x),\quad g(x) = -x^4$

34. $f(x) = 3x^4 - 6x^2,\quad g(x) = 3x^4$

In Exercises 35–50, (a) find all the real zeros of the polynomial function, (b) determine the multiplicity of each zero and the number of turning points of the graph of the function, and (c) use a graphing utility to graph the function and verify your answers.

35. $f(x) = x^2 - 36$ **36.** $f(x) = 81 - x^2$

37. $h(t) = t^2 - 6t + 9$ **38.** $f(x) = x^2 + 10x + 25$

39. $f(x) = \frac{1}{3}x^2 + \frac{1}{3}x - \frac{2}{3}$ **40.** $f(x) = \frac{1}{2}x^2 + \frac{5}{2}x - \frac{3}{2}$

41. $f(x) = 3x^3 - 12x^2 + 3x$ **42.** $g(x) = 5x(x^2 - 2x - 1)$

43. $f(t) = t^3 - 8t^2 + 16t$ **44.** $f(x) = x^4 - x^3 - 30x^2$

45. $g(t) = t^5 - 6t^3 + 9t$ **46.** $f(x) = x^5 + x^3 - 6x$

47. $f(x) = 3x^4 + 9x^2 + 6$ **48.** $f(x) = 2x^4 - 2x^2 - 40$

49. $g(x) = x^3 + 3x^2 - 4x - 12$

50. $f(x) = x^3 - 4x^2 - 25x + 100$

GRAPHICAL ANALYSIS **In Exercises 51–54, (a) use a graphing utility to graph the function, (b) use the graph to approximate any x-intercepts of the graph, (c) set $y = 0$ and solve the resulting equation, and (d) compare the results of part (c) with any x-intercepts of the graph.**

51. $y = 4x^3 - 20x^2 + 25x$

52. $y = 4x^3 + 4x^2 - 8x - 8$

53. $y = x^5 - 5x^3 + 4x$ **54.** $y = \frac{1}{4}x^3(x^2 - 9)$

In Exercises 55–64, find a polynomial function that has the given zeros. (There are many correct answers.)

55. $0, 8$ **56.** $0, -7$

57. $2, -6$ **58.** $-4, 5$

59. $0, -4, -5$ **60.** $0, 1, 10$

61. $4, -3, 3, 0$ **62.** $-2, -1, 0, 1, 2$

63. $1 + \sqrt{3}, 1 - \sqrt{3}$ **64.** $2, 4 + \sqrt{5}, 4 - \sqrt{5}$

In Exercises 65–74, find a polynomial of degree n that has the given zero(s). (There are many correct answers.)

	Zero(s)	*Degree*
65.	$x = -3$	$n = 2$
66.	$x = -12, -6$	$n = 2$
67.	$x = -5, 0, 1$	$n = 3$
68.	$x = -2, 4, 7$	$n = 3$
69.	$x = 0, \sqrt{3}, -\sqrt{3}$	$n = 3$
70.	$x = 9$	$n = 3$
71.	$x = -5, 1, 2$	$n = 4$
72.	$x = -4, -1, 3, 6$	$n = 4$
73.	$x = 0, -4$	$n = 5$
74.	$x = -1, 4, 7, 8$	$n = 5$

In Exercises 75–88, sketch the graph of the function by (a) applying the Leading Coefficient Test, (b) finding the zeros of the polynomial, (c) plotting sufficient solution points, and (d) drawing a continuous curve through the points.

75. $f(x) = x^3 - 25x$ **76.** $g(x) = x^4 - 9x^2$

77. $f(t) = \frac{1}{4}(t^2 - 2t + 15)$

78. $g(x) = -x^2 + 10x - 16$

79. $f(x) = x^3 - 2x^2$ **80.** $f(x) = 8 - x^3$

81. $f(x) = 3x^3 - 15x^2 + 18x$

82. $f(x) = -4x^3 + 4x^2 + 15x$

83. $f(x) = -5x^2 - x^3$ **84.** $f(x) = -48x^2 + 3x^4$

85. $f(x) = x^2(x - 4)$ **86.** $h(x) = \frac{1}{3}x^3(x - 4)^2$

87. $g(t) = -\frac{1}{4}(t - 2)^2(t + 2)^2$

88. $g(x) = \frac{1}{10}(x + 1)^2(x - 3)^3$

In Exercises 89–92, use a graphing utility to graph the function. Use the *zero* or *root* feature to approximate the real zeros of the function. Then determine the multiplicity of each zero.

89. $f(x) = x^3 - 16x$ **90.** $f(x) = \frac{1}{4}x^4 - 2x^2$

91. $g(x) = \frac{1}{5}(x + 1)^2(x - 3)(2x - 9)$

92. $h(x) = \frac{1}{5}(x + 2)^2(3x - 5)^2$

In Exercises 93–96, use the Intermediate Value Theorem and the *table* feature of a graphing utility to find intervals one unit in length in which the polynomial function is guaranteed to have a zero. Adjust the table to approximate the zeros of the function. Use the *zero* or *root* feature of the graphing utility to verify your results.

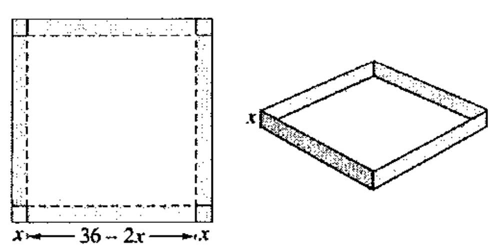
93. $f(x) = x^3 - 3x^2 + 3$

94. $f(x) = 0.11x^3 - 2.07x^2 + 9.81x - 6.88$

95. $g(x) = 3x^4 + 4x^3 - 3$

96. $h(x) = x^4 - 10x^2 + 3$

97. NUMERICAL AND GRAPHICAL ANALYSIS An open box is to be made from a square piece of material, 36 inches on a side, by cutting equal squares with sides of length x from the corners and turning up the sides (see figure).



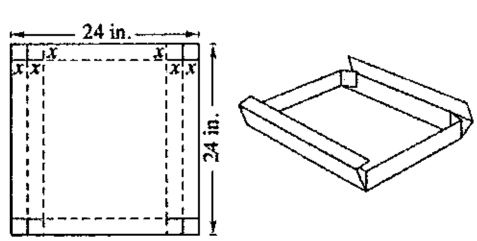
(a) Write a function $V(x)$ that represents the volume of the box.

(b) Determine the domain of the function.

(c) Use a graphing utility to create a table that shows box heights x and the corresponding volumes V. Use the table to estimate the dimensions that will produce a maximum volume.

(d) Use a graphing utility to graph V and use the graph to estimate the value of x for which $V(x)$ is maximum. Compare your result with that of part (c).

98. MAXIMUM VOLUME An open box with locking tabs is to be made from a square piece of material 24 inches on a side. This is to be done by cutting equal squares from the corners and folding along the dashed lines shown in the figure.

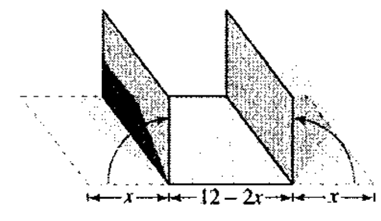


(a) Write a function $V(x)$ that represents the volume of the box.

(b) Determine the domain of the function V.

(c) Sketch a graph of the function and estimate the value of x for which $V(x)$ is maximum.

99. CONSTRUCTION A roofing contractor is fabricating gutters from 12-inch aluminum sheeting. The contractor plans to use an aluminum siding folding press to create the gutter by creasing equal lengths for the sidewalls (see figure).



(a) Let x represent the height of the sidewall of the gutter. Write a function A that represents the cross-sectional area of the gutter.

(b) The length of the aluminum sheeting is 16 feet. Write a function V that represents the volume of one run of gutter in terms of x.

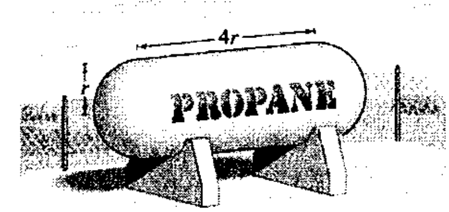
(c) Determine the domain of the function in part (b).

(d) Use a graphing utility to create a table that shows the sidewall heights x and the corresponding volumes V. Use the table to estimate the dimensions that will produce a maximum volume.

(e) Use a graphing utility to graph V. Use the graph to estimate the value of x for which $V(x)$ is a maximum. Compare your result with that of part (d).

(f) Would the value of x change if the aluminum sheeting were of different lengths? Explain.

100. CONSTRUCTION An industrial propane tank is formed by adjoining two hemispheres to the ends of a right circular cylinder. The length of the cylindrical portion of the tank is four times the radius of the hemispherical components (see figure).



(a) Write a function that represents the total volume V of the tank in terms of r.

(b) Find the domain of the function.

(c) Use a graphing utility to graph the function.

(d) The total volume of the tank is to be 120 cubic feet. Use the graph from part (c) to estimate the radius and length of the cylindrical portion of the tank.

101. REVENUE The total revenues R (in millions of dollars) for Krispy Kreme from 2000 through 2007 are shown in the table.

Year	Revenue, R
2000	300.7
2001	394.4
2002	491.5
2003	665.6
2004	707.8
2005	543.4
2006	461.2
2007	429.3

A model that represents these data is given by $R = 3.0711t^4 - 42.803t^3 + 160.59t^2 - 62.6t + 307$, $0 \le t \le 7$, where t represents the year, with $t = 0$ corresponding to 2000. (Source: Krispy Kreme)

(a) Use a graphing utility to create a scatter plot of the data. Then graph the model in the same viewing window.

(b) How well does the model fit the data?

(c) Use a graphing utility to approximate any relative extrema of the model over its domain.

(d) Use a graphing utility to approximate the intervals over which the revenue for Krispy Kreme was increasing and decreasing over its domain.

(e) Use the results of parts (c) and (d) to write a short paragraph about Krispy Kreme's revenue during this time period.

102. REVENUE The total revenues R (in millions of dollars) for Papa John's International from 2000 through 2007 are shown in the table.

Year	Revenue, R
2000	944.7
2001	971.2
2002	946.2
2003	917.4
2004	942.4
2005	968.8
2006	1001.6
2007	1063.6

A model that represents these data is given by $R = -0.5635t^4 + 9.019t^3 - 40.20t^2 + 49.0t + 947$, $0 \le t \le 7$, where t represents the year, with $t = 0$ corresponding to 2000. (Source: Papa John's International)

(a) Use a graphing utility to create a scatter plot of the data. Then graph the model in the same viewing window.

(b) How well does the model fit the data?

(c) Use a graphing utility to approximate any relative extrema of the model over its domain.

(d) Use a graphing utility to approximate the intervals over which the revenue for Papa John's International was increasing and decreasing over its domain.

(e) Use the results of parts (c) and (d) to write a short paragraph about the revenue for Papa John's International during this time period.

103. TREE GROWTH The growth of a red oak tree is approximated by the function

$$G = -0.003t^3 + 0.137t^2 + 0.458t - 0.839$$

where G is the height of the tree (in feet) and t $(2 \le t \le 34)$ is its age (in years).

(a) Use a graphing utility to graph the function. (*Hint:* Use a viewing window in which $-10 \le x \le 45$ and $-5 \le y \le 60$.)

(b) Estimate the age of the tree when it is growing most rapidly. This point is called the *point of diminishing returns* because the increase in size will be less with each additional year.

(c) Using calculus, the point of diminishing returns can also be found by finding the vertex of the parabola given by

$$y = -0.009t^2 + 0.274t + 0.458.$$

Find the vertex of this parabola.

(d) Compare your results from parts (b) and (c).

104. REVENUE The total revenue R (in millions of dollars) for a company is related to its advertising expense by the function

$$R = \frac{1}{100{,}000}(-x^3 + 600x^2), \quad 0 \le x \le 400$$

where x is the amount spent on advertising (in tens of thousands of dollars). Use the graph of this function, shown in the figure on the next page, to estimate the point on the graph at which the function is increasing most rapidly. This point is called the *point of diminishing returns* because any expense above this amount will yield less return per dollar invested in advertising.

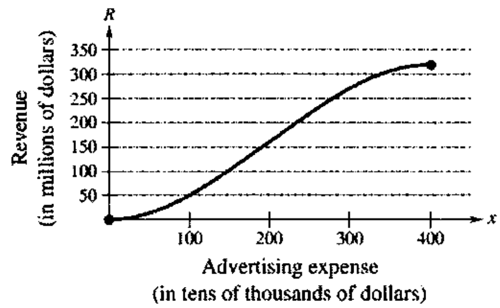


FIGURE FOR 104

EXPLORATION

TRUE OR FALSE? **In Exercises 105–107, determine whether the statement is true or false. Justify your answer.**

105. A fifth-degree polynomial can have five turning points in its graph.

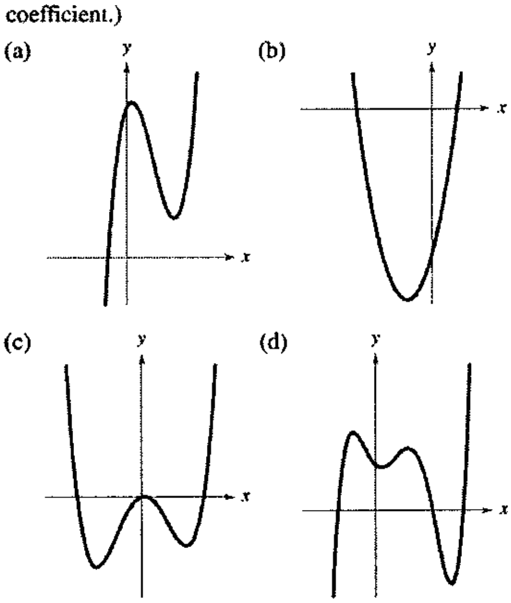
106. It is possible for a sixth-degree polynomial to have only one solution.

107. The graph of the function given by

$$f(x) = 2 + x - x^2 + x^3 - x^4 + x^5 + x^6 - x^7$$

rises to the left and falls to the right.

108. CAPSTONE For each graph, describe a polynomial function that could represent the graph. (Indicate the degree of the function and the sign of its leading coefficient.)



109. GRAPHICAL REASONING Sketch a graph of the function given by $f(x) = x^4$. Explain how the graph of each function g differs (if it does) from the graph of each function f. Determine whether g is odd, even, or neither.

(a) $g(x) = f(x) + 2$ (b) $g(x) = f(x + 2)$

(c) $g(x) = f(-x)$ (d) $g(x) = -f(x)$

(e) $g(x) = f\left(\frac{1}{2}x\right)$ (f) $g(x) = \frac{1}{2}f(x)$

(g) $g(x) = f\left(x^{3/4}\right)$ (h) $g(x) = (f \circ f)(x)$

110. THINK ABOUT IT For each function, identify the degree of the function and whether the degree of the function is even or odd. Identify the leading coefficient and whether the leading coefficient is positive or negative. Use a graphing utility to graph each function. Describe the relationship between the degree of the function and the sign of the leading coefficient of the function and the right-hand and left-hand behavior of the graph of the function.

(a) $f(x) = x^3 - 2x^2 - x + 1$

(b) $f(x) = 2x^5 + 2x^2 - 5x + 1$

(c) $f(x) = -2x^5 - x^2 + 5x + 3$

(d) $f(x) = -x^3 + 5x - 2$

(e) $f(x) = 2x^2 + 3x - 4$

(f) $f(x) = x^4 - 3x^2 + 2x - 1$

(g) $f(x) = x^2 + 3x + 2$

111. THINK ABOUT IT Sketch the graph of each polynomial function. Then count the number of zeros of the function and the numbers of relative minima and relative maxima. Compare these numbers with the degree of the polynomial. What do you observe?

(a) $f(x) = -x^3 + 9x$ (b) $f(x) = x^4 - 10x^2 + 9$

(c) $f(x) = x^5 - 16x$

112. Explore the transformations of the form

$$g(x) = a(x - h)^5 + k.$$

(a) Use a graphing utility to graph the functions $y_1 = -\frac{1}{3}(x - 2)^5 + 1$ and $y_2 = \frac{3}{5}(x + 2)^5 - 3$. Determine whether the graphs are increasing or decreasing. Explain.

(b) Will the graph of g always be increasing or decreasing? If so, is this behavior determined by a, h, or k? Explain.

(c) Use a graphing utility to graph the function given by $H(x) = x^5 - 3x^3 + 2x + 1$. Use the graph and the result of part (b) to determine whether H can be written in the form $H(x) = a(x - h)^5 + k$. Explain.

3.3 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY

1. Two forms of the Division Algorithm are shown below. Identify and label each term or function.

$$f(x) = d(x)q(x) + r(x) \qquad \frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

In Exercises 2–6, fill in the blanks.

2. The rational expression $p(x)/q(x)$ is called ________ if the degree of the numerator is greater than or equal to that of the denominator, and is called ________ if the degree of the numerator is less than that of the denominator.
3. In the Division Algorithm, the rational expression $f(x)/d(x)$ is ________ because the degree of $f(x)$ is greater than or equal to the degree of $d(x)$.
4. An alternative method to long division of polynomials is called ________ ________, in which the divisor must be of the form $x - k$.
5. The ________ Theorem states that a polynomial $f(x)$ has a factor $(x - k)$ if and only if $f(k) = 0$.
6. The ________ Theorem states that if a polynomial $f(x)$ is divided by $x - k$, the remainder is $r = f(k)$.

SKILLS AND APPLICATIONS

ANALYTICAL ANALYSIS **In Exercises 7 and 8, use long division to verify that $y_1 = y_2$.**

7. $y_1 = \dfrac{x^2}{x+2}, \quad y_2 = x - 2 + \dfrac{4}{x+2}$
8. $y_1 = \dfrac{x^4 - 3x^2 - 1}{x^2 + 5}, \quad y_2 = x^2 - 8 + \dfrac{39}{x^2+5}$

GRAPHICAL ANALYSIS **In Exercises 9 and 10, (a) use a graphing utility to graph the two equations in the same viewing window, (b) use the graphs to verify that the expressions are equivalent, and (c) use long division to verify the results algebraically.**

9. $y_1 = \dfrac{x^2 + 2x - 1}{x+3}, \quad y_2 = x - 1 + \dfrac{2}{x+3}$
10. $y_1 = \dfrac{x^4 + x^2 - 1}{x^2+1}, \quad y_2 = x^2 - \dfrac{1}{x^2+1}$

In Exercises 11–26, use long division to divide.

11. $(2x^2 + 10x + 12) \div (x + 3)$
12. $(5x^2 - 17x - 12) \div (x - 4)$
13. $(4x^3 - 7x^2 - 11x + 5) \div (4x + 5)$
14. $(6x^3 - 16x^2 + 17x - 6) \div (3x - 2)$
15. $(x^4 + 5x^3 + 6x^2 - x - 2) \div (x + 2)$
16. $(x^3 + 4x^2 - 3x - 12) \div (x - 3)$
17. $(x^3 - 27) \div (x - 3)$
18. $(x^3 + 125) \div (x + 5)$
19. $(7x + 3) \div (x + 2)$
20. $(8x - 5) \div (2x + 1)$
21. $(x^3 - 9) \div (x^2 + 1)$
22. $(x^5 + 7) \div (x^3 - 1)$
23. $(3x + 2x^3 - 9 - 8x^2) \div (x^2 + 1)$
24. $(5x^3 - 16 - 20x + x^4) \div (x^2 - x - 3)$
25. $\dfrac{x^4}{(x-1)^3}$
26. $\dfrac{2x^3 - 4x^2 - 15x + 5}{(x-1)^2}$

In Exercises 27–46, use synthetic division to divide.

27. $(3x^3 - 17x^2 + 15x - 25) \div (x - 5)$
28. $(5x^3 + 18x^2 + 7x - 6) \div (x + 3)$
29. $(6x^3 + 7x^2 - x + 26) \div (x - 3)$
30. $(2x^3 + 14x^2 - 20x + 7) \div (x + 6)$
31. $(4x^3 - 9x + 8x^2 - 18) \div (x + 2)$
32. $(9x^3 - 16x - 18x^2 + 32) \div (x - 2)$
33. $(-x^3 + 75x - 250) \div (x + 10)$
34. $(3x^3 - 16x^2 - 72) \div (x - 6)$
35. $(5x^3 - 6x^2 + 8) \div (x - 4)$
36. $(5x^3 + 6x + 8) \div (x + 2)$
37. $\dfrac{10x^4 - 50x^3 - 800}{x - 6}$
38. $\dfrac{x^5 - 13x^4 - 120x + 80}{x + 3}$
39. $\dfrac{x^3 + 512}{x + 8}$
40. $\dfrac{x^3 - 729}{x - 9}$
41. $\dfrac{-3x^4}{x - 2}$
42. $\dfrac{-3x^4}{x + 2}$
43. $\dfrac{180x - x^4}{x - 6}$
44. $\dfrac{5 - 3x + 2x^2 - x^3}{x + 1}$
45. $\dfrac{4x^3 + 16x^2 - 23x - 15}{x + \frac{1}{2}}$
46. $\dfrac{3x^3 - 4x^2 + 5}{x - \frac{3}{2}}$

In Exercises 47–54, write the function in the form $f(x) = (x - k)q(x) + r$ for the given value of k, and demonstrate that $f(k) = r$.

47. $f(x) = x^3 - x^2 - 14x + 11, \quad k = 4$

48. $f(x) = x^3 - 5x^2 - 11x + 8, \quad k = -2$

49. $f(x) = 15x^4 + 10x^3 - 6x^2 + 14, \quad k = -\frac{2}{3}$

50. $f(x) = 10x^3 - 22x^2 - 3x + 4, \quad k = \frac{1}{5}$

51. $f(x) = x^3 + 3x^2 - 2x - 14, \quad k = \sqrt{2}$

52. $f(x) = x^3 + 2x^2 - 5x - 4, \quad k = -\sqrt{5}$

53. $f(x) = -4x^3 + 6x^2 + 12x + 4, \quad k = 1 - \sqrt{3}$

54. $f(x) = -3x^3 + 8x^2 + 10x - 8, \quad k = 2 + \sqrt{2}$

In Exercises 55–58, use the Remainder Theorem and synthetic division to find each function value. Verify your answers using another method.

55. $f(x) = 2x^3 - 7x + 3$

(a) $f(1)$ (b) $f(-2)$ (c) $f(\frac{1}{2})$ (d) $f(2)$

56. $g(x) = 2x^6 + 3x^4 - x^2 + 3$

(a) $g(2)$ (b) $g(1)$ (c) $g(3)$ (d) $g(-1)$

57. $h(x) = x^3 - 5x^2 - 7x + 4$

(a) $h(3)$ (b) $h(2)$ (c) $h(-2)$ (d) $h(-5)$

58. $f(x) = 4x^4 - 16x^3 + 7x^2 + 20$

(a) $f(1)$ (b) $f(-2)$ (c) $f(5)$ (d) $f(-10)$

In Exercises 59–66, use synthetic division to show that x is a solution of the third-degree polynomial equation, and use the result to factor the polynomial completely. List all real solutions of the equation.

59. $x^3 - 7x + 6 = 0, \quad x = 2$

60. $x^3 - 28x - 48 = 0, \quad x = -4$

61. $2x^3 - 15x^2 + 27x - 10 = 0, \quad x = \frac{1}{2}$

62. $48x^3 - 80x^2 + 41x - 6 = 0, \quad x = \frac{2}{3}$

63. $x^3 + 2x^2 - 3x - 6 = 0, \quad x = \sqrt{3}$

64. $x^3 + 2x^2 - 2x - 4 = 0, \quad x = \sqrt{2}$

65. $x^3 - 3x^2 + 2 = 0, \quad x = 1 + \sqrt{3}$

66. $x^3 - x^2 - 13x - 3 = 0, \quad x = 2 - \sqrt{5}$

In Exercises 67–74, (a) verify the given factors of the function f, (b) find the remaining factor(s) of f, (c) use your results to write the complete factorization of f, (d) list all real zeros of f, and (e) confirm your results by using a graphing utility to graph the function.

Function	*Factors*
67. $f(x) = 2x^3 + x^2 - 5x + 2$	$(x + 2), (x - 1)$
68. $f(x) = 3x^3 + 2x^2 - 19x + 6$	$(x + 3), (x - 2)$
69. $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$	$(x - 5), (x + 4)$
70. $f(x) = 8x^4 - 14x^3 - 71x^2 - 10x + 24$	$(x + 2), (x - 4)$
71. $f(x) = 6x^3 + 41x^2 - 9x - 14$	$(2x + 1), (3x - 2)$
72. $f(x) = 10x^3 - 11x^2 - 72x + 45$	$(2x + 5), (5x - 3)$
73. $f(x) = 2x^3 - x^2 - 10x + 5$	$(2x - 1), (x + \sqrt{5})$
74. $f(x) = x^3 + 3x^2 - 48x - 144$	$(x + 4\sqrt{3}), (x + 3)$

GRAPHICAL ANALYSIS In Exercises 75–80, (a) use the *zero* or *root* feature of a graphing utility to approximate the zeros of the function accurate to three decimal places, (b) determine one of the exact zeros, and (c) use synthetic division to verify your result from part (b), and then factor the polynomial completely.

75. $f(x) = x^3 - 2x^2 - 5x + 10$

76. $g(x) = x^3 - 4x^2 - 2x + 8$

77. $h(t) = t^3 - 2t^2 - 7t + 2$

78. $f(s) = s^3 - 12s^2 + 40s - 24$

79. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

80. $g(x) = 6x^4 - 11x^3 - 51x^2 + 99x - 27$

In Exercises 81–84, simplify the rational expression by using long division or synthetic division.

81. $\dfrac{4x^3 - 8x^2 + x + 3}{2x - 3}$

82. $\dfrac{x^3 + x^2 - 64x - 64}{x + 8}$

83. $\dfrac{x^4 + 6x^3 + 11x^2 + 6x}{x^2 + 3x + 2}$

84. $\dfrac{x^4 + 9x^3 - 5x^2 - 36x + 4}{x^2 - 4}$

85. **DATA ANALYSIS: HIGHER EDUCATION** The amounts A (in billions of dollars) donated to support higher education in the United States from 2000 through 2007 are shown in the table, where t represents the year, with $t = 0$ corresponding to 2000.

Year, t	Amount, A
0	23.2
1	24.2
2	23.9
3	23.9
4	24.4
5	25.6
6	28.0
7	29.8

(a) Use a graphing utility to create a scatter plot of the data.

(b) Use the *regression* feature of the graphing utility to find a cubic model for the data. Graph the model in the same viewing window as the scatter plot.

(c) Use the model to create a table of estimated values of A. Compare the model with the original data.

(d) Use synthetic division to evaluate the model for the year 2010. Even though the model is relatively accurate for estimating the given data, would you use this model to predict the amount donated to higher education in the future? Explain.

86. DATA ANALYSIS: HEALTH CARE The amounts A (in billions of dollars) of national health care expenditures in the United States from 2000 through 2007 are shown in the table, where t represents the year, with $t = 0$ corresponding to 2000.

Year, t	Amount, A
0	30.5
1	32.2
2	34.2
3	38.0
4	42.7
5	47.9
6	52.7
7	57.6

(a) Use a graphing utility to create a scatter plot of the data.

(b) Use the *regression* feature of the graphing utility to find a cubic model for the data. Graph the model in the same viewing window as the scatter plot.

(c) Use the model to create a table of estimated values of A. Compare the model with the original data.

(d) Use synthetic division to evaluate the model for the year 2010.

EXPLORATION

TRUE OR FALSE? **In Exercises 87–89, determine whether the statement is true or false. Justify your answer.**

87. If $(7x + 4)$ is a factor of some polynomial function f, then $\frac{4}{7}$ is a zero of f.

88. $(2x - 1)$ is a factor of the polynomial

$$6x^6 + x^5 - 92x^4 + 45x^3 + 184x^2 + 4x - 48.$$

89. The rational expression

$$\frac{x^3 + 2x^2 - 13x + 10}{x^2 - 4x - 12}$$

is improper.

90. Use the form $f(x) = (x - k)q(x) + r$ to create a cubic function that (a) passes through the point $(2, 5)$ and rises to the right, and (b) passes through the point $(-3, 1)$ and falls to the right. (There are many correct answers.)

THINK ABOUT IT **In Exercises 91 and 92, perform the division by assuming that n is a positive integer.**

91. $\dfrac{x^{3n} + 9x^{2n} + 27x^n + 27}{x^n + 3}$ **92.** $\dfrac{x^{3n} - 3x^{2n} + 5x^n - 6}{x^n - 2}$

93. WRITING Briefly explain what it means for a divisor to divide evenly into a dividend.

94. WRITING Briefly explain how to check polynomial division, and justify your reasoning. Give an example.

EXPLORATION **In Exercises 95 and 96, find the constant c such that the denominator will divide evenly into the numerator.**

95. $\dfrac{x^3 + 4x^2 - 3x + c}{x - 5}$ **96.** $\dfrac{x^5 - 2x^2 + x + c}{x + 2}$

97. THINK ABOUT IT Find the value of k such that $x - 4$ is a factor of $x^3 - kx^2 + 2kx - 8$.

98. THINK ABOUT IT Find the value of k such that $x - 3$ is a factor of $x^3 - kx^2 + 2kx - 12$.

99. WRITING Complete each polynomial division. Write a brief description of the pattern that you obtain, and use your result to find a formula for the polynomial division $(x^n - 1)/(x - 1)$. Create a numerical example to test your formula.

(a) $\dfrac{x^2 - 1}{x - 1} =$ (b) $\dfrac{x^3 - 1}{x - 1} =$

(c) $\dfrac{x^4 - 1}{x - 1} =$

100. CAPSTONE Consider the division

$$f(x) \div (x - k)$$

where

$$f(x) = (x + 3)^2(x - 3)(x + 1)^3.$$

(a) What is the remainder when $k = -3$? Explain.

(b) If it is necessary to find $f(2)$, is it easier to evaluate the function directly or to use synthetic division? Explain.

3.4 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. The ________ ________ of ________ states that if $f(x)$ is a polynomial of degree n $(n > 0)$, then f has at least one zero in the complex number system.
2. The ________ ________ ________ states that if $f(x)$ is a polynomial of degree n $(n > 0)$, then f has precisely n linear factors, $f(x) = a_n(x - c_1)(x - c_2) \cdot \cdot \cdot (x - c_n)$, where $c_1, c_2, \ldots, c_n$ are complex numbers.
3. The test that gives a list of the possible rational zeros of a polynomial function is called the ________ ________ Test.
4. If $a + bi$ is a complex zero of a polynomial with real coefficients, then so is its ________, $a - bi$.
5. Every polynomial of degree $n > 0$ with real coefficients can be written as the product of ________ and ________ factors with real coefficients, where the ________ factors have no real zeros.
6. A quadratic factor that cannot be factored further as a product of linear factors containing real numbers is said to be ________ over the ________.
7. The theorem that can be used to determine the possible numbers of positive real zeros and negative real zeros of a function is called ________ ________ of ________.
8. A real number b is a(n) ________ bound for the real zeros of f if no real zeros are less than b, and is a(n) ________ bound if no real zeros are greater than b.

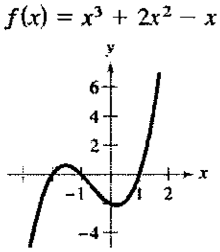
SKILLS AND APPLICATIONS

In Exercises 9–14, find all the zeros of the function.

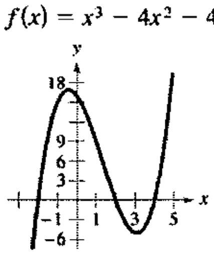
9. $f(x) = x(x - 6)^2$
10. $f(x) = x^2(x + 3)(x^2 - 1)$
11. $g(x) = (x - 2)(x + 4)^3$
12. $f(x) = (x + 5)(x - 8)^2$
13. $f(x) = (x + 6)(x + i)(x - i)$
14. $h(t) = (t - 3)(t - 2)(t - 3i)(t + 3i)$

In Exercises 15–18, use the Rational Zero Test to list all possible rational zeros of f. Verify that the zeros of f shown on the graph are contained in the list.

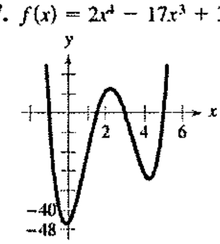
15. $f(x) = x^3 + 2x^2 - x - 2$



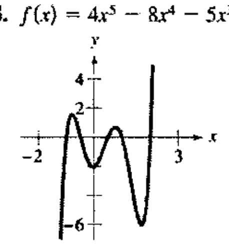
16. $f(x) = x^3 - 4x^2 - 4x + 16$



17. $f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45$



18. $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$



In Exercises 19–28, find all the rational zeros of the function.

19. $f(x) = x^3 - 6x^2 + 11x - 6$
20. $f(x) = x^3 - 7x - 6$
21. $g(x) = x^3 - 4x^2 - x + 4$
22. $h(x) = x^3 - 9x^2 + 20x - 12$
23. $h(t) = t^3 + 8t^2 + 13t + 6$
24. $p(x) = x^3 - 9x^2 + 27x - 27$
25. $C(x) = 2x^3 + 3x^2 - 1$
26. $f(x) = 3x^3 - 19x^2 + 33x - 9$
27. $f(x) = 9x^4 - 9x^3 - 58x^2 + 4x + 24$
28. $f(x) = 2x^4 - 15x^3 + 23x^2 + 15x - 25$

In Exercises 29–32, find all real solutions of the polynomial equation.

29. $z^4 + z^3 + z^2 + 3z - 6 = 0$

30. $x^4 - 13x^2 - 12x = 0$

31. $2y^4 + 3y^3 - 16y^2 + 15y - 4 = 0$

32. $x^5 - x^4 - 3x^3 + 5x^2 - 2x = 0$

In Exercises 33–36, (a) list the possible rational zeros of f, (b) sketch the graph of f so that some of the possible zeros in part (a) can be disregarded, and then (c) determine all real zeros of f.

33. $f(x) = x^3 + x^2 - 4x - 4$

34. $f(x) = -3x^3 + 20x^2 - 36x + 16$

35. $f(x) = -4x^3 + 15x^2 - 8x - 3$

36. $f(x) = 4x^3 - 12x^2 - x + 15$

In Exercises 37–40, (a) list the possible rational zeros of f, (b) use a graphing utility to graph f so that some of the possible zeros in part (a) can be disregarded, and then (c) determine all real zeros of f.

37. $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

38. $f(x) = 4x^4 - 17x^2 + 4$

39. $f(x) = 32x^3 - 52x^2 + 17x + 3$

40. $f(x) = 4x^3 + 7x^2 - 11x - 18$

GRAPHICAL ANALYSIS In Exercises 41–44, (a) use the *zero* or *root* feature of a graphing utility to approximate the zeros of the function accurate to three decimal places, (b) determine one of the exact zeros (use synthetic division to verify your result), and (c) factor the polynomial completely.

41. $f(x) = x^4 - 3x^2 + 2$

42. $P(t) = t^4 - 7t^2 + 12$

43. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

44. $g(x) = 6x^4 - 11x^3 - 51x^2 + 99x - 27$

In Exercises 45–50, find a polynomial function with real coefficients that has the given zeros. (There are many correct answers.)

45. $1, 5i$

46. $4, -3i$

47. $2, 5 + i$

48. $5, 3 - 2i$

49. $\frac{2}{3}, -1, 3 + \sqrt{2}i$

50. $-5, -5, 1 + \sqrt{3}i$

In Exercises 51–54, write the polynomial (a) as the product of factors that are irreducible over the *rationals*, (b) as the product of linear and quadratic factors that are irreducible over the *reals*, and (c) in completely factored form.

51. $f(x) = x^4 + 6x^2 - 27$

52. $f(x) = x^4 - 2x^3 - 3x^2 + 12x - 18$
(*Hint:* One factor is $x^2 - 6$.)

53. $f(x) = x^4 - 4x^3 + 5x^2 - 2x - 6$
(*Hint:* One factor is $x^2 - 2x - 2$.)

54. $f(x) = x^4 - 3x^3 - x^2 - 12x - 20$
(*Hint:* One factor is $x^2 + 4$.)

In Exercises 55–62, use the given zero to find all the zeros of the function.

	Function	*Zero*
55.	$f(x) = x^3 - x^2 + 4x - 4$	$2i$
56.	$f(x) = 2x^3 + 3x^2 + 18x + 27$	$3i$
57.	$f(x) = 2x^4 - x^3 + 49x^2 - 25x - 25$	$5i$
58.	$g(x) = x^3 - 7x^2 - x + 87$	$5 + 2i$
59.	$g(x) = 4x^3 + 23x^2 + 34x - 10$	$-3 + i$
60.	$h(x) = 3x^3 - 4x^2 + 8x + 8$	$1 - \sqrt{3}i$
61.	$f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22$	$-3 + \sqrt{2}i$
62.	$f(x) = x^3 + 4x^2 + 14x + 20$	$-1 - 3i$

In Exercises 63–80, find all the zeros of the function and write the polynomial as a product of linear factors.

63. $f(x) = x^2 + 36$

64. $f(x) = x^2 - x + 56$

65. $h(x) = x^2 - 2x + 17$

66. $g(x) = x^2 + 10x + 17$

67. $f(x) = x^4 - 16$

68. $f(y) = y^4 - 256$

69. $f(z) = z^2 - 2z + 2$

70. $h(x) = x^3 - 3x^2 + 4x - 2$

71. $g(x) = x^3 - 3x^2 + x + 5$

72. $f(x) = x^3 - x^2 + x + 39$

73. $h(x) = x^3 - x + 6$

74. $h(x) = x^3 + 9x^2 + 27x + 35$

75. $f(x) = 5x^3 - 9x^2 + 28x + 6$

76. $g(x) = 2x^3 - x^2 + 8x + 21$

77. $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

78. $h(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$

79. $f(x) = x^4 + 10x^2 + 9$

80. $f(x) = x^4 + 29x^2 + 100$

In Exercises 81–86, find all the zeros of the function. When there is an extended list of possible rational zeros, use a graphing utility to graph the function in order to discard any rational zeros that are obviously not zeros of the function.

81. $f(x) = x^3 + 24x^2 + 214x + 740$

82. $f(s) = 2s^3 - 5s^2 + 12s - 5$

83. $f(x) = 16x^3 - 20x^2 - 4x + 15$

84. $f(x) = 9x^3 - 15x^2 + 11x - 5$

85. $f(x) = 2x^4 + 5x^3 + 4x^2 + 5x + 2$

86. $g(x) = x^5 - 8x^4 + 28x^3 - 56x^2 + 64x - 32$

In Exercises 87–94, use Descartes's Rule of Signs to determine the possible numbers of positive and negative zeros of the function.

87. $g(x) = 2x^3 - 3x^2 - 3$ **88.** $h(x) = 4x^2 - 8x + 3$

89. $h(x) = 2x^3 + 3x^2 + 1$ **90.** $h(x) = 2x^4 - 3x + 2$

91. $g(x) = 5x^5 - 10x$

92. $f(x) = 4x^3 - 3x^2 + 2x - 1$

93. $f(x) = -5x^3 + x^2 - x + 5$

94. $f(x) = 3x^3 + 2x^2 + x + 3$

In Exercises 95–98, use synthetic division to verify the upper and lower bounds of the real zeros of f.

95. $f(x) = x^3 + 3x^2 - 2x + 1$

(a) Upper: $x = 1$ (b) Lower: $x = -4$

96. $f(x) = x^3 - 4x^2 + 1$

(a) Upper: $x = 4$ (b) Lower: $x = -1$

97. $f(x) = x^4 - 4x^3 + 16x - 16$

(a) Upper: $x = 5$ (b) Lower: $x = -3$

98. $f(x) = 2x^4 - 8x + 3$

(a) Upper: $x = 3$ (b) Lower: $x = -4$

In Exercises 99–102, find all the real zeros of the function.

99. $f(x) = 4x^3 - 3x - 1$

100. $f(z) = 12z^3 - 4z^2 - 27z + 9$

101. $f(y) = 4y^3 + 3y^2 + 8y + 6$

102. $g(x) = 3x^3 - 2x^2 + 15x - 10$

In Exercises 103–106, find all the rational zeros of the polynomial function.

103. $P(x) = x^4 - \frac{25}{4}x^2 + 9 = \frac{1}{4}(4x^4 - 25x^2 + 36)$

104. $f(x) = x^3 - \frac{3}{2}x^2 - \frac{23}{2}x + 6 = \frac{1}{2}(2x^3 - 3x^2 - 23x + 12)$

105. $f(x) = x^3 - \frac{1}{4}x^2 - x + \frac{1}{4} = \frac{1}{4}(4x^3 - x^2 - 4x + 1)$

106. $f(z) = z^3 + \frac{11}{6}z^2 - \frac{1}{2}z - \frac{1}{3} = \frac{1}{6}(6z^3 + 11z^2 - 3z - 2)$

In Exercises 107–110, match the cubic function with the numbers of rational and irrational zeros.

(a) Rational zeros: 0; irrational zeros: 1

(b) Rational zeros: 3; irrational zeros: 0

(c) Rational zeros: 1; irrational zeros: 2

(d) Rational zeros: 1; irrational zeros: 0

107. $f(x) = x^3 - 1$ **108.** $f(x) = x^3 - 2$

109. $f(x) = x^3 - x$ **110.** $f(x) = x^3 - 2x$

111. GEOMETRY An open box is to be made from a rectangular piece of material, 15 centimeters by 9 centimeters, by cutting equal squares from the corners and turning up the sides.

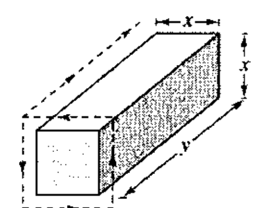
(a) Let x represent the length of the sides of the squares removed. Draw a diagram showing the squares removed from the original piece of material and the resulting dimensions of the open box.

(b) Use the diagram to write the volume V of the box as a function of x. Determine the domain of the function.

(c) Sketch the graph of the function and approximate the dimensions of the box that will yield a maximum volume.

(d) Find values of x such that $V = 56$. Which of these values is a physical impossibility in the construction of the box? Explain.

112. GEOMETRY A rectangular package to be sent by a delivery service (see figure) can have a maximum combined length and girth (perimeter of a cross section) of 120 inches.



(a) Write a function $V(x)$ that represents the volume of the package.

(b) Use a graphing utility to graph the function and approximate the dimensions of the package that will yield a maximum volume.

(c) Find values of x such that $V = 13{,}500$. Which of these values is a physical impossibility in the construction of the package? Explain.

113. ADVERTISING COST A company that produces MP3 players estimates that the profit P (in dollars) for selling a particular model is given by

$$P = -76x^3 + 4830x^2 - 320{,}000, \quad 0 \le x \le 60$$

where x is the advertising expense (in tens of thousands of dollars). Using this model, find the smaller of two advertising amounts that will yield a profit of \$2,500,000.

114. ADVERTISING COST A company that manufactures bicycles estimates that the profit P (in dollars) for selling a particular model is given by

$$P = -45x^3 + 2500x^2 - 275{,}000, \quad 0 \le x \le 50$$

where x is the advertising expense (in tens of thousands of dollars). Using this model, find the smaller of two advertising amounts that will yield a profit of \$800,000.

115. GEOMETRY A bulk food storage bin with dimensions 2 feet by 3 feet by 4 feet needs to be increased in size to hold five times as much food as the current bin. (Assume each dimension is increased by the same amount.)

(a) Write a function that represents the volume V of the new bin.

(b) Find the dimensions of the new bin.

116. GEOMETRY A manufacturer wants to enlarge an existing manufacturing facility such that the total floor area is 1.5 times that of the current facility. The floor area of the current facility is rectangular and measures 250 feet by 160 feet. The manufacturer wants to increase each dimension by the same amount.

(a) Write a function that represents the new floor area A.

(b) Find the dimensions of the new floor.

(c) Another alternative is to increase the current floor's length by an amount that is twice an increase in the floor's width. The total floor area is 1.5 times that of the current facility. Repeat parts (a) and (b) using these criteria.

117. COST The ordering and transportation cost C (in thousands of dollars) for the components used in manufacturing a product is given by

$$C = 100\left(\frac{200}{x^2} + \frac{x}{x + 30}\right), \quad x \geq 1$$

where x is the order size (in hundreds). In calculus, it can be shown that the cost is a minimum when

$$3x^3 - 40x^2 - 2400x - 36{,}000 = 0.$$

Use a calculator to approximate the optimal order size to the nearest hundred units.

118. HEIGHT OF A BASEBALL A baseball is thrown upward from a height of 6 feet with an initial velocity of 48 feet per second, and its height h (in feet) is

$$h(t) = -16t^2 + 48t + 6, \quad 0 \leq t \leq 3$$

where t is the time (in seconds). You are told the ball reaches a height of 64 feet. Is this possible?

119. PROFIT The demand equation for a certain product is $p = 140 - 0.0001x$, where p is the unit price (in dollars) of the product and x is the number of units produced and sold. The cost equation for the product is $C = 80x + 150{,}000$, where C is the total cost (in dollars) and x is the number of units produced. The total profit obtained by producing and selling x units is $P = R - C = xp - C$. You are working in the marketing department of the company that produces this product, and you are asked to determine a price p that will yield a profit of 9 million dollars. Is this possible? Explain.

120. ATHLETICS The attendance A (in millions) at NCAA women's college basketball games for the years 2000 through 2007 is shown in the table. (Source: National Collegiate Athletic Association, Indianapolis, IN)

Year	Attendance, A
2000	8.7
2001	8.8
2002	9.5
2003	10.2
2004	10.0
2005	9.9
2006	9.9
2007	10.9

(a) Use a graphing utility to create a scatter plot of the data. Let t represent the year, with $t = 0$ corresponding to 2000.

(b) Use the *regression* feature of the graphing utility to find a quartic model for the data.

(c) Graph the model and the scatter plot in the same viewing window. How well does the model fit the data?

(d) According to the model in part (b), in what year(s) was the attendance at least 10 million?

(e) According to the model, will the attendance continue to increase? Explain.

EXPLORATION

TRUE OR FALSE? In Exercises 121 and 122, decide whether the statement is true or false. Justify your answer.

121. It is possible for a third-degree polynomial function with integer coefficients to have no real zeros.

122. If $x = -i$ is a zero of the function given by

$$f(x) = x^3 + ix^2 + ix - 1$$

then $x = i$ must also be a zero of f.

THINK ABOUT IT In Exercises 123–128, determine (if possible) the zeros of the function g if the function f has zeros at $x = r_1$, $x = r_2$, and $x = r_3$.

123. $g(x) = -f(x)$

124. $g(x) = 3f(x)$

125. $g(x) = f(x - 5)$

126. $g(x) = f(2x)$

127. $g(x) = 3 + f(x)$

128. $g(x) = f(-x)$

129. THINK ABOUT IT A third-degree polynomial function f has real zeros $-2, \frac{1}{2}$, and 3, and its leading coefficient is negative. Write an equation for f. Sketch the graph of f. How many different polynomial functions are possible for f?

130. CAPSTONE Use a graphing utility to graph the function given by $f(x) = x^4 - 4x^2 + k$ for different values of k. Find values of k such that the zeros of f satisfy the specified characteristics. (Some parts do not have unique answers.)

(a) Four real zeros

(b) Two real zeros, each of multiplicity 2

(c) Two real zeros and two complex zeros

(d) Four complex zeros

(e) Will the answers to parts (a) through (d) change for the function g, where $g(x) = f(x - 2)$?

(f) Will the answers to parts (a) through (d) change for the function g, where $g(x) = f(2x)$?

131. THINK ABOUT IT Sketch the graph of a fifth-degree polynomial function whose leading coefficient is positive and that has a zero at $x = 3$ of multiplicity 2.

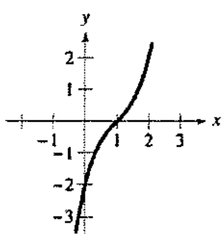
132. WRITING Compile a list of all the various techniques for factoring a polynomial that have been covered so far in the text. Give an example illustrating each technique, and write a paragraph discussing when the use of each technique is appropriate.

133. THINK ABOUT IT Let $y = f(x)$ be a quartic polynomial with leading coefficient $a = 1$ and $f(i) = f(2i) = 0$. Write an equation for f.

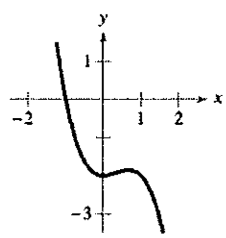
134. THINK ABOUT IT Let $y = f(x)$ be a cubic polynomial with leading coefficient $a = -1$ and $f(2) = f(i) = 0$. Write an equation for f.

In Exercises 135 and 136, the graph of a cubic polynomial function $y = f(x)$ is shown. It is known that one of the zeros is $1 + i$. Write an equation for f.

135.



136.



137. Use the information in the table to answer each question.

Interval	Value of $f(x)$
$(-\infty, -2)$	Positive
$(-2, 1)$	Negative
$(1, 4)$	Negative
$(4, \infty)$	Positive

(a) What are the three real zeros of the polynomial function f?

(b) What can be said about the behavior of the graph of f at $x = 1$?

(c) What is the least possible degree of f? Explain. Can the degree of f ever be odd? Explain.

(d) Is the leading coefficient of f positive or negative? Explain.

(e) Write an equation for f. (There are many correct answers.)

(f) Sketch a graph of the equation you wrote in part (e).

138. (a) Find a quadratic function f (with integer coefficients) that has $\pm\sqrt{b}\,i$ as zeros. Assume that b is a positive integer.

(b) Find a quadratic function f (with integer coefficients) that has $a \pm bi$ as zeros. Assume that b is a positive integer.

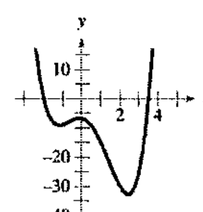
139. GRAPHICAL REASONING The graph of one of the following functions is shown below. Identify the function shown in the graph. Explain why each of the others is not the correct function. Use a graphing utility to verify your result.

(a) $f(x) = x^2(x + 2)(x - 3.5)$

(b) $g(x) = (x + 2)(x - 3.5)$

(c) $h(x) = (x + 2)(x - 3.5)(x^2 + 1)$

(d) $k(x) = (x + 1)(x + 2)(x - 3.5)$



This page contains answers for this chapter only

Chapter 3

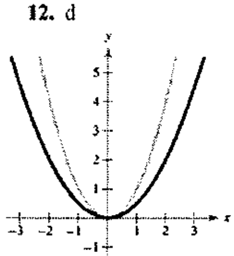
Section 3.1 *(page 266)*

1. polynomial **3.** quadratic; parabola

5. positive; minimum **7.** e **8.** c **9.** b **10.** a

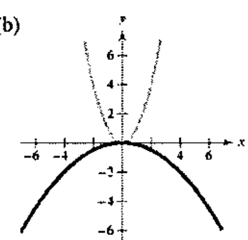
11. f **12.** d

13. (a)



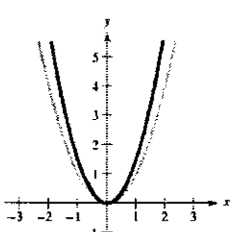
Vertical shrink

(b)



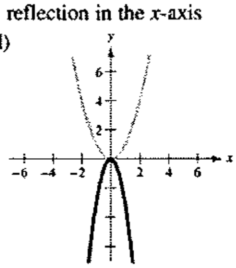
Vertical shrink and reflection in the x-axis

(c)



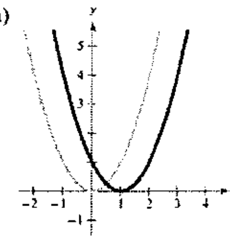
Vertical stretch

(d)



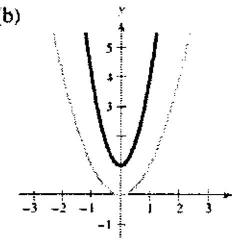
Vertical stretch and reflection in the x-axis

15. (a)



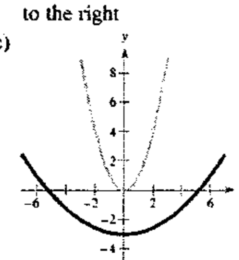
Horizontal shift one unit to the right

(b)



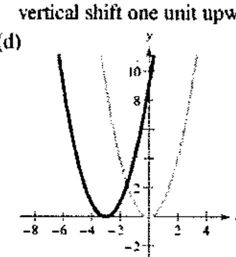
Horizontal shrink and vertical shift one unit upward

(c)



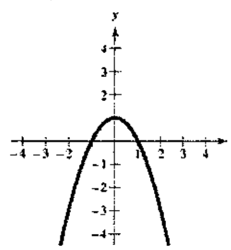
Horizontal stretch and vertical shift three units downward

(d)



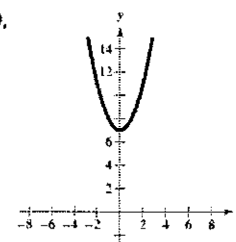
Horizontal shift three units to the left

17.



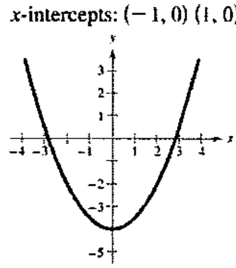
Vertex: $(0, 1)$
Axis of symmetry: y-axis
x-intercepts: $(-1, 0)$ $(1, 0)$

19.



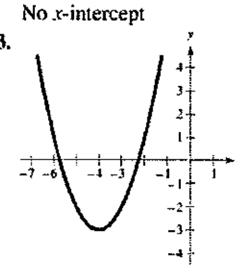
Vertex: $(0, 7)$
Axis of symmetry: y-axis
No x-intercept

21.

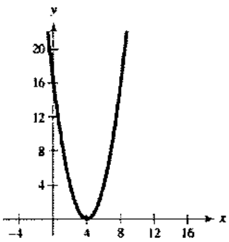


Vertex: $(0, -4)$
Axis of symmetry: y-axis
x-intercepts: $(\pm 2\sqrt{2}, 0)$

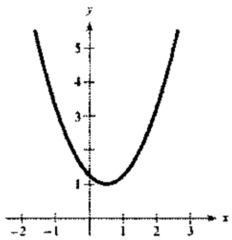
23.



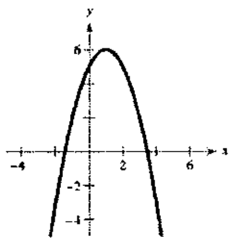
Vertex: $(-4, -3)$
Axis of symmetry: $x = -4$
x-intercepts: $(-4 \pm \sqrt{3}, 0)$

25.


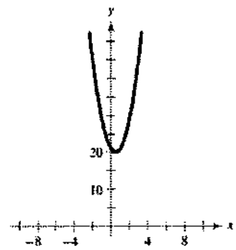
Vertex: $(4, 0)$
Axis of symmetry: $x = 4$
x-intercept: $(4, 0)$

27.


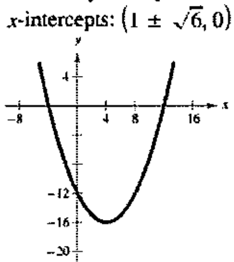
Vertex: $\left(\frac{1}{2}, 1\right)$
Axis of symmetry: $x = \frac{1}{2}$
No x-intercept

29.


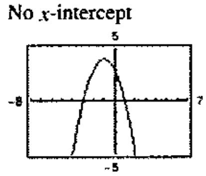
Vertex: $(1, 6)$
Axis of symmetry: $x = 1$
x-intercepts: $(1 \pm \sqrt{6}, 0)$

31.


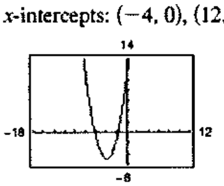
Vertex: $\left(\frac{1}{2}, 20\right)$
Axis of symmetry: $x = \frac{1}{2}$
No x-intercept

33.


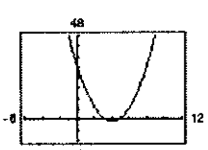
Vertex: $(4, -16)$
Axis of symmetry: $x = 4$
x-intercepts: $(-4, 0), (12, 0)$

35.


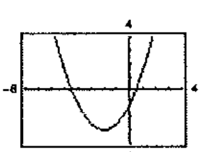
Vertex: $(-1, 4)$
Axis of symmetry: $x = -1$
x-intercepts: $(1, 0), (-3, 0)$

37.


Vertex: $(-4, -5)$
Axis of symmetry: $x = -4$
x-intercepts: $(-4 \pm \sqrt{5}, 0)$

39.


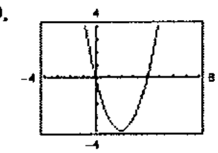
Vertex: $(4, -1)$
Axis of symmetry: $x = 4$
x-intercepts: $\left(4 \pm \frac{1}{2}\sqrt{2}, 0\right)$

41.


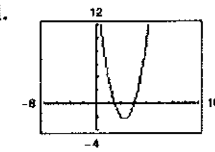
Vertex: $(-2, -3)$
Axis of symmetry: $x = -2$
x-intercepts: $(-2 \pm \sqrt{6}, 0)$

43. $y = -(x + 1)^2 + 4$ **45.** $y = -2(x + 2)^2 + 2$
47. $f(x) = (x + 2)^2 + 5$ **49.** $f(x) = 4(x - 1)^2 - 2$

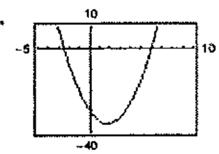
51. $f(x) = \frac{3}{4}(x - 5)^2 + 12$ **53.** $f(x) = -\frac{24}{49}\left(x + \frac{1}{4}\right)^2 + \frac{3}{2}$
55. $f(x) = -\frac{16}{3}\left(x + \frac{5}{2}\right)^2$ **57.** $(5, 0), (-1, 0)$

59.


$(0, 0), (4, 0)$

61.


$(3, 0), (6, 0)$

63.


$\left(-\frac{5}{2}, 0\right), (6, 0)$

65. $f(x) = x^2 - 2x - 3$
$g(x) = -x^2 + 2x + 3$

67. $f(x) = x^2 - 10x$
$g(x) = -x^2 + 10x$

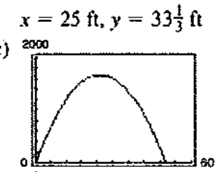
69. $f(x) = 2x^2 + 7x + 3$
$g(x) = -2x^2 - 7x - 3$

71. 55, 55 **73.** 12, 6 **75.** 16 ft **77.** 20 fixtures

79. (a) \$14,000,000; \$14,375,000; \$13,500,000
(b) \$24; \$14,400,000
Answers will vary.

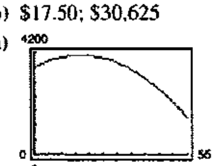
81. (a) $A = \dfrac{8x(50 - x)}{3}$
(b)

x	5	10	15	20	25	30
a	600	$1066\frac{2}{3}$	1400	1600	$1666\frac{2}{3}$	1600

$x = 25$ ft, $y = 33\frac{1}{3}$ ft
(c)


$x = 25$ ft, $y = 33\frac{1}{3}$ ft
(d) $A = -\frac{8}{3}(x - 25)^2 + \frac{5000}{3}$ (e) They are identical.

83. (a) $R = -100x^2 + 3500x,\ 15 \le x \le 20$
(b) \$17.50; \$30,625

85. (a)


(b) 4075 cigarettes; Yes, the warning had an effect because the maximum consumption occurred in 1966.
(c) 7366 cigarettes per year; 20 cigarettes per day

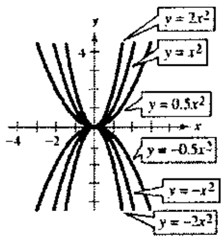
87. True. The equation has no real solutions, so the graph has no x-intercepts.

89. True. The graph of a quadratic function with a negative leading coefficient will be a downward-opening parabola.

91. $b = \pm 20$ **93.** $b = \pm 8$

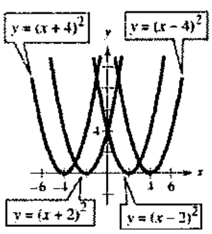
95. $f(x) = a\left(x + \dfrac{b}{2a}\right)^2 + \dfrac{4ac - b^2}{4a}$

97. (a)



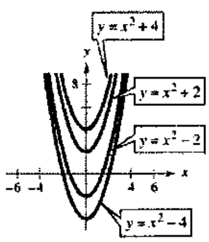
As $|a|$ increases, the parabola becomes narrower. For $a > 0$, the parabola opens upward. For $a < 0$, the parabola opens downward.

(b)



For $h < 0$, the vertex will be on the negative x-axis. For $h > 0$, the vertex will be on the positive x-axis. As $|h|$ increases, the parabola moves away from the origin.

(c)



As $|k|$ increases, the vertex moves upward (for $k > 0$) or downward (for $k < 0$), away from the origin.

99. Yes. A graph of a quadratic equation whose vertex is on the x-axis has only one x-intercept.

Section 3.2 *(page 279)*

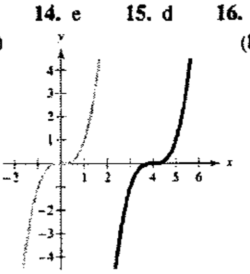
1. continuous **3.** x^n

5. (a) solution; (b) $(x - a)$; (c) x-intercept **7.** standard

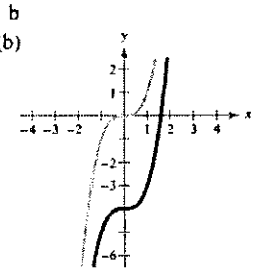
9. c **10.** g **11.** h **12.** f

13. a **14.** e **15.** d **16.** b

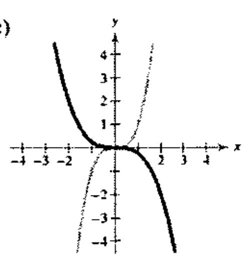
17. (a)



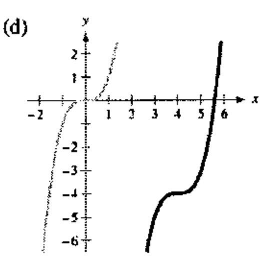
(b)



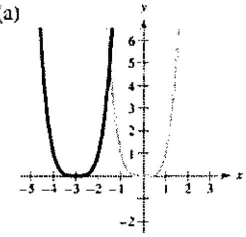
(c)



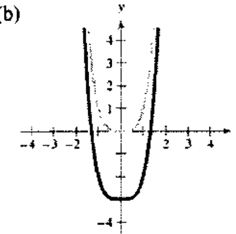
(d)



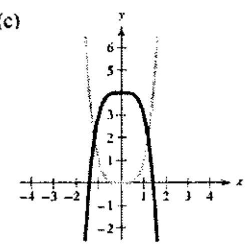
19. (a)



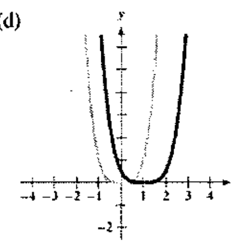
(b)



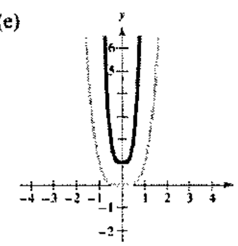
(c)



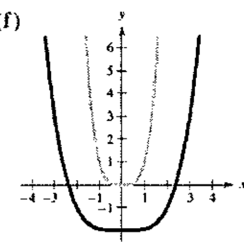
(d)



(e)



(f)



21. Falls to the left, rises to the right

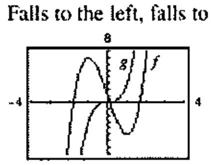
23. Falls to the left, falls to the right

25. Rises to the left, falls to the right

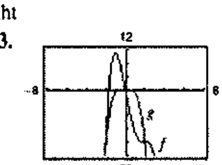
27. Rises to the left, falls to the right

29. Falls to the left, falls to the right

31.



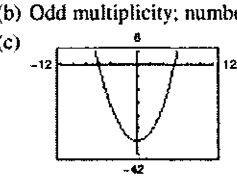
33.



35. (a) ± 6

(b) Odd multiplicity; number of turning points: 1

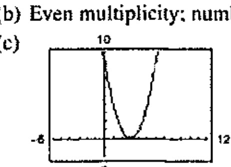
(c)

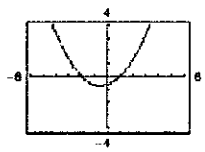


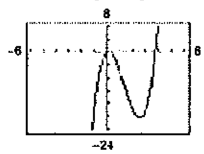
37. (a) 3

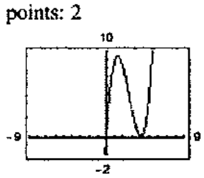
(b) Even multiplicity; number of turning points: 1

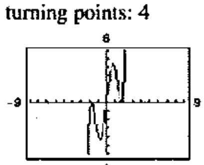
(c)

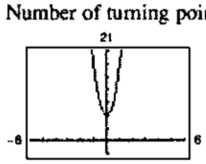


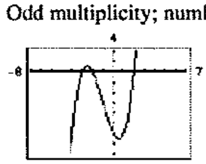
39. (a) $-2, 1$
(b) Odd multiplicity; number of turning points: 1
(c)


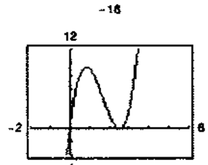
41. (a) $0, 2 \pm \sqrt{3}$
(b) Odd multiplicity; number of turning points: 2
(c)


43. (a) 0, 4
(b) 0, odd multiplicity; 4, even multiplicity; number of turning points: 2
(c)


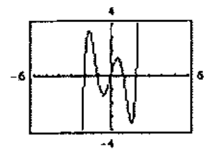
45. (a) $0, \pm\sqrt{3}$
(b) 0, odd multiplicity; $\pm\sqrt{3}$, even multiplicity; number of turning points: 4
(c)


47. (a) No real zeros
(b) Number of turning points: 1
(c)


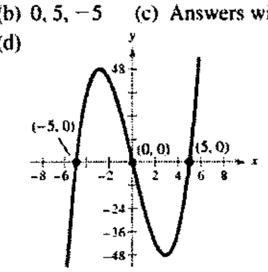
49. (a) $\pm 2, -3$
(b) Odd multiplicity; number of turning points: 2
(c)


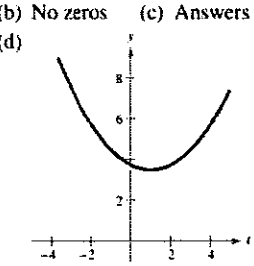
51. (a)


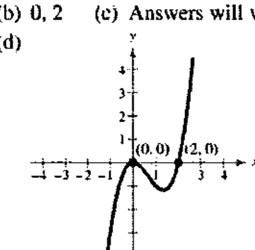
(b) x-intercepts: $(0, 0), (\frac{5}{2}, 0)$ (c) $x = 0, \frac{5}{2}$
(d) The answers in part (c) match the x-intercepts.

53. (a)


(b) x-intercepts: $(0, 0), (\pm 1, 0), (\pm 2, 0)$
(c) $x = 0, 1, -1, 2, -2$
(d) The answers in part (c) match the x-intercepts.

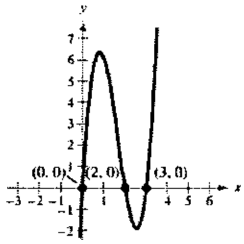
55. $f(x) = x^2 - 8x$ **57.** $f(x) = x^2 + 4x - 12$
59. $f(x) = x^3 + 9x^2 + 20x$
61. $f(x) = x^4 - 4x^3 - 9x^2 + 36x$ **63.** $f(x) = x^2 - 2x - 2$
65. $f(x) = x^2 + 6x + 9$ **67.** $f(x) = x^3 + 4x^2 - 5x$
69. $f(x) = x^3 - 3x$
71. $f(x) = x^4 + x^3 - 15x^2 + 23x - 10$
73. $f(x) = x^5 + 16x^4 + 96x^3 + 256x^2 + 256x$
75. (a) Falls to the left, rises to the right
(b) $0, 5, -5$ (c) Answers will vary.
(d)


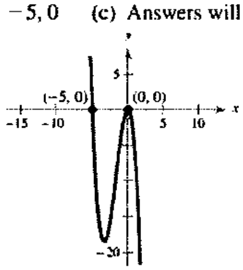
77. (a) Rises to the left, rises to the right
(b) No zeros (c) Answers will vary.
(d)


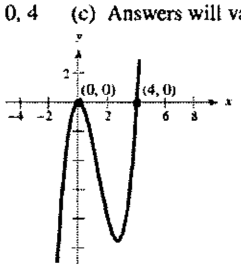
79. (a) Falls to the left, rises to the right
(b) 0, 2 (c) Answers will vary.
(d)


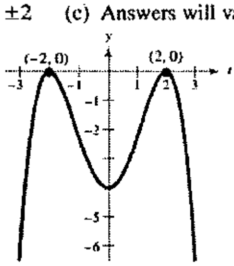
81. (a) Falls to the left, rises to the right
(b) 0, 2, 3 (c) Answers will vary.

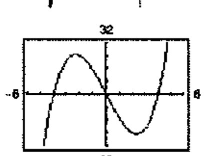
CHAPTER 3

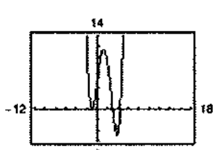
(d)


83. (a) Rises to the left, falls to the right
(b) $-5, 0$ (c) Answers will vary.
(d)


85. (a) Falls to the left, rises to the right
(b) 0, 4 (c) Answers will vary.
(d)


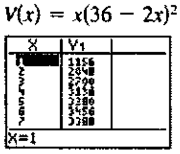
87. (a) Falls to the left, falls to the right
(b) ± 2 (c) Answers will vary.
(d)


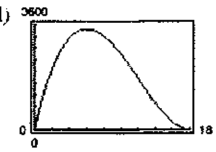
89.

Zeros: $0, \pm 4$,
odd multiplicity

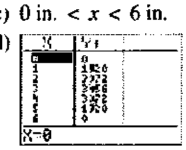
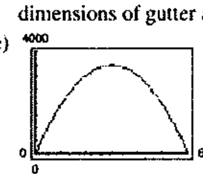
91.

Zeros: -1,
even multiplicity;
$3, \frac{9}{2}$, odd multiplicity

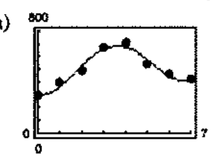
93. $[-1, 0], [1, 2], [2, 3]$; about $-0.879, 1.347, 2.532$

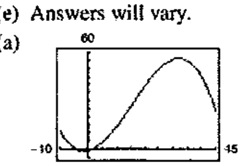
95. $[-2, -1], [0, 1]$; about $-1.585, 0.779$

97. (a) $V(x) = x(36 - 2x)^2$ (b) Domain: $0 < x < 18$
(c)

6 in. × 24 in. × 24 in.

(d)

$x = 6$; The results are the same.

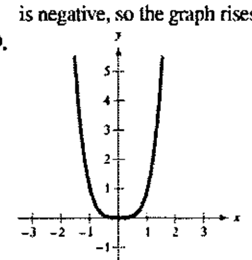
99. (a) $A = -2x^2 + 12x$ (b) $V = -384x^2 + 2304x$
(c) 0 in. $< x <$ 6 in.
(d)

When $x = 3$, the volume is maximum at $V = 3456$; dimensions of gutter are 3 in. × 6 in. × 3 in.
(e)

The maximum value is the same.
(f) No. Answers will vary.

101. (a)

(b) The model fits the data well.
(c) Relative minima: (0.21, 300.54), (6.62, 410.74)
Relative maximum: (3.62, 681.72)
(d) Increasing: (0.21, 3.62), (6.62, 7)
Decreasing: (0, 0.21), (3.62, 6.62)
(e) Answers will vary.

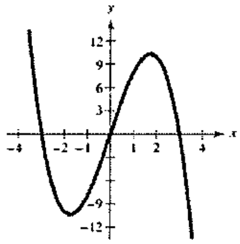
103. (a)

(b) $t \approx 15$
(c) Vertex: (15.22, 2.54)
(d) The results are approximately equal.

105. False. A fifth-degree polynomial can have at most four turning points.

107. True. The degree of the function is odd and its leading coefficient is negative, so the graph rises to the left and falls to the right.

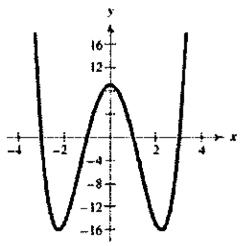
109.

(a) Vertical shift of two units; Even
(b) Horizontal shift of two units; Neither
(c) Reflection in the y-axis; Even
(d) Reflection in the x-axis; Even
(e) Horizontal stretch; Even
(f) Vertical shrink; Even
(g) $g(x) = x^3, x \geq 0$; Neither
(h) $g(x) = x^{16}$; Even

111. (a)



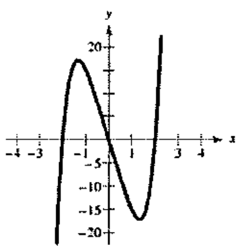
Zeros: 3
Relative minimum: 1
Relative maximum: 1
The number of zeros is the same as the degree, and the number of extrema is one less than the degree.

(b)



Zeros: 4
Relative minima: 2
Relative maximum: 1
The number of zeros is the same as the degree, and the number of extrema is one less than the degree.

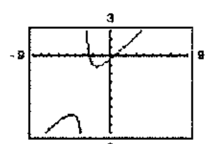
(c)



Zeros: 3
Relative minimum: 1
Relative maximum: 1
The number of zeros and the number of extrema are both less than the degree.

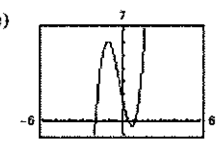
Section 3.3 *(page 290)*

1. $f(x)$: dividend; $d(x)$: divisor; $q(x)$: quotient; $r(x)$: remainder
3. improper **5.** Factor **7.** Answers will vary.
9. (a) and (b)

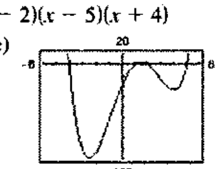


(c) Answers will vary.

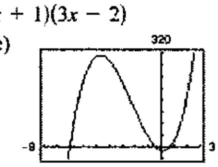
11. $2x + 4,\ x \neq -3$ **13.** $x^2 - 3x + 1,\ x \neq -\frac{5}{4}$
15. $x^3 + 3x^2 - 1,\ x \neq -2$ **17.** $x^2 + 3x + 9,\ x \neq 3$
19. $7 - \dfrac{11}{x+2}$ **21.** $x - \dfrac{x+9}{x^2+1}$ **23.** $2x - 8 + \dfrac{x-1}{x^2+1}$
25. $x + 3 + \dfrac{6x^2 - 8x + 3}{(x-1)^3}$ **27.** $3x^2 - 2x + 5,\ x \neq 5$
29. $6x^2 + 25x + 74 + \dfrac{248}{x-3}$ **31.** $4x^2 - 9,\ x \neq -2$
33. $-x^2 + 10x - 25,\ x \neq -10$
35. $5x^2 + 14x + 56 + \dfrac{232}{x-4}$
37. $10x^3 + 10x^2 + 60x + 360 + \dfrac{1360}{x-6}$
39. $x^2 - 8x + 64,\ x \neq -8$
41. $-3x^3 - 6x^2 - 12x - 24 - \dfrac{48}{x-2}$
43. $-x^3 - 6x^2 - 36x - 36 - \dfrac{216}{x-6}$
45. $4x^2 + 14x - 30,\ x \neq -\frac{1}{2}$
47. $f(x) = (x-4)(x^2+3x-2) + 3,\ f(4) = 3$
49. $f(x) = \left(x + \frac{2}{3}\right)(15x^3 - 6x + 4) + \frac{34}{3},\ f\left(-\frac{2}{3}\right) = \frac{34}{3}$
51. $f(x) = \left(x - \sqrt{2}\right)\left[x^2 + \left(3 + \sqrt{2}\right)x + 3\sqrt{2}\right] - 8,$
$f\left(\sqrt{2}\right) = -8$
53. $f(x) = \left(x - 1 + \sqrt{3}\right)\left[-4x^2 + \left(2 + 4\sqrt{3}\right)x + \left(2 + 2\sqrt{3}\right)\right],$
$f\left(1 - \sqrt{3}\right) = 0$
55. (a) -2 (b) 1 (c) $-\frac{1}{4}$ (d) 5
57. (a) -35 (b) -22 (c) -10 (d) -211
59. $(x-2)(x+3)(x-1)$; Solutions: $2, -3, 1$
61. $(2x-1)(x-5)(x-2)$; Solutions: $\frac{1}{2}, 5, 2$
63. $\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)(x+2)$; Solutions: $-\sqrt{3}, \sqrt{3}, -2$
65. $(x-1)\left(x - 1 - \sqrt{3}\right)\left(x - 1 + \sqrt{3}\right)$;
Solutions: $1,\ 1 + \sqrt{3},\ 1 - \sqrt{3}$
67. (a) Answers will vary. (b) $2x - 1$
(c) $f(x) = (2x-1)(x+2)(x-1)$
(d) $\frac{1}{2}, -2, 1$ (e)



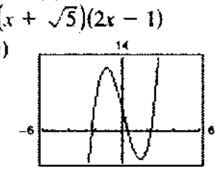
69. (a) Answers will vary. (b) $(x-1), (x-2)$
(c) $f(x) = (x-1)(x-2)(x-5)(x+4)$
(d) $1, 2, 5, -4$ (e)



71. (a) Answers will vary. (b) $x + 7$
(c) $f(x) = (x+7)(2x+1)(3x-2)$
(d) $-7, -\frac{1}{2}, \frac{2}{3}$ (e)



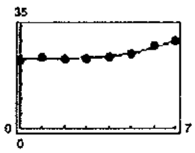
73. (a) Answers will vary. (b) $x - \sqrt{5}$
(c) $f(x) = \left(x - \sqrt{5}\right)\left(x + \sqrt{5}\right)(2x-1)$
(d) $\pm\sqrt{5}, \frac{1}{2}$ (e)



75. (a) Zeros are 2 and about ± 2.236.
(b) $x = 2$ (c) $f(x) = (x-2)\left(x - \sqrt{5}\right)\left(x + \sqrt{5}\right)$
77. (a) Zeros are -2, about 0.268, and about 3.732.
(b) $t = -2$
(c) $h(t) = (t+2)\left[t - \left(2 + \sqrt{3}\right)\right]\left[t - \left(2 - \sqrt{3}\right)\right]$
79. (a) Zeros are 0, 3, 4, and about ± 1.414.
(b) $x = 0$ (c) $h(x) = x(x-4)(x-3)\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)$

81. $2x^2 - x - 1,\ x \neq \frac{3}{2}$ **83.** $x^2 + 3x,\ x \neq -2, -1$

85. (a) and (b)



$A = 0.0349t^3 - 0.168t^2 + 0.42t + 23.4$

(c)

t	0	1	2	3
$A(t)$	23.4	23.7	23.8	24.1

t	4	5	6	7
$A(t)$	24.6	25.7	27.4	30.1

(d) $45.7 billion; No, because the model will approach infinity quickly.

87. False. $-\frac{4}{7}$ is a zero of f.

89. True. The degree of the numerator is greater than the degree of the denominator.

91. $x^{2n} + 6x^n + 9,\ x^n \neq -3$ **93.** The remainder is 0.

95. $c = -210$ **97.** $k = 7$

99. (a) $x + 1,\ x \neq 1$ (b) $x^2 + x + 1,\ x \neq 1$
(c) $x^3 + x^2 + x + 1,\ x \neq 1$
In general, $\dfrac{x^n - 1}{x - 1} = x^{n-1} + x^{n-2} + \cdots + x + 1,\ x \neq 1$

Section 3.4 *(page 303)*

1. Fundamental Theorem of Algebra **3.** Rational Zero

5. linear; quadratic; quadratic **7.** Descartes's Rule of Signs

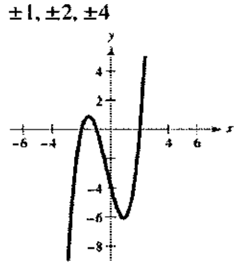
9. 0, 6 **11.** 2, −4 **13.** $-6, \pm i$ **15.** $\pm 1, \pm 2$

17. $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{9}{2}, \pm\frac{15}{2}, \pm\frac{45}{2}$

19. 1, 2, 3 **21.** 1, −1, 4 **23.** −6, −1

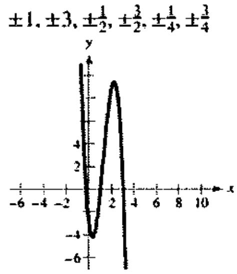
25. $\frac{1}{2}, -1$ **27.** $-2, 3, \pm\frac{2}{3}$ **29.** −2, 1 **31.** $-4, \frac{1}{2}, 1, 1$

33. (a) $\pm 1, \pm 2, \pm 4$
(b)



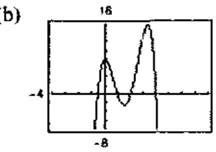
(c) −2, −1, 2

35. (a) $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$
(b)



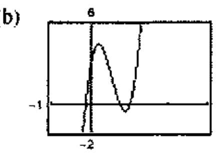
(c) $-\frac{1}{4}, 1, 3$

37. (a) $\pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{2}$
(b)



(c) $-\frac{1}{2}, 1, 2, 4$

39. (a) $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{1}{16}, \pm\frac{3}{16}, \pm\frac{1}{32}, \pm\frac{3}{32}$
(b)



(c) $1, \frac{3}{4}, -\frac{1}{8}$

41. (a) ±1, about ±1.414 (b) $\pm 1, \pm\sqrt{2}$
(c) $f(x) = (x + 1)(x - 1)(x + \sqrt{2})(x - \sqrt{2})$

43. (a) 0, 3, 4, about ±1.414 (b) $0, 3, 4, \pm\sqrt{2}$
(c) $h(x) = x(x - 3)(x - 4)(x + \sqrt{2})(x - \sqrt{2})$

45. $x^3 - x^2 + 25x - 25$ **47.** $x^3 - 12x^2 + 46x - 52$

49. $3x^4 - 17x^3 + 25x^2 + 23x - 22$

51. (a) $(x^2 + 9)(x^2 - 3)$ (b) $(x^2 + 9)(x + \sqrt{3})(x - \sqrt{3})$
(c) $(x + 3i)(x - 3i)(x + \sqrt{3})(x - \sqrt{3})$

53. (a) $(x^2 - 2x - 2)(x^2 - 2x + 3)$
(b) $(x - 1 + \sqrt{3})(x - 1 - \sqrt{3})(x^2 - 2x + 3)$
(c) $(x - 1 + \sqrt{3})(x - 1 - \sqrt{3})(x - 1 + \sqrt{2}i)$
$(x - 1 - \sqrt{2}i)$

55. $\pm 2i, 1$ **57.** $\pm 5i, -\frac{1}{2}, 1$ **59.** $-3 \pm i, \frac{1}{4}$

61. $2, -3 \pm \sqrt{2}i, 1$ **63.** $\pm 6i;\ (x + 6i)(x - 6i)$

65. $1 \pm 4i;\ (x - 1 - 4i)(x - 1 + 4i)$

67. $\pm 2, \pm 2i;\ (x - 2)(x + 2)(x - 2i)(x + 2i)$

69. $1 \pm i;\ (z - 1 + i)(z - 1 - i)$

71. $-1, 2 \pm i;\ (x + 1)(x - 2 + i)(x - 2 - i)$

73. $-2, 1 \pm \sqrt{2}i;\ (x + 2)(x - 1 + \sqrt{2}i)(x - 1 - \sqrt{2}i)$

75. $-\frac{1}{5}, 1 \pm \sqrt{5}i;\ (5x + 1)(x - 1 + \sqrt{5}i)(x - 1 - \sqrt{5}i)$

77. $2, \pm 2i;\ (x - 2)^2(x + 2i)(x - 2i)$

79. $\pm i, \pm 3i;\ (x + i)(x - i)(x + 3i)(x - 3i)$

81. $-10, -7 \pm 5i$ **83.** $-\frac{3}{4}, 1 \pm \frac{1}{2}i$ **85.** $-2, -\frac{1}{2}, \pm i$

87. One positive zero **89.** One negative zero

91. One positive zero, one negative zero

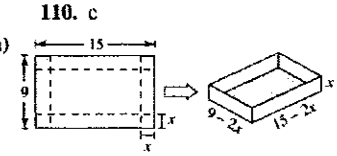
93. One or three positive zeros

95–97. Answers will vary. **99.** $1, -\frac{1}{2}$ **101.** $-\frac{3}{4}$

103. $\pm 2, \pm\frac{3}{2}$ **105.** $\pm 1, \frac{1}{4}$ **107.** d **108.** a

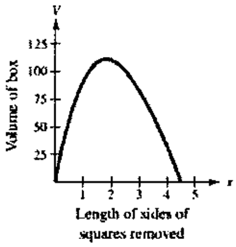
109. b **110.** c

111. (a)



(b) $V(x) = x(9 - 2x)(15 - 2x)$
Domain: $0 < x < \frac{9}{2}$

(c)



1.82 cm × 5.36 cm × 11.36 cm

(d) $\frac{1}{2}, \frac{7}{2}, 8$; 8 is not in the domain of V.

113. $x \approx 38.4$, or \$384,000

115. (a) $V(x) = x^3 + 9x^2 + 26x + 24 = 120$

(b) 4 ft × 5 ft × 6 ft

117. $x \approx 40$, or 4000 units

119. No. Setting $p = 9{,}000{,}000$ and solving the resulting equation yields imaginary roots.

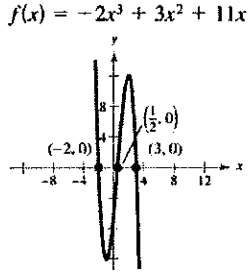
121. False. The most complex zeros it can have is two, and the Linear Factorization Theorem guarantees that there are three linear factors, so one zero must be real.

123. r_1, r_2, r_3 **125.** $5 + r_1, 5 + r_2, 5 + r_3$

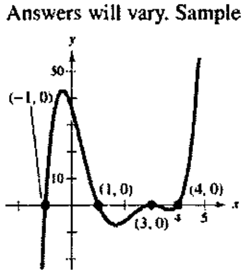
127. The zeros cannot be determined.

129. Answers will vary. There are infinitely many possible functions for f. Sample equation and graph:

$f(x) = -2x^3 + 3x^2 + 11x - 6$



131. Answers will vary. Sample graph:



133. $f(x) = x^4 + 5x^2 + 4$ **135.** $f(x) = x^3 - 3x^2 + 4x - 2$

137. (a) $-2, 1, 4$

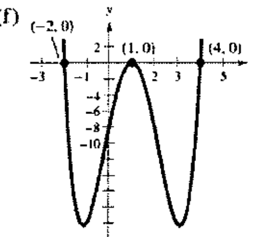
(b) The graph touches the x-axis at $x = 1$.

(c) The least possible degree of the function is 4, because there are at least four real zeros (1 is repeated) and a function can have at most the number of real zeros equal to the degree of the function. The degree cannot be odd by the definition of multiplicity.

(d) Positive. From the information in the table, it can be concluded that the graph will eventually rise to the left and rise to the right.

(e) $f(x) = x^4 - 4x^3 - 3x^2 + 14x - 8$

(f)



139. (a) Not correct because f has (0, 0) as an intercept.

(b) Not correct because the function must be at least a fourth-degree polynomial.

(c) Correct function

(d) Not correct because k has $(-1, 0)$ as an intercept.

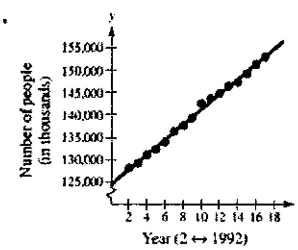
Section 3.5 *(page 314)*

1. variation; regression **3.** least squares regression

5. directly proportional **7.** directly proportional

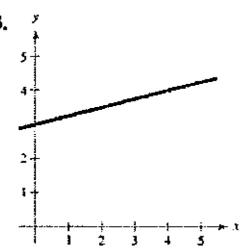
9. combined

11.



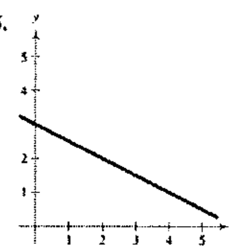
The model is a good fit for the actual data.

13.



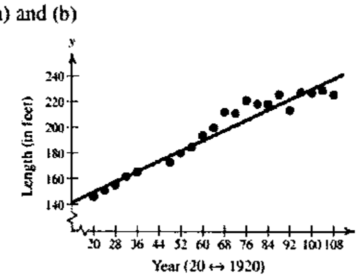
$y = \frac{1}{4}x + 3$

15.



$y = -\frac{1}{2}x + 3$

17. (a) and (b)



$y \approx t + 130$

(c) $y = 1.01t + 130.82$ (d) The models are similar.

(e) Part (b): 242 ft; Part (c): 243.94 ft

(f) Answers will vary.

CHAPTER 3