6.4 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

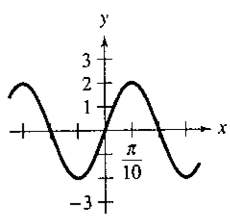
VOCABULARY: Fill in the blanks.

1. One period of a sine or cosine function is called one ________ of the sine or cosine curve.
2. The ________ of a sine or cosine curve represents half the distance between the maximum and minimum values of the function.
3. For the function given by $y = a\sin(bx - c)$, $\frac{c}{b}$ represents the ________ ________ of the graph of the function.
4. For the function given by $y = d + a\cos(bx - c)$, d represents a ________ ________ of the graph of the function.

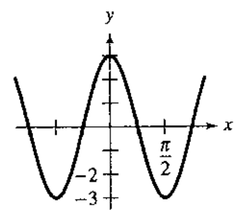
SKILLS AND APPLICATIONS

In Exercises 5–18, find the period and amplitude.

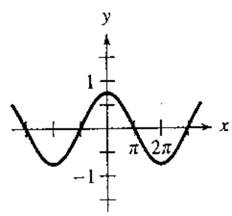
5. $y = 2\sin 5x$



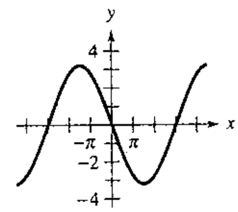
6. $y = 3\cos 2x$



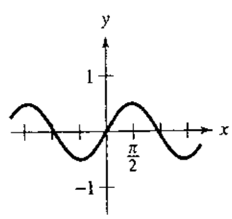
7. $y = \frac{3}{4}\cos\frac{x}{2}$



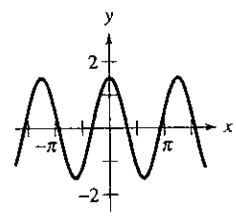
8. $y = -3\sin\frac{x}{3}$



9. $y = \frac{1}{2}\sin\frac{\pi x}{3}$



10. $y = \frac{3}{2}\cos\frac{\pi x}{2}$



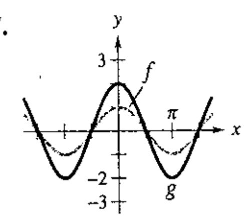
11. $y = -4\sin x$
12. $y = -\cos\frac{2x}{3}$
13. $y = 3\sin 10x$
14. $y = \frac{1}{5}\sin 6x$
15. $y = \frac{5}{3}\cos\frac{4x}{5}$
16. $y = \frac{5}{2}\cos\frac{x}{4}$
17. $y = \frac{1}{4}\sin 2\pi x$
18. $y = \frac{2}{3}\cos\frac{\pi x}{10}$

In Exercises 19–26, describe the relationship between the graphs of f and g. Consider amplitude, period, and shifts.

19. $f(x) = \sin x$
    $g(x) = \sin(x - \pi)$
20. $f(x) = \cos x$
    $g(x) = \cos(x + \pi)$
21. $f(x) = \cos 2x$
    $g(x) = -\cos 2x$
22. $f(x) = \sin 3x$
    $g(x) = \sin(-3x)$
23. $f(x) = \cos x$
    $g(x) = \cos 2x$
24. $f(x) = \sin x$
    $g(x) = \sin 3x$
25. $f(x) = \sin 2x$
    $g(x) = 3 + \sin 2x$
26. $f(x) = \cos 4x$
    $g(x) = -2 + \cos 4x$

In Exercises 27–30, describe the relationship between the graphs of f and g. Consider amplitude, period, and shifts.

27.



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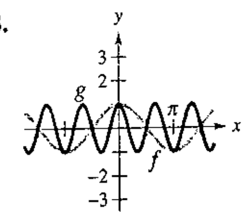

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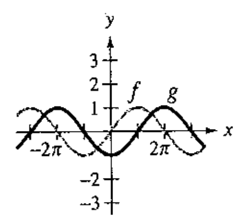

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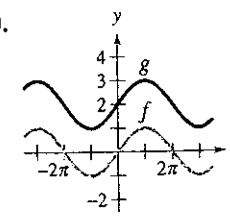


In Exercises 31–38, graph f and g on the same set (coordinate axes. (Include two full periods.)

31. $f(x) = -2\sin x$
    $g(x) = 4\sin x$
32. $f(x) = \sin x$
    $g(x) = \sin\frac{x}{3}$
33. $f(x) = \cos x$
    $g(x) = 2 + \cos x$
34. $f(x) = 2\cos 2x$
    $g(x) = -\cos 4x$

35. $f(x) = -\frac{1}{2}\sin\frac{x}{2}$

$g(x) = 3 - \frac{1}{2}\sin\frac{x}{2}$

36. $f(x) = 4\sin \pi x$

$g(x) = 4\sin \pi x - 3$

37. $f(x) = 2\cos x$

$g(x) = 2\cos(x - \pi)$

38. $f(x) = -\cos x$

$g(x) = -\cos(x - \pi)$

In Exercises 39–60, sketch the graph of the function. (Include two full periods.)

39. $y = 5\sin x$

40. $y = \frac{1}{4}\sin x$

41. $y = \frac{1}{3}\cos x$

42. $y = 4\cos x$

43. $y = \cos\frac{x}{2}$

44. $y = \sin 4x$

45. $y = \cos 2\pi x$

46. $y = \sin\frac{\pi x}{4}$

47. $y = -\sin\frac{2\pi x}{3}$

48. $y = -10\cos\frac{\pi x}{6}$

49. $y = \sin\left(x - \frac{\pi}{2}\right)$

50. $y = \sin(x - 2\pi)$

51. $y = 3\cos(x - \pi)$

52. $y = 4\cos\left(x + \frac{\pi}{4}\right)$

53. $y = 2 - \sin\frac{2\pi x}{3}$

54. $y = -3 + 5\cos\frac{\pi t}{12}$

55. $y = 2 - \frac{1}{10}\cos 60\pi x$

56. $y = 2\cos x - 3$

57. $y = 3\cos(x - \pi) - 3$

58. $y = 4\cos\left(x - \frac{\pi}{4}\right) - 4$

59. $y = \frac{2}{3}\cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$

60. $y = -3\cos(6x + \pi)$

In Exercises 61–66, g is related to a parent function $f(x) = \sin(x)$ or $f(x) = \cos(x)$. (a) Describe the sequence of transformations from f to g. (b) Sketch the graph of g. (c) Use function notation to write g in terms of f.

61. $g(x) = \sin(4x - \pi)$

62. $g(x) = \sin(2x - \pi)$

63. $g(x) = \cos(x - \pi) + 2$

64. $g(x) = 1 - \cos(x - \pi)$

65. $g(x) = 2\sin(4x - \pi) - 3$

66. $g(x) = 4 - \sin(2x - \pi)$

In Exercises 67–72, use a graphing utility to graph the function. Include two full periods. Be sure to choose an appropriate viewing window.

67. $y = -2\sin(4x + \pi)$

68. $y = -4\sin\left(\frac{2}{3}x - \frac{\pi}{3}\right)$

69. $y = \cos\left(2\pi x - \frac{\pi}{2}\right) + 1$

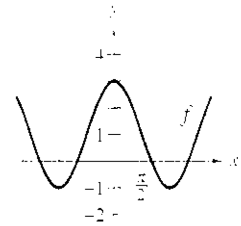
70. $y = 3\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) - 2$

71. $y = -0.1\sin\left(\frac{\pi x}{10} + \pi\right)$

72. $y = \frac{1}{100}\sin 120\pi t$

GRAPHICAL REASONING In Exercises 73–76, find a and d for the function $f(x) = a\cos x + d$ such that the graph of f matches the figure.

73.



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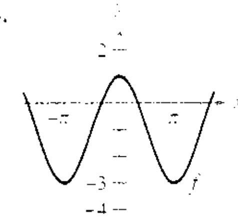

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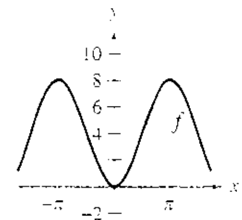

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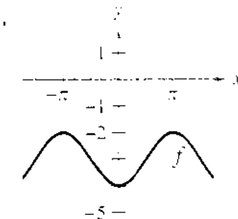

GRAPHICAL REASONING In Exercises 77–80, find a, b, and c for the function $f(x) = a\sin(bx - c)$ such that the graph of f matches the figure.

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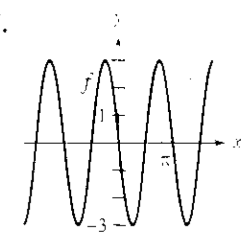

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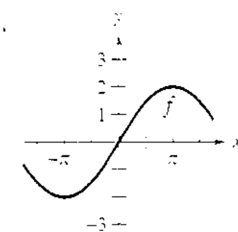

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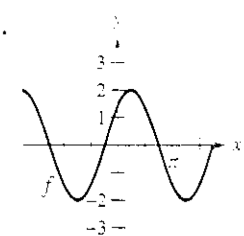
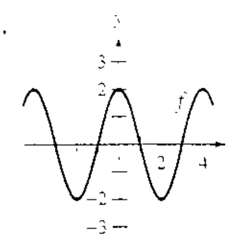


80.



In Exercises 81 and 82, use a graphing utility to graph y_1 and y_2 in the interval $[-2\pi, 2\pi]$. Use the graphs to find real numbers x such that $y_1 = y_2$.

81. $y_1 = \sin x$

$y_2 = -\frac{1}{2}$

82. $y_1 = \cos x$

$y_2 = -1$

In Exercises 83–86, write an equation for the function that is described by the given characteristics.

83. A sine curve with a period of π, an amplitude of 2, a right phase shift of $\pi/2$, and a vertical translation up 1 unit

84. A sine curve with a period of 4π, an amplitude of 3, a left phase shift of $\pi/4$, and a vertical translation down 1 unit

85. A cosine curve with a period of π, an amplitude of 1, a left phase shift of π, and a vertical translation down $\frac{3}{2}$ units

86. A cosine curve with a period of 4π, an amplitude of 3, a right phase shift of $\pi/2$, and a vertical translation up 2 units

87. RESPIRATORY CYCLE For a person at rest, the velocity v (in liters per second) of airflow during a respiratory cycle (the time from the beginning of one breath to the beginning of the next) is given by $v = 0.85 \sin \frac{\pi t}{3}$, where t is the time (in seconds). (Inhalation occurs when $v > 0$, and exhalation occurs when $v < 0$.)

(a) Find the time for one full respiratory cycle.

(b) Find the number of cycles per minute.

(c) Sketch the graph of the velocity function.

88. RESPIRATORY CYCLE After exercising for a few minutes, a person has a respiratory cycle for which the velocity of airflow is approximated by $v = 1.75 \sin \frac{\pi t}{2}$, where t is the time (in seconds). (Inhalation occurs when $v > 0$, and exhalation occurs when $v < 0$.)

(a) Find the time for one full respiratory cycle.

(b) Find the number of cycles per minute.

(c) Sketch the graph of the velocity function.

89. DATA ANALYSIS: METEOROLOGY The table shows the maximum daily high temperatures in Las Vegas L and International Falls I (in degrees Fahrenheit) for month t, with $t = 1$ corresponding to January. (Source: National Climatic Data Center)

Month, t	Las Vegas, L	International Falls, I
1	57.1	13.8
2	63.0	22.4
3	69.5	34.9
4	78.1	51.5
5	87.8	66.6
6	98.9	74.2
7	104.1	78.6
8	101.8	76.3
9	93.8	64.7
10	80.8	51.7
11	66.0	32.5
12	57.3	18.1

(a) A model for the temperature in Las Vegas is given by

$$L(t) = 80.60 + 23.50 \cos\left(\frac{\pi t}{6} - 3.67\right).$$

Find a trigonometric model for International Falls.

(b) Use a graphing utility to graph the data points and the model for the temperatures in Las Vegas. How well does the model fit the data?

(c) Use a graphing utility to graph the data points and the model for the temperatures in International Falls. How well does the model fit the data?

(d) Use the models to estimate the average maximum temperature in each city. Which term of the models did you use? Explain.

(e) What is the period of each model? Are the periods what you expected? Explain.

(f) Which city has the greater variability in temperature throughout the year? Which factor of the models determines this variability? Explain.

90. HEALTH The function given by

$$P = 100 - 20 \cos \frac{5\pi t}{3}$$

approximates the blood pressure P (in millimeters of mercury) at time t (in seconds) for a person at rest.

(a) Find the period of the function.

(b) Find the number of heartbeats per minute.

91. PIANO TUNING When tuning a piano, a technician strikes a tuning fork for the A above middle C and sets up a wave motion that can be approximated by $y = 0.001 \sin 880\pi t$, where t is the time (in seconds).

(a) What is the period of the function?

(b) The frequency f is given by $f = 1/p$. What is the frequency of the note?

92. DATA ANALYSIS: ASTRONOMY The percents y (in decimal form) of the moon's face that was illuminated on day x in the year 2009, where $x = 1$ represents January 1, are shown in the table. (Source: U.S. Naval Observatory)

x	y
4	0.5
11	1.0
18	0.5
26	0.0
33	0.5
40	1.0

6.5 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

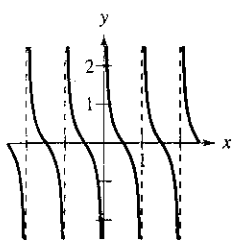
VOCABULARY: Fill in the blanks.

1. The tangent, cotangent, and cosecant functions are ________ , so the graphs of these functions have symmetry with respect to the ________.
2. The graphs of the tangent, cotangent, secant, and cosecant functions all have ________ asymptotes.
3. To sketch the graph of a secant or cosecant function, first make a sketch of its corresponding ________ function.
4. For the functions given by $f(x) = g(x) \cdot \sin x$, $g(x)$ is called the ________ factor of the function $f(x)$.
5. The period of $y = \tan x$ is ________.
6. The domain of $y = \cot x$ is all real numbers such that ________.
7. The range of $y = \sec x$ is ________.
8. The period of $y = \csc x$ is ________.

SKILLS AND APPLICATIONS

In Exercises 9–14, match the function with its graph. State the period of the function. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

(a)



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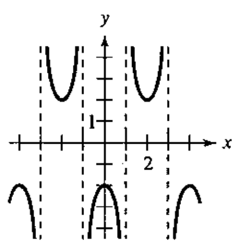

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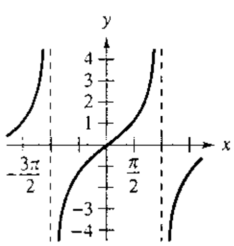

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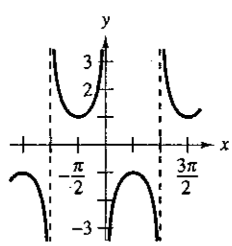

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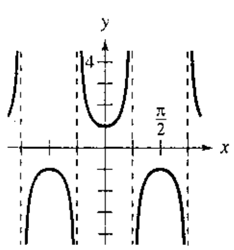

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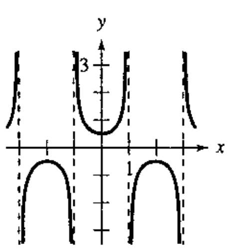


9. $y = \sec 2x$
10. $y = \tan \dfrac{x}{2}$
11. $y = \dfrac{1}{2} \cot \pi x$
12. $y = -\csc x$
13. $y = \dfrac{1}{2} \sec \dfrac{\pi x}{2}$
14. $y = -2 \sec \dfrac{\pi x}{2}$

In Exercises 15–38, sketch the graph of the function. Include two full periods.

15. $y = \dfrac{1}{3} \tan x$
16. $y = \tan 4x$
17. $y = -2 \tan 3x$
18. $y = -3 \tan \pi x$
19. $y = -\frac{1}{2} \sec x$
20. $y = \frac{1}{4} \sec x$
21. $y = \csc \pi x$
22. $y = 3 \csc 4x$
23. $y = \frac{1}{2} \sec \pi x$
24. $y = -2 \sec 4x + 2$
25. $y = \csc \dfrac{x}{2}$
26. $y = \csc \dfrac{x}{3}$
27. $y = 3 \cot 2x$
28. $y = 3 \cot \dfrac{\pi x}{2}$
29. $y = 2 \sec 3x$
30. $y = -\frac{1}{2} \tan x$
31. $y = \tan \dfrac{\pi x}{4}$
32. $y = \tan(x + \pi)$
33. $y = 2 \csc(x - \pi)$
34. $y = \csc(2x - \pi)$
35. $y = 2 \sec(x + \pi)$
36. $y = -\sec \pi x + 1$
37. $y = \dfrac{1}{4} \csc\left(x + \dfrac{\pi}{4}\right)$
38. $y = 2 \cot\left(x + \dfrac{\pi}{2}\right)$

In Exercises 39–48, use a graphing utility to graph the function. Include two full periods.

39. $y = \tan \dfrac{x}{3}$
40. $y = -\tan 2x$
41. $y = -2 \sec 4x$
42. $y = \sec \pi x$
43. $y = \tan\left(x - \dfrac{\pi}{4}\right)$
44. $y = \dfrac{1}{4} \cot\left(x - \dfrac{\pi}{2}\right)$
45. $y = -\csc(4x - \pi)$
46. $y = 2 \sec(2x - \pi)$
47. $y = 0.1 \tan\left(\dfrac{\pi x}{4} + \dfrac{\pi}{4}\right)$
48. $y = \dfrac{1}{3} \sec\left(\dfrac{\pi x}{2} + \dfrac{\pi}{2}\right)$

In Exercises 49–56, use a graph to solve the equation on the interval $[-2\pi, 2\pi]$.

49. $\tan x = 1$ **50.** $\tan x = \sqrt{3}$

51. $\cot x = -\dfrac{\sqrt{3}}{3}$ **52.** $\cot x = 1$

53. $\sec x = -2$ **54.** $\sec x = 2$

55. $\csc x = \sqrt{2}$ **56.** $\csc x = -\dfrac{2\sqrt{3}}{3}$

In Exercises 57–64, use the graph of the function to determine whether the function is even, odd, or neither. Verify your answer algebraically.

57. $f(x) = \sec x$ **58.** $f(x) = \tan x$

59. $g(x) = \cot x$ **60.** $g(x) = \csc x$

61. $f(x) = x + \tan x$ **62.** $f(x) = x^2 - \sec x$

63. $g(x) = x \csc x$ **64.** $g(x) = x^2 \cot x$

65. GRAPHICAL REASONING Consider the functions given by

$$f(x) = 2 \sin x \quad \text{and} \quad g(x) = \frac{1}{2} \csc x$$

on the interval $(0, \pi)$.

(a) Graph f and g in the same coordinate plane.

(b) Approximate the interval in which $f > g$.

(c) Describe the behavior of each of the functions as x approaches π. How is the behavior of g related to the behavior of f as x approaches π?

66. GRAPHICAL REASONING Consider the functions given by

$$f(x) = \tan \frac{\pi x}{2} \quad \text{and} \quad g(x) = \frac{1}{2} \sec \frac{\pi x}{2}$$

on the interval $(-1, 1)$.

(a) Use a graphing utility to graph f and g in the same viewing window.

(b) Approximate the interval in which $f < g$.

(c) Approximate the interval in which $2f < 2g$. How does the result compare with that of part (b)? Explain.

In Exercises 67–72, use a graphing utility to graph the two equations in the same viewing window. Use the graphs to determine whether the expressions are equivalent. Verify the results algebraically.

67. $y_1 = \sin x \csc x, \quad y_2 = 1$

68. $y_1 = \sin x \sec x, \quad y_2 = \tan x$

69. $y_1 = \dfrac{\cos x}{\sin x}, \quad y_2 = \cot x$

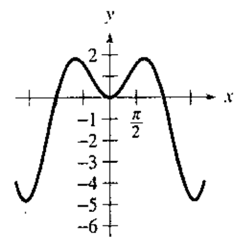
70. $y_1 = \tan x \cot^2 x, \quad y_2 = \cot x$

71. $y_1 = 1 + \cot^2 x, \quad y_2 = \csc^2 x$

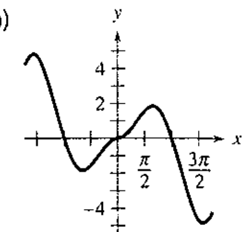
72. $y_1 = \sec^2 x - 1, \quad y_2 = \tan^2 x$

In Exercises 73–76, match the function with its grap
Describe the behavior of the function as x approaches zer
[The graphs are labeled (a), (b), (c), and (d).]

(a)



(b)



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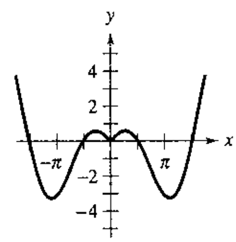
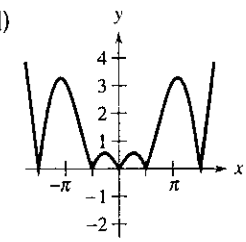


(d)



73. $f(x) = |x \cos x|$ **74.** $f(x) = x \sin x$

75. $g(x) = |x| \sin x$ **76.** $g(x) = |x| \cos x$

CONJECTURE In Exercises 77–80, graph the function and g. Use the graphs to make a conjecture about t relationship between the functions.

77. $f(x) = \sin x + \cos\left(x + \dfrac{\pi}{2}\right), \quad g(x) = 0$

78. $f(x) = \sin x - \cos\left(x + \dfrac{\pi}{2}\right), \quad g(x) = 2 \sin x$

79. $f(x) = \sin^2 x, \quad g(x) = \frac{1}{2}(1 - \cos 2x)$

80. $f(x) = \cos^2 \dfrac{\pi x}{2}, \quad g(x) = \dfrac{1}{2}(1 + \cos \pi x)$

In Exercises 81–84, use a graphing utility to graph t function and the damping factor of the function in the sa viewing window. Describe the behavior of the function a increases without bound.

81. $g(x) = e^{-x^2/2} \sin x$ **82.** $f(x) = e^{-x} \cos x$

83. $f(x) = 2^{-x/4} \cos \pi x$ **84.** $h(x) = 2^{-x^2/4} \sin x$

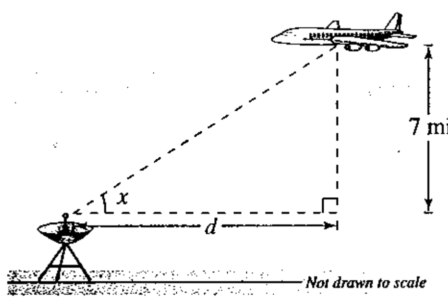
In Exercises 85–90, use a graphing utility to graph t function. Describe the behavior of the function as approaches zero.

85. $y = \dfrac{6}{x} + \cos x, \quad x > 0$ **86.** $y = \dfrac{4}{x} + \sin 2x, \quad x >$

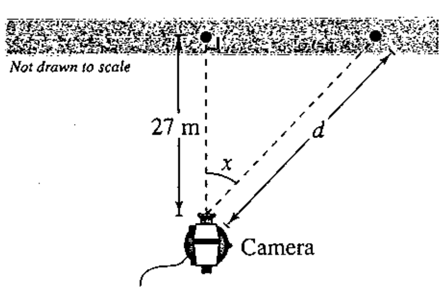
87. $g(x) = \dfrac{\sin x}{x}$ **88.** $f(x) = \dfrac{1 - \cos x}{x}$

89. $f(x) = \sin \dfrac{1}{x}$ **90.** $h(x) = x \sin \dfrac{1}{x}$

91. DISTANCE A plane flying at an altitude of 7 miles above a radar antenna will pass directly over the radar antenna (see figure). Let d be the ground distance from the antenna to the point directly under the plane and let x be the angle of elevation to the plane from the antenna. (d is positive as the plane approaches the antenna.) Write d as a function of x and graph the function over the interval $0 < x < \pi$.



92. TELEVISION COVERAGE A television camera is on a reviewing platform 27 meters from the street on which a parade will be passing from left to right (see figure). Write the distance d from the camera to a particular unit in the parade as a function of the angle x, and graph the function over the interval $-\pi/2 < x < \pi/2$. (Consider x as negative when a unit in the parade approaches from the left.)



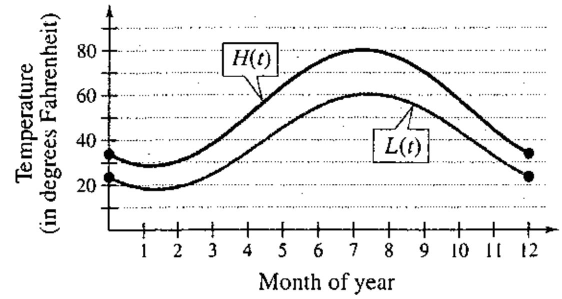
93. METEOROLOGY The normal monthly high temperatures H (in degrees Fahrenheit) in Erie, Pennsylvania are approximated by

$$H(t) = 56.94 - 20.86 \cos(\pi t/6) - 11.58 \sin(\pi t/6)$$

and the normal monthly low temperatures L are approximated by

$$L(t) = 41.80 - 17.13 \cos(\pi t/6) - 13.39 \sin(\pi t/6)$$

where t is the time (in months), with $t = 1$ corresponding to January (see figure). (Source: National Climatic Data Center)



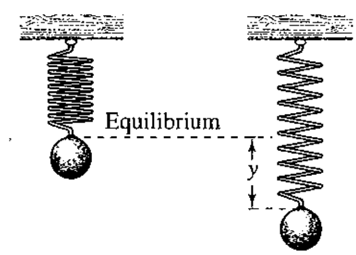
(a) What is the period of each function?

(b) During what part of the year is the difference between the normal high and normal low temperatures greatest? When is it smallest?

(c) The sun is northernmost in the sky around June 21, but the graph shows the warmest temperatures at a later date. Approximate the lag time of the temperatures relative to the position of the sun.

94. SALES The projected monthly sales S (in thousands of units) of lawn mowers (a seasonal product) are modeled by $S = 74 + 3t - 40 \cos(\pi t/6)$, where t is the time (in months), with $t = 1$ corresponding to January. Graph the sales function over 1 year.

95. HARMONIC MOTION An object weighing W pounds is suspended from the ceiling by a steel spring (see figure). The weight is pulled downward (positive direction) from its equilibrium position and released. The resulting motion of the weight is described by the function $y = \frac{1}{2}e^{-t/4} \cos 4t$, $t > 0$, where y is the distance (in feet) and t is the time (in seconds).



(a) Use a graphing utility to graph the function.

(b) Describe the behavior of the displacement function for increasing values of time t.

EXPLORATION

TRUE OR FALSE? In Exercises 96 and 97, determine whether the statement is true or false. Justify your answer.

96. The graph of $y = \csc x$ can be obtained on a calculator by graphing the reciprocal of $y = \sin x$.

97. The graph of $y = \sec x$ can be obtained on a calculator by graphing a translation of the reciprocal of $y = \sin x$.

6.6 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

	Function	*Alternative Notation*	*Domain*	*Range*
1.	$y = \arcsin x$	________	________	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
2.	________	$y = \cos^{-1} x$	$-1 \le x \le 1$	________
3.	$y = \arctan x$	________	________	________

4. Without restrictions, no trigonometric function has an ________ function.

SKILLS AND APPLICATIONS

In Exercises 5–20, evaluate the expression without using a calculator.

5. $\arcsin \frac{1}{2}$
6. $\arcsin 0$
7. $\arccos \frac{1}{2}$
8. $\arccos 0$
9. $\arctan \frac{\sqrt{3}}{3}$
10. $\arctan(1)$
11. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
12. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
13. $\arctan(-\sqrt{3})$
14. $\arctan \sqrt{3}$
15. $\arccos\left(-\frac{1}{2}\right)$
16. $\arcsin \frac{\sqrt{2}}{2}$
17. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
18. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$
19. $\tan^{-1} 0$
20. $\cos^{-1} 1$

In Exercises 21 and 22, use a graphing utility to graph f, g, and $y = x$ in the same viewing window to verify geometrically that g is the inverse function of f. (Be sure to restrict the domain of f properly.)

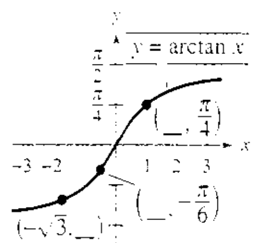
21. $f(x) = \sin x, \quad g(x) = \arcsin x$
22. $f(x) = \tan x, \quad g(x) = \arctan x$

In Exercises 23–40, use a calculator to evaluate the expression. Round your result to two decimal places.

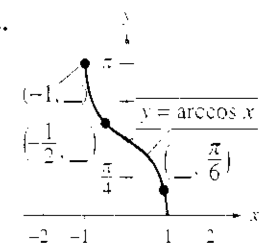
23. $\arccos 0.37$
24. $\arcsin 0.65$
25. $\arcsin(-0.75)$
26. $\arccos(-0.7)$
27. $\arctan(-3)$
28. $\arctan 25$
29. $\sin^{-1} 0.31$
30. $\cos^{-1} 0.26$
31. $\arccos(-0.41)$
32. $\arcsin(-0.125)$
33. $\arctan 0.92$
34. $\arctan 2.8$
35. $\arcsin \frac{7}{8}$
36. $\arccos\left(-\frac{1}{3}\right)$
37. $\tan^{-1} \frac{19}{4}$
38. $\tan^{-1}\left(-\frac{95}{7}\right)$
39. $\tan^{-1}(-\sqrt{372})$
40. $\tan^{-1}(-\sqrt{2165})$

In Exercises 41 and 42, determine the missing coordinates of the points on the graph of the function.

41.

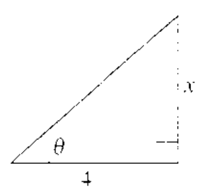


42.



In Exercises 43–48, use an inverse trigonometric function to write θ as a function of x.

43.



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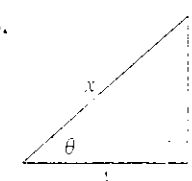

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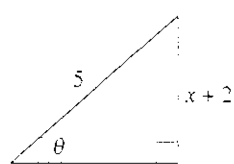

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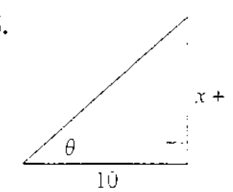

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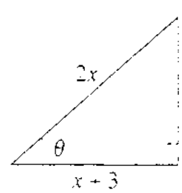
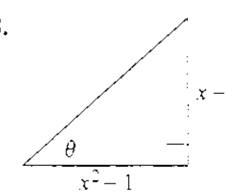


48.



In Exercises 49–54, use the properties of inverse trigonometric functions to evaluate the expression.

49. $\sin(\arcsin 0.3)$
50. $\tan(\arctan 45)$
51. $\cos[\arccos(-0.1)]$
52. $\sin[\arcsin(-0.2)]$
53. $\arcsin(\sin 3\pi)$
54. $\arccos\left(\cos \frac{7\pi}{2}\right)$

In Exercises 55–66, find the exact value of the expression. (*Hint:* Sketch a right triangle.)

55. $\sin\left(\arctan \frac{3}{4}\right)$
56. $\sec\left(\arcsin \frac{4}{5}\right)$
57. $\cos(\tan^{-1} 2)$
58. $\sin\left(\cos^{-1} \frac{\sqrt{5}}{5}\right)$
59. $\cos\left(\arcsin \frac{5}{13}\right)$
60. $\csc\left[\arctan\left(-\frac{5}{12}\right)\right]$
61. $\sec\left[\arctan\left(-\frac{3}{5}\right)\right]$
62. $\tan\left[\arcsin\left(-\frac{3}{4}\right)\right]$
63. $\sin\left[\arccos\left(-\frac{2}{3}\right)\right]$
64. $\cot\left(\arctan \frac{5}{8}\right)$
65. $\csc\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$
66. $\sec\left[\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$

In Exercises 67–76, write an algebraic expression that is equivalent to the expression. (*Hint:* Sketch a right triangle, as demonstrated in Example 7.)

67. $\cot(\arctan x)$
68. $\sin(\arctan x)$
69. $\cos(\arcsin 2x)$
70. $\sec(\arctan 3x)$
71. $\sin(\arccos x)$
72. $\sec[\arcsin(x - 1)]$
73. $\tan\left(\arccos \frac{x}{3}\right)$
74. $\cot\left(\arctan \frac{1}{x}\right)$
75. $\csc\left(\arctan \frac{x}{\sqrt{2}}\right)$
76. $\cos\left(\arcsin \frac{x - h}{r}\right)$

In Exercises 77 and 78, use a graphing utility to graph f and g in the same viewing window to verify that the two functions are equal. Explain why they are equal. Identify any asymptotes of the graphs.

77. $f(x) = \sin(\arctan 2x), \quad g(x) = \dfrac{2x}{\sqrt{1 + 4x^2}}$
78. $f(x) = \tan\left(\arccos \dfrac{x}{2}\right), \quad g(x) = \dfrac{\sqrt{4 - x^2}}{x}$

In Exercises 79–82, fill in the blank.

79. $\arctan \dfrac{9}{x} = \arcsin(\quad\quad), \quad x \neq 0$
80. $\arcsin \dfrac{\sqrt{36 - x^2}}{6} = \arccos(\quad\quad), \quad 0 \leq x \leq 6$
81. $\arccos \dfrac{3}{\sqrt{x^2 - 2x + 10}} = \arcsin(\quad\quad)$
82. $\arccos \dfrac{x - 2}{2} = \arctan(\quad\quad), \quad |x - 2| \leq 2$

In Exercises 83 and 84, sketch a graph of the functic compare the graph of g with the graph of $f(x) =$ arcsi

83. $g(x) = \arcsin(x - 1)$
84. $g(x) = \arcsin \dfrac{x}{2}$

In Exercises 85–90, sketch a graph of the function.

85. $y = 2 \arccos x$
86. $g(t) = \arccos(t + 2)$
87. $f(x) = \arctan 2x$
88. $f(x) = \dfrac{\pi}{2} + \arctan x$
89. $h(v) = \tan(\arccos v)$
90. $f(x) = \arccos \dfrac{x}{4}$

In Exercises 91–96, use a graphing utility to gra function.

91. $f(x) = 2 \arccos(2x)$
92. $f(x) = \pi \arcsin(4x)$
93. $f(x) = \arctan(2x - 3)$
94. $f(x) = -3 + \arctan(\pi x)$
95. $f(x) = \pi - \sin^{-1}\left(\dfrac{2}{3}\right)$
96. $f(x) = \dfrac{\pi}{2} + \cos^{-1}\left(\dfrac{1}{\pi}\right)$

In Exercises 97 and 98, write the function in terms of tl function by using the identity

$$A \cos \omega t + B \sin \omega t = \sqrt{A^2 + B^2} \sin\left(\omega t + \arctan \frac{A}{B}\right)$$

Use a graphing utility to graph both forms of the fu What does the graph imply?

97. $f(t) = 3 \cos 2t + 3 \sin 2t$
98. $f(t) = 4 \cos \pi t + 3 \sin \pi t$

In Exercises 99–104, fill in the blank. If not p state the reason. (*Note:* The notation $x \to c^+$ indicat x approaches c from the right and $x \to c^-$ indicates approaches c from the left.)

99. As $x \to 1^-$, the value of $\arcsin x \to$ ______.
100. As $x \to 1^-$, the value of $\arccos x \to$ ______.

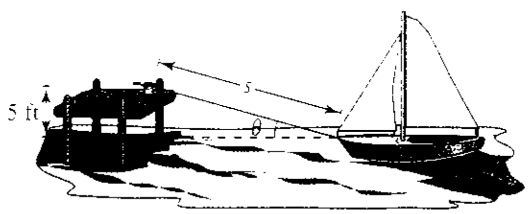
101. As $x \to \infty$, the value of $\arctan x \to$ ______.

102. As $x \to -1^{+}$, the value of $\arcsin x \to$ ______.

103. As $x \to -1^{+}$, the value of $\arccos x \to$ ______.

104. As $x \to -\infty$, the value of $\arctan x \to$ ______.

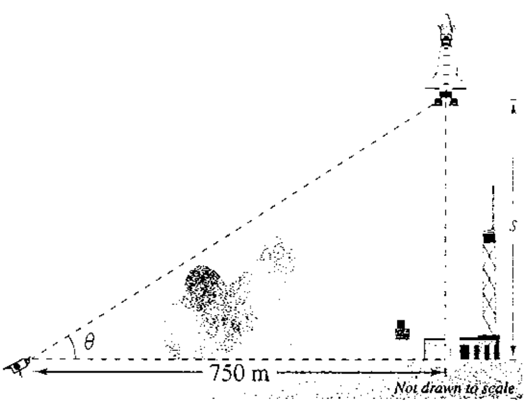
105. **DOCKING A BOAT** A boat is pulled in by means of a winch located on a dock 5 feet above the deck of the boat (see figure). Let θ be the angle of elevation from the boat to the winch and let s be the length of the rope from the winch to the boat.



(a) Write θ as a function of s.

(b) Find θ when $s = 40$ feet and $s = 20$ feet.

106. **PHOTOGRAPHY** A television camera at ground level is filming the lift-off of a space shuttle at a point 750 meters from the launch pad (see figure). Let θ be the angle of elevation to the shuttle and let s be the height of the shuttle.

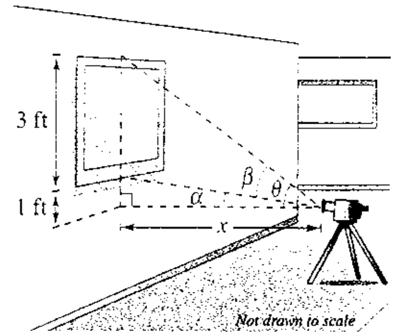


(a) Write θ as a function of s.

(b) Find θ when $s = 300$ meters and $s = 1200$ meters.

107. **PHOTOGRAPHY** A photographer is taking a picture of a three-foot-tall painting hung in an art gallery. The camera lens is 1 foot below the lower edge of the painting (see figure). The angle β subtended by the camera lens x feet from the painting is

$$\beta = \arctan \frac{3x}{x^2 + 4}, \quad x > 0.$$

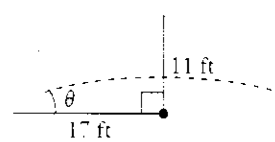


(a) Use a graphing utility to graph β as a function of x.

(b) Move the cursor along the graph to approximate the distance from the picture when β is maximum.

(c) Identify the asymptote of the graph and discuss its meaning in the context of the problem.

108. **GRANULAR ANGLE OF REPOSE** Different types of granular substances naturally settle at different angles when stored in cone-shaped piles. This angle θ is called the *angle of repose* (see figure). When rock salt is stored in a cone-shaped pile 11 feet high, the diameter of the pile's base is about 34 feet.



(a) Find the angle of repose for rock salt.

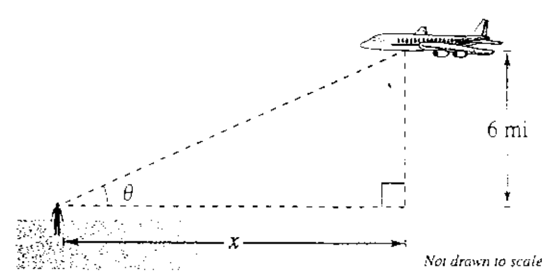
(b) How tall is a pile of rock salt that has a base diameter of 40 feet?

109. **GRANULAR ANGLE OF REPOSE** When whole corn is stored in a cone-shaped pile 20 feet high, the diameter of the pile's base is about 82 feet.

(a) Find the angle of repose for whole corn.

(b) How tall is a pile of corn that has a base diameter of 100 feet?

110. **ANGLE OF ELEVATION** An airplane flies at an altitude of 6 miles toward a point directly over an observer. Consider θ and x as shown in the figure.



(a) Write θ as a function of x.

(b) Find θ when $x = 7$ miles and $x = 1$ mile.

7.1 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blank to complete the trigonometric identity.

1. $\dfrac{\sin u}{\cos u} =$ ________
2. $\dfrac{1}{\csc u} =$ ________
3. $\dfrac{1}{\tan u} =$ ________
4. $\dfrac{1}{\cos u} =$ ________
5. $1 +$ ________ $= \csc^2 u$
6. $1 + \tan^2 u =$ ________
7. $\sin\left(\dfrac{\pi}{2} - u\right) =$ ________
8. $\sec\left(\dfrac{\pi}{2} - u\right) =$ ________
9. $\cos(-u) =$ ________
10. $\tan(-u) =$ ________

SKILLS AND APPLICATIONS

In Exercises 11–24, use the given values to evaluate (if possible) all six trigonometric functions.

11. $\sin x = \dfrac{1}{2}, \quad \cos x = \dfrac{\sqrt{3}}{2}$
12. $\tan x = \dfrac{\sqrt{3}}{3}, \quad \cos x = -\dfrac{\sqrt{3}}{2}$
13. $\sec \theta = \sqrt{2}, \quad \sin \theta = -\dfrac{\sqrt{2}}{2}$
14. $\csc \theta = \frac{25}{7}, \quad \tan \theta = \frac{7}{24}$
15. $\tan x = \frac{8}{15}, \quad \sec x = -\frac{17}{15}$
16. $\cot \phi = -3, \quad \sin \phi = \dfrac{\sqrt{10}}{10}$
17. $\sec \phi = \dfrac{3}{2}, \quad \csc \phi = -\dfrac{3\sqrt{5}}{5}$
18. $\cos\left(\dfrac{\pi}{2} - x\right) = \dfrac{3}{5}, \quad \cos x = \dfrac{4}{5}$
19. $\sin(-x) = -\dfrac{1}{3}, \quad \tan x = -\dfrac{\sqrt{2}}{4}$
20. $\sec x = 4, \quad \sin x > 0$
21. $\tan \theta = 2, \quad \sin \theta < 0$
22. $\csc \theta = -5, \quad \cos \theta < 0$
23. $\sin \theta = -1, \quad \cot \theta = 0$
24. $\tan \theta$ is undefined, $\quad \sin \theta > 0$

In Exercises 25–30, match the trigonometric expression with one of the following.

(a) $\sec x$ (b) -1 (c) $\cot x$
(d) 1 (e) $-\tan x$ (f) $\sin x$

25. $\sec x \cos x$
26. $\tan x \csc x$
27. $\cot^2 x - \csc^2 x$
28. $(1 - \cos^2 x)(\csc x)$
29. $\dfrac{\sin(-x)}{\cos(-x)}$
30. $\dfrac{\sin[(\pi/2) - x]}{\cos[(\pi/2) - x]}$

In Exercises 31–36, match the trigonometric expression with one of the following.

(a) $\csc x$ (b) $\tan x$ (c) $\sin^2 x$
(d) $\sin x \tan x$ (e) $\sec^2 x$ (f) $\sec^2 x + \tan^2 x$

31. $\sin x \sec x$
32. $\cos^2 x(\sec^2 x - 1)$
33. $\sec^4 x - \tan^4 x$
34. $\cot x \sec x$
35. $\dfrac{\sec^2 x - 1}{\sin^2 x}$
36. $\dfrac{\cos^2[(\pi/2) - x]}{\cos x}$

In Exercises 37–58, use the fundamental identities to simplify the expression. There is more than one correct form of each answer.

37. $\cot \theta \sec \theta$
38. $\cos \beta \tan \beta$
39. $\tan(-x) \cos x$
40. $\sin x \cot(-x)$
41. $\sin \phi(\csc \phi - \sin \phi)$
42. $\sec^2 x(1 - \sin^2 x)$
43. $\dfrac{\cot x}{\csc x}$
44. $\dfrac{\csc \theta}{\sec \theta}$
45. $\dfrac{1 - \sin^2 x}{\csc^2 x - 1}$
46. $\dfrac{1}{\tan^2 x + 1}$
47. $\dfrac{\tan \theta \cot \theta}{\sec \theta}$
48. $\dfrac{\sin \theta \csc \theta}{\tan \theta}$
49. $\sec \alpha \cdot \dfrac{\sin \alpha}{\tan \alpha}$
50. $\dfrac{\tan^2 \theta}{\sec^2 \theta}$
51. $\cos\left(\dfrac{\pi}{2} - x\right) \sec x$
52. $\cot\left(\dfrac{\pi}{2} - x\right) \cos x$
53. $\dfrac{\cos^2 y}{1 - \sin y}$
54. $\cos t(1 + \tan^2 t)$
55. $\sin \beta \tan \beta + \cos \beta$
56. $\csc \phi \tan \phi + \sec \phi$
57. $\cot u \sin u + \tan u \cos u$
58. $\sin \theta \sec \theta + \cos \theta \csc \theta$

In Exercises 59–70, factor the expression and use the fundamental identities to simplify. There is more than one correct form of each answer.

59. $\tan^2 x - \tan^2 x \sin^2 x$
60. $\sin^2 x \csc^2 x - \sin^2 x$
61. $\sin^2 x \sec^2 x - \sin^2 x$
62. $\cos^2 x + \cos^2 x \tan^2 x$
63. $\dfrac{\sec^2 x - 1}{\sec x - 1}$
64. $\dfrac{\cos^2 x - 4}{\cos x - 2}$
65. $\tan^4 x + 2\tan^2 x + 1$
66. $1 - 2\cos^2 x + \cos^4 x$
67. $\sin^4 x - \cos^4 x$
68. $\sec^4 x - \tan^4 x$
69. $\csc^3 x - \csc^2 x - \csc x + 1$
70. $\sec^3 x - \sec^2 x - \sec x + 1$

In Exercises 71–74, perform the multiplication and use the fundamental identities to simplify. There is more than one correct form of each answer.

71. $(\sin x + \cos x)^2$
72. $(\cot x + \csc x)(\cot x - \csc x)$
73. $(2\csc x + 2)(2\csc x - 2)$
74. $(3 - 3\sin x)(3 + 3\sin x)$

In Exercises 75–80, perform the addition or subtraction and use the fundamental identities to simplify. There is more than one correct form of each answer.

75. $\dfrac{1}{1 + \cos x} + \dfrac{1}{1 - \cos x}$
76. $\dfrac{1}{\sec x + 1} - \dfrac{1}{\sec x - 1}$
77. $\dfrac{\cos x}{1 + \sin x} + \dfrac{1 + \sin x}{\cos x}$
78. $\dfrac{\tan x}{1 + \sec x} + \dfrac{1 + \sec x}{\tan x}$
79. $\tan x + \dfrac{\cos x}{1 + \sin x}$
80. $\tan x - \dfrac{\sec^2 x}{\tan x}$

In Exercises 81–84, rewrite the expression so that it is not in fractional form. There is more than one correct form of each answer.

81. $\dfrac{\sin^2 y}{1 - \cos y}$
82. $\dfrac{5}{\tan x + \sec x}$
83. $\dfrac{3}{\sec x - \tan x}$
84. $\dfrac{\tan^2 x}{\csc x + 1}$

NUMERICAL AND GRAPHICAL ANALYSIS In Exercises 85–88, use a graphing utility to complete the table and graph the functions. Make a conjecture about y_1 and y_2.

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1							
y_2							

85. $y_1 = \cos\left(\dfrac{\pi}{2} - x\right), \quad y_2 = \sin x$
86. $y_1 = \sec x - \cos x, \quad y_2 = \sin x \tan x$
87. $y_1 = \dfrac{\cos x}{1 - \sin x}, \quad y_2 = \dfrac{1 + \sin x}{\cos x}$
88. $y_1 = \sec^4 x - \sec^2 x, \quad y_2 = \tan^2 x + \tan^4 x$

In Exercises 89–92, use a graphing utility to determine whic of the six trigonometric functions is equal to the expressior Verify your answer algebraically.

89. $\cos x \cot x + \sin x$
90. $\sec x \csc x - \tan x$
91. $\dfrac{1}{\sin x}\left(\dfrac{1}{\cos x} - \cos x\right)$
92. $\dfrac{1}{2}\left(\dfrac{1 + \sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1 + \sin\theta}\right)$

In Exercises 93–104, use the trigonometric substitution t write the algebraic expression as a trigonometric function c θ, where $0 < \theta < \pi/2$.

93. $\sqrt{9 - x^2}, \quad x = 3\cos\theta$
94. $\sqrt{64 - 16x^2}, \quad x = 2\cos\theta$
95. $\sqrt{16 - x^2}, \quad x = 4\sin\theta$
96. $\sqrt{49 - x^2}, \quad x = 7\sin\theta$
97. $\sqrt{x^2 - 9}, \quad x = 3\sec\theta$
98. $\sqrt{x^2 - 4}, \quad x = 2\sec\theta$
99. $\sqrt{x^2 + 25}, \quad x = 5\tan\theta$
100. $\sqrt{x^2 + 100}, \quad x = 10\tan\theta$
101. $\sqrt{4x^2 + 9}, \quad 2x = 3\tan\theta$
102. $\sqrt{9x^2 + 25}, \quad 3x = 5\tan\theta$
103. $\sqrt{2 - x^2}, \quad x = \sqrt{2}\sin\theta$
104. $\sqrt{10 - x^2}, \quad x = \sqrt{10}\sin\theta$

In Exercises 105–108, use the trigonometric substitution t write the algebraic equation as a trigonometric equation of (where $-\pi/2 < \theta < \pi/2$. Then find $\sin\theta$ and $\cos\theta$.

105. $3 = \sqrt{9 - x^2}, \quad x = 3\sin\theta$
106. $3 = \sqrt{36 - x^2}, \quad x = 6\sin\theta$
107. $2\sqrt{2} = \sqrt{16 - 4x^2}, \quad x = 2\cos\theta$
108. $-5\sqrt{3} = \sqrt{100 - x^2}, \quad x = 10\cos\theta$

In Exercises 109–112, use a graphing utility to solve th equation for θ, where $0 \le \theta < 2\pi$.

109. $\sin\theta = \sqrt{1 - \cos^2\theta}$
110. $\cos\theta = -\sqrt{1 - \sin^2\theta}$
111. $\sec\theta = \sqrt{1 + \tan^2\theta}$
112. $\csc\theta = \sqrt{1 + \cot^2\theta}$

In Exercises 113–118, rewrite the expression as a single logarithm and simplify the result.

113. $\ln|\cos x| - \ln|\sin x|$ **114.** $\ln|\sec x| - \ln|\sin x|$

115. $\ln|\sin x| - \ln|\cot x|$ **116.** $\ln|\tan x| - \ln|\csc x|$

117. $\ln|\cot t| - \ln(1 - \tan^2 t)$

118. $\ln(\cos^2 t) - \ln(1 - \tan^2 t)$

In Exercises 119–122, use a calculator to demonstrate the identity for each value of θ.

119. $\csc^2 \theta - \cot^2 \theta = 1$

(a) $\theta = 132°$ (b) $\theta = \dfrac{2\pi}{7}$

120. $\tan^2 \theta + 1 = \sec^2 \theta$

(a) $\theta = 346°$ (b) $\theta = 3.1$

121. $\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin \theta$

(a) $\theta = 80°$ (b) $\theta = 0.8$

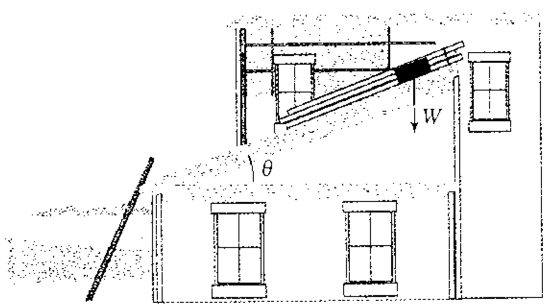
122. $\sin(-\theta) = -\sin \theta$

(a) $\theta = 250°$ (b) $\theta = \frac{1}{2}$

123. FRICTION The forces acting on an object weighing W units on an inclined plane positioned at an angle of θ with the horizontal (see figure) are modeled by

$$\mu W \cos \theta = W \sin \theta$$

where μ is the coefficient of friction. Solve the equation for μ and simplify the result.



124. RATE OF CHANGE The rate of change of the function $f(x) = -x + \tan x$ is given by the expression $-1 + \sec^2 x$. Show that this expression can also be written as $\tan^2 x$.

125. RATE OF CHANGE The rate of change of the function $f(x) = \sec x + \cos x$ is given by the expression $\sec x \tan x - \sin x$. Show that this expression can also be written as $\sin x \tan^2 x$.

126. RATE OF CHANGE The rate of change of the function $f(x) = -\csc x - \sin x$ is given by the expression $\csc x \cot x - \cos x$. Show that this expression can also be written as $\cos x \cot^2 x$.

EXPLORATION

TRUE OR FALSE? In Exercises 127 and 128, determine whether the statement is true or false. Justify your answer.

127. The even and odd trigonometric identities are helpful for determining whether the value of a trigonometric function is positive or negative.

128. A cofunction identity can be used to transform a tangent function so that it can be represented by a cosecant function.

In Exercises 129–132, fill in the blanks. (*Note:* The notation $x \to c^+$ indicates that x approaches c from the right and $x \to c^-$ indicates that x approaches c from the left.)

129. As $x \to \dfrac{\pi}{2}^-$, $\sin x \to$ ___ and $\csc x \to$ ___.

130. As $x \to 0^+$, $\cos x \to$ ___ and $\sec x \to$ ___.

131. As $x \to \dfrac{\pi}{2}^-$, $\tan x \to$ ___ and $\cot x \to$ ___.

132. As $x \to \pi^+$, $\sin x \to$ ___ and $\csc x \to$ ___.

In Exercises 133–138, determine whether or not the equation is an identity, and give a reason for your answer.

133. $\cos \theta = \sqrt{1 - \sin^2 \theta}$ **134.** $\cot \theta = \sqrt{\csc^2 \theta + 1}$

135. $\dfrac{(\sin k\theta)}{(\cos k\theta)} = \tan \theta$, k is a constant.

136. $\dfrac{1}{(5 \cos \theta)} = 5 \sec \theta$

137. $\sin \theta \csc \theta = 1$ **138.** $\csc^2 \theta = 1$

139. Use the trigonometric substitution $u = a \sin \theta$, where $-\pi/2 < \theta < \pi/2$ and $a > 0$, to simplify the expression $\sqrt{a^2 - u^2}$.

140. Use the trigonometric substitution $u = a \tan \theta$, where $-\pi/2 < \theta < \pi/2$ and $a > 0$, to simplify the expression $\sqrt{a^2 + u^2}$.

141. Use the trigonometric substitution $u = a \sec \theta$, where $0 < \theta < \pi/2$ and $a > 0$, to simplify the expression $\sqrt{u^2 - a^2}$.

142. CAPSTONE

(a) Use the definitions of sine and cosine to derive the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$.

(b) Use the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$ to derive the other Pythagorean identities, $1 + \tan^2 \theta = \sec^2 \theta$ and $1 + \cot^2 \theta = \csc^2 \theta$. Discuss how to remember these identities and other fundamental identities.

7.2 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY

In Exercises 1 and 2, fill in the blanks.

1. An equation that is true for all real values in its domain is called an ________.
2. An equation that is true for only some values in its domain is called a ________ ________.

In Exercises 3–8, fill in the blank to complete the trigonometric identity.

3. $\dfrac{1}{\cot u} =$ ________
4. $\dfrac{\cos u}{\sin u} =$ ________
5. $\sin^2 u +$ ________ $= 1$
6. $\cos\left(\dfrac{\pi}{2} - u\right) =$ ________
7. $\csc(-u) =$ ________
8. $\sec(-u) =$ ________

SKILLS AND APPLICATIONS

In Exercises 9–50, verify the identity.

9. $\tan t \cot t = 1$
10. $\sec y \cos y = 1$
11. $\cot^2 y(\sec^2 y - 1) = 1$
12. $\cos x + \sin x \tan x = \sec x$
13. $(1 + \sin \alpha)(1 - \sin \alpha) = \cos^2 \alpha$
14. $\cos^2 \beta - \sin^2 \beta = 2\cos^2 \beta - 1$
15. $\cos^2 \beta - \sin^2 \beta = 1 - 2\sin^2 \beta$
16. $\sin^2 \alpha - \sin^4 \alpha = \cos^2 \alpha - \cos^4 \alpha$
17. $\dfrac{\tan^2 \theta}{\sec \theta} = \sin \theta \tan \theta$
18. $\dfrac{\cot^3 t}{\csc t} = \cos t(\csc^2 t - 1)$
19. $\dfrac{\cot^2 t}{\csc t} = \dfrac{1 - \sin^2 t}{\sin t}$
20. $\dfrac{1}{\tan \beta} + \tan \beta = \dfrac{\sec^2 \beta}{\tan \beta}$
21. $\sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \cos^3 x \sqrt{\sin x}$
22. $\sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x) = \sec^5 x \tan^3 x$
23. $\dfrac{\cot x}{\sec x} = \csc x - \sin x$
24. $\dfrac{\sec \theta - 1}{1 - \cos \theta} = \sec \theta$
25. $\csc x - \sin x = \cos x \cot x$
26. $\sec x - \cos x = \sin x \tan x$
27. $\dfrac{1}{\tan x} + \dfrac{1}{\cot x} = \tan x + \cot x$
28. $\dfrac{1}{\sin x} - \dfrac{1}{\csc x} = \csc x - \sin x$
29. $\dfrac{1 + \sin \theta}{\cos \theta} + \dfrac{\cos \theta}{1 + \sin \theta} = 2\sec \theta$
30. $\dfrac{\cos \theta \cot \theta}{1 - \sin \theta} - 1 = \csc \theta$
31. $\dfrac{1}{\cos x + 1} + \dfrac{1}{\cos x - 1} = -2\csc x \cot x$
32. $\cos x - \dfrac{\cos x}{1 - \tan x} = \dfrac{\sin x \cos x}{\sin x - \cos x}$
33. $\tan\left(\dfrac{\pi}{2} - \theta\right)\tan \theta = 1$
34. $\dfrac{\cos[(\pi/2) - x]}{\sin[(\pi/2) - x]} = \tan x$
35. $\dfrac{\tan x \cot x}{\cos x} = \sec x$
36. $\dfrac{\csc(-x)}{\sec(-x)} = -\cot x$
37. $(1 + \sin y)[1 + \sin(-y)] = \cos^2 y$
38. $\dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{\cot x + \cot y}{\cot x \cot y - 1}$
39. $\dfrac{\tan x + \cot y}{\tan x \cot y} = \tan y + \cot x$
40. $\dfrac{\cos x - \cos y}{\sin x + \sin y} + \dfrac{\sin x - \sin y}{\cos x + \cos y} = 0$
41. $\sqrt{\dfrac{1 + \sin \theta}{1 - \sin \theta}} = \dfrac{1 + \sin \theta}{|\cos \theta|}$
42. $\sqrt{\dfrac{1 - \cos \theta}{1 + \cos \theta}} = \dfrac{1 - \cos \theta}{|\sin \theta|}$
43. $\cos^2 \beta + \cos^2\left(\dfrac{\pi}{2} - \beta\right) = 1$
44. $\sec^2 y - \cot^2\left(\dfrac{\pi}{2} - y\right) = 1$
45. $\sin t \csc\left(\dfrac{\pi}{2} - t\right) = \tan t$
46. $\sec^2\left(\dfrac{\pi}{2} - x\right) - 1 = \cot^2 x$
47. $\tan(\sin^{-1} x) = \dfrac{x}{\sqrt{1 - x^2}}$
48. $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$
49. $\tan\left(\sin^{-1}\dfrac{x - 1}{4}\right) = \dfrac{x - 1}{\sqrt{16 - (x - 1)^2}}$
50. $\tan\left(\cos^{-1}\dfrac{x + 1}{2}\right) = \dfrac{\sqrt{4 - (x + 1)^2}}{x + 1}$

ERROR ANALYSIS In Exercises 51 and 52, describe the error(s).

51. $(1 + \tan x)[1 + \cot(-x)]$

$= (1 + \tan x)(1 + \cot x)$

$= 1 + \cot x + \tan x + \tan x \cot x$

$= 1 + \cot x + \tan x + 1$

$= 2 + \cot x + \tan x$

52. $\dfrac{1 + \sec(-\theta)}{\sin(-\theta) + \tan(-\theta)} = \dfrac{1 - \sec\theta}{\sin\theta - \tan\theta}$

$= \dfrac{1 - \sec\theta}{(\sin\theta)[1 - (1/\cos\theta)]}$

$= \dfrac{1 - \sec\theta}{\sin\theta(1 - \sec\theta)}$

$= \dfrac{1}{\sin\theta} = \csc\theta$

In Exercises 53–60, (a) use a graphing utility to graph each side of the equation to determine whether the equation is an identity, (b) use the *table* feature of a graphing utility to determine whether the equation is an identity, and (c) confirm the results of parts (a) and (b) algebraically.

53. $(1 + \cot^2 x)(\cos^2 x) = \cot^2 x$

54. $\csc x(\csc x - \sin x) + \dfrac{\sin x - \cos x}{\sin x} + \cot x = \csc^2 x$

55. $2 + \cos^2 x - 3\cos^4 x = \sin^2 x(3 + 2\cos^2 x)$

56. $\tan^4 x + \tan^2 x - 3 = \sec^2 x(4\tan^2 x - 3)$

57. $\csc^4 x - 2\csc^2 x + 1 = \cot^4 x$

58. $(\sin^4\beta - 2\sin^2\beta + 1)\cos\beta = \cos^5\beta$

59. $\dfrac{1 + \cos x}{\sin x} = \dfrac{\sin x}{1 - \cos x}$ **60.** $\dfrac{\cot\alpha}{\csc\alpha + 1} = \dfrac{\csc\alpha + 1}{\cot\alpha}$

In Exercises 61–64, verify the identity.

61. $\tan^5 x = \tan^3 x\sec^2 x - \tan^3 x$

62. $\sec^4 x\tan^2 x = (\tan^2 x + \tan^4 x)\sec^2 x$

63. $\cos^3 x\sin^2 x = (\sin^2 x - \sin^4 x)\cos x$

64. $\sin^4 x + \cos^4 x = 1 - 2\cos^2 x + 2\cos^4 x$

In Exercises 65–68, use the cofunction identities to evaluate the expression without using a calculator.

65. $\sin^2 25° + \sin^2 65°$ **66.** $\cos^2 55° + \cos^2 35°$

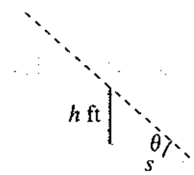
67. $\cos^2 20° + \cos^2 52° + \cos^2 38° + \cos^2 70°$

68. $\tan^2 63° + \cot^2 16° - \sec^2 74° - \csc^2 27°$

69. RATE OF CHANGE The rate of change of the function $f(x) = \sin x + \csc x$ with respect to change in the variable x is given by the expression $\cos x - \csc x\cot x$. Show that the expression for the rate of change can also be $-\cos x\cot^2 x$.

70. SHADOW LENGTH The length s of a shadow cast by a vertical gnomon (a device used to tell time) of height h when the angle of the sun above the horizon is θ (see figure) can be modeled by the equation

$$s = \frac{h\sin(90° - \theta)}{\sin\theta}.$$



(a) Verify that the equation for s is equal to $h\cot\theta$.

(b) Use a graphing utility to complete the table. Let $h = 5$ feet.

θ	15°	30°	45°	60°	75°	90°
s						

(c) Use your table from part (b) to determine the angles of the sun that result in the maximum and minimum lengths of the shadow.

(d) Based on your results from part (c), what time of day do you think it is when the angle of the sun above the horizon is 90°?

EXPLORATION

TRUE OR FALSE? In Exercises 71 and 72, determine whether the statement is true or false. Justify your answer.

71. There can be more than one way to verify a trigonometric identity.

72. The equation $\sin^2\theta + \cos^2\theta = 1 + \tan^2\theta$ is an identity because $\sin^2(0) + \cos^2(0) = 1$ and $1 + \tan^2(0) = 1$.

THINK ABOUT IT In Exercises 73–77, explain why the equation is not an identity and find one value of the variable for which the equation is not true.

73. $\sin\theta = \sqrt{1 - \cos^2\theta}$ **74.** $\tan\theta = \sqrt{\sec^2\theta - 1}$

75. $1 - \cos\theta = \sin\theta$ **76.** $\csc\theta - 1 = \cot\theta$

77. $1 + \tan\theta = \sec\theta$

78. CAPSTONE Write a short paper in your own words explaining to a classmate the difference between a trigonometric identity and a conditional equation. Include suggestions on how to verify a trigonometric identity.