CHAPTER 6
Trigonometry

Section 6.1 Angles and Their Measure

You should know the following basic facts about angles, their measurement, and their applications.

- Types of Angles:
  - (a) Acute: Measure between $0°$ and $90°$.
  - (b) Right: Measure $90°$.
  - (c) Obtuse: Measure between $90°$ and $180°$.
  - (d) Straight: Measure $180°$.
- Two positive angles, α and β are complementary if $\alpha + \beta = 90°$. They are supplementary if $\alpha + \beta = 180°$.
- Two angles in standard position that have the same terminal side are called coterminal angles.
- To convert degrees to radians, use $1° = \pi/180$ radians.
- To convert radians to degrees, use 1 radian $= (180/\pi)°$.
- $1' =$ one minute $= 1/60$ of $1°$.
- $1'' =$ one second $= 1/60$ of $1' = 1/3600$ of $1°$.
- The length of a circular arc is $s = r\theta$ where θ is measured in radians.
- Speed $=$ distance/time
- Angular speed $= \theta/t = s/rt$

1. Trigonometry

2. angle

3. coterminal

4. degree

5. acute; obtuse

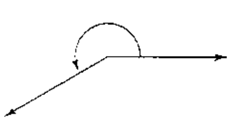
6. complementary; supplementary

7. radian

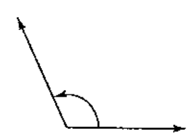
8. linear

9. angular

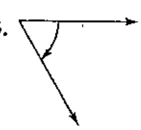
10. $A = \frac{1}{2}r^2\theta$

11. 

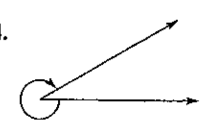
The angle shown is approximately $210°$.

12. 

The angle shown is approximately $120°$.

13. 

The angle shown is approximately $-60°$.

14. 

The angle shown is approximately $15°$.

15. (a) Since $90° < 130° < 180°$; $130°$ lies in Quadrant II.

    (b) Since $270° < 285° < 360°$; $285°$ lies in Quadrant IV.

16. (a) Since $0° < 8.3° < 90°$; $8.3°$ lies in Quadrant I.

    (b) Since $180° < 257° 30' < 270°$; $257° 30'$ lies in Quadrant III.

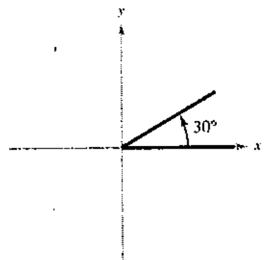
17. (a) Since $-180° < -132°50' < -90°$; $-132° 50'$ lies in Quadrant III.

(b) Since $-360° < -336° < -270°$; $-336°$ lies in Quadrant I.

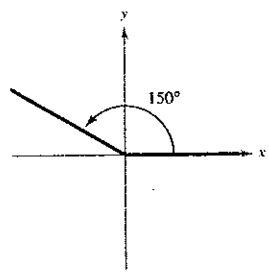
18. (a) Since $-270° < -260° < -180°$; $-260°$ lies in Quadrant II.

(b) Since $-90° < -3.4° < 0°$; $-3.4°$ lies in Quadrant IV.

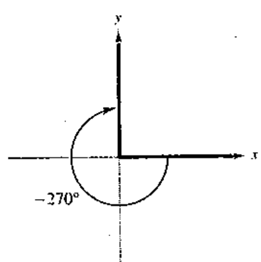
19. (a) $30°$



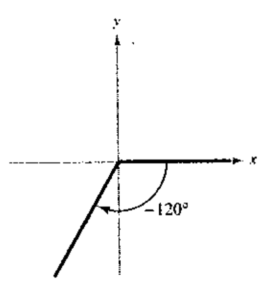
(b) $150°$



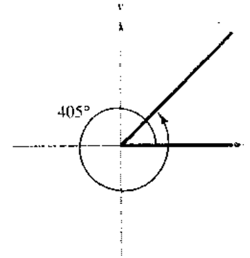
20. (a) $-270°$



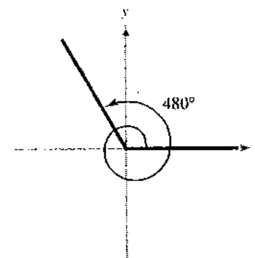
(b) $-120°$



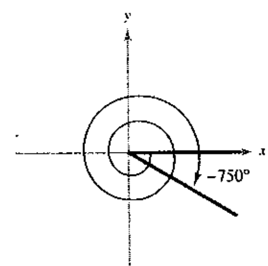
21. (a) $405°$



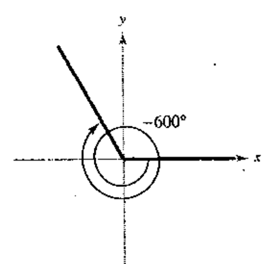
(b) $480°$



22. (a) $-750°$



(b) $-600°$



23. (a) Coterminal angles for $45°$

$45° + 360° = 405°$

$45° - 360° = -315°$

(b) Coterminal angles for $-36°$

$-36° + 360° = 324°$

$-36° - 360° = -396°$

24. (a) $120° + 360° = 480°$

$120° - 360° = -240°$

(b) $-420° + 720° = 300°$

$-420° + 360° = -60°$

25. (a) Coterminal angles for $300°$

$300° + 360° = 660°$

$300° - 360° = -60°$

(b) Coterminal angles for $740°$

$740° - 2(360°) = 20°$

$20° - 360° = -340°$

26. (a) $-520° + 720° = 200°$

$-520° + 360° = -160°$

(b) $230° + 360° = 590°$

$230° - 360° = -130°$

27. (a) $54°45' = 54° + \left(\frac{45}{60}\right)^{\circ} = 54.75°$

(b) $-128°30' = -128° - \left(\frac{30}{60}\right)^{\circ} = -128.5°$

28. (a) $245°10' = 245° + \left(\frac{10}{60}\right)^{\circ}$

$= 245° + 0.167°$

$= 245.167°$

(b) $2°12' = 2° + \left(\frac{12}{60}\right)^{\circ} = 2° + 0.2° = 2.2°$

29. (a) $85°18'30'' = \left(85 + \frac{18}{60} + \frac{30}{3600}\right)^{\circ} \approx 85.308°$

(b) $330°25'' = \left(330 + \frac{25}{3600}\right)^{\circ} \approx 330.007°$

30. (a) $-135°36'' = -135° - \left(\frac{36}{3600}\right)^{\circ}$

$= -135° - 0.01°$

$= -135.01°$

(b) $-408°16'20'' = -\left(408° + \left(\frac{16}{60}\right)^{\circ} + \left(\frac{20}{3600}\right)^{\circ}\right)$

$\approx -(408° + 0.2667° + 0.0056°)$

$= -408.272°$

31. (a) $240.6° = 240° + 0.6(60)' = 240°36'$

(b) $-145.8° = -[145° + 0.8(60')] = -145°48'$

32. (a) $-345.12° = -(345° + (0.12)(60'))$

$= -(345° + 7' + 0.2(60''))$

$= -345°7'12''$

(b) $0.45° = 0° + (0.45)(60') = 0° + 27' = 0°27'$

33. (a) $2.5° = 2° + 0.5(60)' = 2°30'$

(b) $-3.58° = -\left[3° + 0.58(60)'\right]$

$= -[3°34.8']$

$= -\left[3°34' + 0.8(60)''\right]$

$= -3°34'48''$

34. (a) $0.355 = -(0° + (0.355)(60'))$

$= -(0° + 21' + (0.3)(60''))$

$= -(0° + 21' + 18'')$

$= -0°21'18''$

(b) $0.7865 = 0° + (0.7865)(60')$

$= 0° + 47' + (0.19)(60'')$

$\approx 0° + 47' + 11.4''$

$= 0°47'11.4''$

35. (a) Complement: $90° - 18° = 72°$

Supplement: $180° - 18° = 162°$

(b) Complement: $90° - 85° = 5°$

Supplement: $180° - 85° = 95°$

36. (a) Complement: $90° - 46° = 44°$

Supplement: $180° - 46° = 134°$

(b) Complement: Not possible. $93°$ is greater than $90°$.

Supplement: $180° - 93° = 87°$

37. (a) Complement: $90° - 24° = 66°$

Supplement: $180° - 24° = 156°$

(b) Complement: Not possible. $126°$ is greater than $90°$.

Supplement: $180° - 126° = 54°$

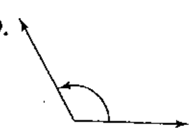
38. (a) Complement: $90° - 87° = 3°$

Supplement: $180° - 87° = 93°$

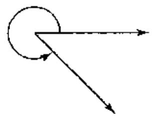
(b) Complement: Not possible. $(166° > 90°)$

Supplement: $180° - 166° = 14°$

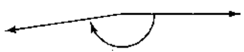
39.



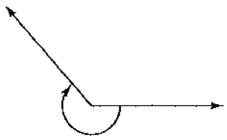
The angle shown is approximately 2 radians.

40. 

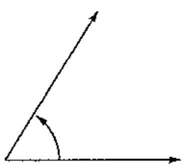
The angle shown is approximately 5.5 radians.

41. 

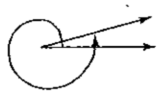
The angle shown is approximately -3 radians.

42. 

The angle shown is approximately -4 radians.

43. 

The angle shown is approximately 1 radian.

44. 

The angle shown is approximately 6.5 radians.

45. (a) Because $0 < \frac{\pi}{4} < \frac{\pi}{2}$; $\frac{\pi}{4}$ lies in Quadrant I.

(b) Because $\pi < \frac{5\pi}{4} < \frac{3\pi}{2}$; $\frac{5\pi}{4}$ lies in Quadrant III.

46. (a) Because $\pi < \frac{11\pi}{8} < \frac{3\pi}{2}$; $\frac{11\pi}{8}$ lies in Quadrant III.

(b) Because $\pi < \frac{9\pi}{8} < \frac{3\pi}{2}$; $\frac{9\pi}{8}$ lies in Quadrant III.

47. (a) Because $0 < \frac{\pi}{5} < \frac{\pi}{2}$; $\frac{\pi}{5}$ lies in Quadrant I.

(b) Because $\pi < \frac{7\pi}{5} < \frac{3\pi}{2}$; $\frac{7\pi}{5}$ lies in Quadrant III.

48. (a) Because $-\frac{\pi}{2} < -\frac{\pi}{12} < 0$; $-\frac{\pi}{12}$ lies in Quadrant IV.

(b) Because $-\frac{3\pi}{2} < -\frac{11\pi}{9} < -\pi$; $-\frac{11\pi}{9}$ lies in Quadrant II.

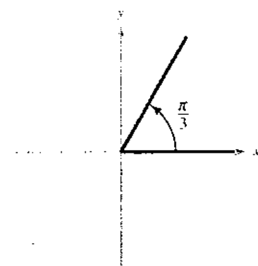
49. (a) Because $-\frac{\pi}{2} < -1 < 0$; -1 lies in Quadrant IV.

(b) Because $-\pi < -2 < -\frac{\pi}{2}$; -2 lies in Quadrant III.

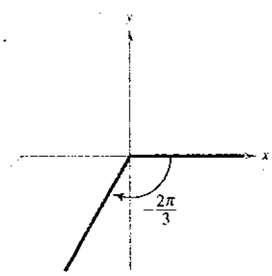
50. (a) Because $\frac{3\pi}{2} < 6.02 < 2\pi$; 6.02 lies in Quadrant IV.

(b) Because $\frac{\pi}{2} < 2.25 < \pi$; 2.25 lies in Quadrant II.

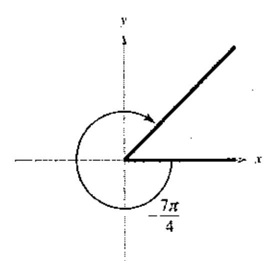
51. (a) $\frac{\pi}{3}$



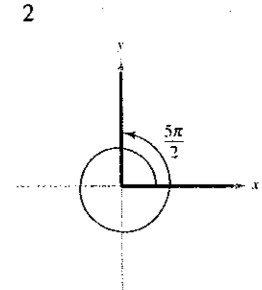
(b) $-\frac{2\pi}{3}$



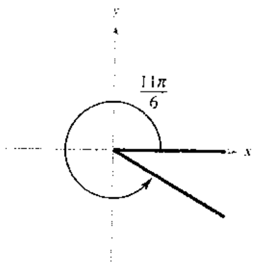
52. (a) $-\frac{7\pi}{4}$



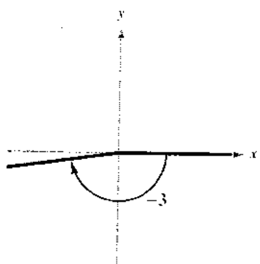
(b) $\frac{5\pi}{2}$



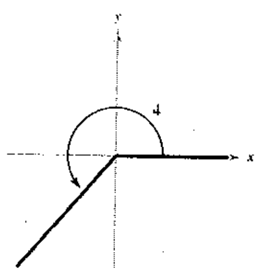
53. (a) $\dfrac{11\pi}{6}$



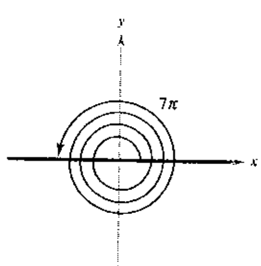
(b) -3



54. (a) 4



(b) 7π



55. (a) Coterminal angles for $\dfrac{\pi}{6}$

$$\frac{\pi}{6} + 2\pi = \frac{13\pi}{6}$$
$$\frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$$

(b) Coterminal angles for $\dfrac{5\pi}{6}$

$$\frac{5\pi}{6} + 2\pi = \frac{17\pi}{6}$$
$$\frac{5\pi}{6} - 2\pi = -\frac{7\pi}{6}$$

56. (a) $\dfrac{7\pi}{6} + 2\pi = \dfrac{19\pi}{6}$

$$\frac{7\pi}{6} - 2\pi = -\frac{5\pi}{6}$$

(b) $-\dfrac{11\pi}{6} + 2\pi = \dfrac{\pi}{6}$

$$-\frac{11\pi}{6} - 2\pi = -\frac{23\pi}{6}$$

57. (a) Coterminal angles for $\dfrac{2\pi}{3}$

$$\frac{2\pi}{3} + 2\pi = \frac{8\pi}{3}$$
$$\frac{2\pi}{3} - 2\pi = -\frac{4\pi}{3}$$

(b) Coterminal angles for $-\dfrac{\pi}{12}$

$$-\frac{\pi}{12} + 2\pi = \frac{23\pi}{12}$$
$$-\frac{\pi}{12} - 2\pi = -\frac{25\pi}{12}$$

58. (a) Coterminal angles for $-\dfrac{3\pi}{4}$

$$-\frac{3\pi}{4} + 2\pi = \frac{5\pi}{4}$$
$$-\frac{3\pi}{4} - 2\pi = -\frac{11\pi}{4}$$

(b) Coterminal angles for $-\dfrac{7\pi}{4}$

$$-\frac{7\pi}{4} + 2\pi = \frac{\pi}{4}$$
$$-\frac{7\pi}{4} - 2\pi = -\frac{15\pi}{4}$$

59. (a) Coterminal angles for $-\dfrac{9\pi}{4}$

$$-\frac{9\pi}{4} + 4\pi = \frac{7\pi}{4}$$
$$-\frac{9\pi}{4} + 2\pi = -\frac{\pi}{4}$$

(b) Coterminal angles for $-\dfrac{2\pi}{15}$

$$-\frac{2\pi}{15} + 2\pi = \frac{28\pi}{15}$$
$$-\frac{2\pi}{15} - 2\pi = -\frac{32\pi}{15}$$

103. $\theta = \frac{s}{r} = \frac{450}{6378} \approx 0.071 \text{ radian} \approx 4.04°$

104. $r = 6378$ kilometers

$\theta = \frac{s}{r} = \frac{400}{6378} \approx 0.062716$ radian

$0.062716\left(\frac{180°}{\pi}\right) \approx 3.59°$

The difference in latitude is about $3.59°$.

105. $\theta = \frac{s}{r} = \frac{2.5}{6} = \frac{25}{60} = \frac{5}{12}$ radian

106. $\theta = \frac{s}{r} = \frac{24}{5} = 4.8 \text{ radians} = 4.8\left(\frac{180°}{\pi}\right) \approx 275°$

107. (a) 65 miles per hour $= 65(5280)/60$
$= 5720$ feet per minute

The circumference of the tire is $C = 2.5\pi$ feet.

The number of revolutions per minute is $r = 5720/2.5\pi \approx 728.3$ rev/minute.

(b) The angular speed is θ/t.

$\theta = \frac{5720}{2.5\pi}(2\pi) = 4576$ radians

Angular speed $= \frac{4576 \text{ radians}}{1 \text{ minute}}$
$= 4576$ radians/minute

108. (a) 2-inch diameter pulley

$1700 \text{ rpm} = 1700 \cdot 2\pi$ radians/minute
≈ 10681.4 radians/minute

Since $r = 1$, the belt moves 10681.4 inches/minute.

On the 4-inch diameter pulley:

$r = 2$

$s = 10681.4 = 2 \cdot \theta$

$\theta = \frac{10681.4}{2} = 5340.7$

This pulley is turning at 5340.7 radians/minute.

(b) $\frac{5340.7}{2\pi} = 850$ rpm

109. (a) Angular speed $= \frac{(5200)(2\pi) \text{ radians}}{1 \text{ minute}}$
$= 10{,}400\pi$ radians per minute

(b) Linear speed $= \frac{\left(\frac{7.25}{2} \text{ in.}\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)(5200)(2\pi) \text{ feet}}{1 \text{ minute}}$
$= 3141\frac{2}{3}\pi$ feet per minute
≈ 164.5 feet per second

110. (a) 4 rpm $= 4(2\pi)$ radians/minute
$= 8\pi$
≈ 25.13274 radians/minute

(b) $r = 25$ ft

$\frac{r\theta}{t} = 200\pi$ ft/minute

Linear speed $\approx 25(25.13274)$ ft/minute
≈ 628.32 ft/minute

111. (a) $(200)(2\pi) \le$ Angular speed $\le (500)(2\pi)$ radians per minute

Interval: $[400\pi, 1000\pi]$ radians per minute

(b) $(6)(200)(2\pi) \le$ Linear speed $\le (6)(500)(2\pi)$ centimeters per minute

Interval: $[2400\pi, 6000\pi]$ centimeters per minute

112. (a) Road speed (linear speed) $= \frac{\left(\frac{25}{2} \text{ in.}\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)\left(\frac{1 \text{ mi}}{5280 \text{ ft}}\right)(480)(2\pi)}{1 \text{ minute}\left(\frac{1 \text{ hour}}{60 \text{ minutes}}\right)} \approx 35.70$ mi/h

(b) $\frac{55 \text{ mi}}{1 \text{ h}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{12 \text{ in.}}{1 \text{ ft}} \times \frac{1 \text{ h}}{60 \text{ min}} = \frac{58{,}080 \text{ in.}}{1 \text{ min}}$

The circumference of the machine is $C = 2\pi\left(\frac{25}{2}\right) = 25\pi$ inches.

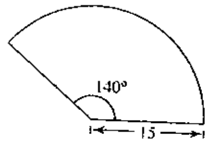
The number of revolutions per minute is $r = 58{,}080/25\pi \approx 739.50$ revolutions/min.

113. $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}(15)^2(140°)\left(\frac{\pi}{180°}\right)$

$= 87.5\pi \text{ m}^2$

$\approx 274.89 \text{ m}^2$



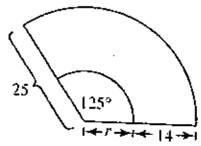
114. $A = \frac{1}{2}\theta\left(R^2 - r^2\right)$

$R = 25$

$r = 25 - 14 = 11$

$A = \frac{1}{2}\left(\frac{125}{180}\right)^{\circ}\pi \cdot \left(25^2 - 11^2\right) \approx 175\pi$

$\approx 549.8 \text{ in.}^2$

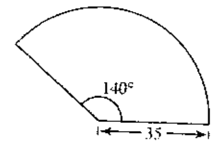


115. $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}(35)^2(140°)\left(\frac{\pi}{180°}\right)$

$\approx 476.39\pi$ square meters

≈ 1496.62 square meters



116. (a) Arc length of larger sprocket in feet:

$s = r\theta$

$s = \frac{1}{3}(2\pi) = \frac{2\pi}{3}$ feet

Therefore, the chain moves $\frac{2\pi}{3}$ feet as does the smaller rear sprocket.

Thus, the angle θ of the smaller sprocket is

$$\theta = \frac{s}{r} = \frac{\frac{2\pi}{3}\text{ ft}}{\frac{2}{12}\text{ ft}} = 4\pi \quad \left(r = 2 \text{ inches} = \frac{2}{12}\text{ feet}\right)$$

and the arc length of the tire in feet is:

$s = \theta r$

$s = (4\pi)\left(\frac{14}{12}\right) = \frac{14\pi}{3}$ feet

$$\text{Speed} = \frac{s}{t} = \frac{\frac{14\pi}{3}}{1 \text{ sec}} = \frac{14\pi}{3} \text{ feet per second}$$

$$\frac{14\pi \text{ feet}}{3 \text{ seconds}} \times \frac{3600 \text{ seconds}}{1 \text{ hour}} \times \frac{1 \text{ mile}}{5280 \text{ feet}} \approx 10 \text{ miles per hour}$$

(b) Since the arc length of the tire is $\frac{14\pi}{3}$ feet and the cyclist is pedaling at a rate of one revolution per second, we have:

$$\text{Distance} = \left(\frac{14\pi}{3}\frac{\text{feet}}{\text{revolutions}}\right)\left(\frac{1 \text{ mile}}{5280 \text{ feet}}\right)(n \text{ revolutions})$$

$$= \frac{7\pi}{7920}n \text{ miles}$$

(c) Distance = Rate · Time

$$= \left(\frac{14\pi}{3} \text{ feed/second}\right)\left(\frac{1 \text{ mile}}{5280 \text{ feet}}\right)(t \text{ seconds})$$

$$= \frac{7\pi}{7920}t \text{ miles}$$

(d) The functions are both linear.

117. False. A measurement of 4π radians corresponds to two complete revolutions from the initial to the terminal side of an angle.

118. True. If α and β are coterminal angles, then $\alpha = \beta + n(360°)$ or $\alpha = \beta + n(2\pi)$, where n is an integer. The difference between α and β is $\alpha - \beta = n(360°)$, or $\alpha - \beta = n(2\pi)$ if expressed in radians.

119. False. The terminal side of $-1260°$ lies on the negative x-axis.

120. Sample answers:

(a) An angle is standard position is an angle in which the origin is the vertex and the initial side coincides with the positive x-axis.

(b) Positive angles are generated by counterclockwise rotation. Negative angles are generated by clockwise rotation.

(c) Coterminal angles are angles α and β that have the same initial and terminal sides.

(d) Degree is a common unit of angle measurement equivalent to a rotation of $\frac{1}{360}$ of a complete revolution about the vertex. Radians also measure angles. One radian is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of a circle.

(e) An acute angle is an angle measuring between $0°$ and $90°$. An obtuse angle is an angle measuring between $90°$ and $180°$.

(f) Two positive angles are complementary if their sum is $90°$. Two positive angles are supplementary if their sum is $180°$.

121. The speed increases, since the linear speed is proportional to the radius.

122. $1 \text{ radian} = \left(\dfrac{180°}{\pi}\right) \approx 57.3°$, so one radian is much larger than one degree.

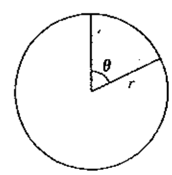
123. Since the arc length s is given by $s = r\theta$, if the central angle θ is fixed while the radius r increases, then s increases in proportion to r.

124. Area of circle $= \pi r^2$

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Measure of central angle of sector}}{\text{Measure of central angle of circle}}$$

$$\frac{\text{Area of sector}}{\pi r^2} = \frac{\theta}{2\pi}$$

$$\text{Area of sector} = \left(\pi r^2\right)\left(\frac{\theta}{2\pi}\right) = \frac{1}{2}r^2\theta$$

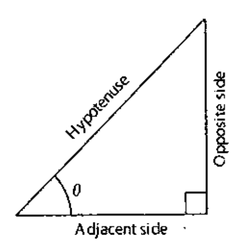


Section 6.2 Right Triangle Trigonometry

- You should know the right triangle definition of trigonometric functions.

(a) $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ (b) $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ (c) $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

(d) $\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$ (e) $\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$ (f) $\cot\theta = \dfrac{\text{adj}}{\text{opp}}$



—continued—

- You should know the following identities.

(a) $\sin\theta = \dfrac{1}{\csc\theta}$ (b) $\csc\theta = \dfrac{1}{\sin\theta}$ (c) $\cos\theta = \dfrac{1}{\sec\theta}$

(d) $\sec\theta = \dfrac{1}{\cos\theta}$ (e) $\tan\theta = \dfrac{1}{\cot\theta}$ (f) $\cot\theta = \dfrac{1}{\tan\theta}$

(g) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ (h) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ (i) $\sin^2\theta + \cos^2\theta = 1$

(j) $1 + \tan^2\theta = \sec^2\theta$ (k) $1 + \cot^2\theta = \csc^2\theta$

- You should know that two acute angles α and β are complementary if $\alpha + \beta = 90°$, and that the cofunctions of complementary angles are equal.
- You should know the trigonometric function values of 30°, 45°, and 60°, or be able to construct triangles from which you can determine them.

1. (i) $\dfrac{\text{hypotenuse}}{\text{adjacent}} = \sec\theta$ (e)

(ii) $\dfrac{\text{adjacent}}{\text{opposite}} = \cot\theta$ (f)

(iii) $\dfrac{\text{hypotenuse}}{\text{opposite}} = \csc\theta$ (d)

(iv) $\dfrac{\text{adjacent}}{\text{hypotenuse}} = \cos\theta$ (b)

(v) $\dfrac{\text{opposite}}{\text{hypotenuse}} = \sin\theta$ (a)

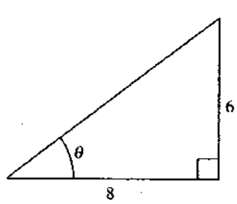
(vi) $\dfrac{\text{opposite}}{\text{adjacent}} = \tan\theta$ (c)

2. opposite, adjacent; hypotenuse

3. Complementary

4. elevation; depression

5. hyp $= \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

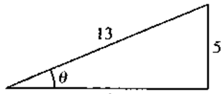


$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{6}{10} = \dfrac{3}{5}$ $\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{10}{6} = \dfrac{5}{3}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{8}{10} = \dfrac{4}{5}$ $\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{10}{8} = \dfrac{5}{4}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{6}{8} = \dfrac{3}{4}$ $\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{8}{6} = \dfrac{4}{3}$

6. adj $= \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = 12$

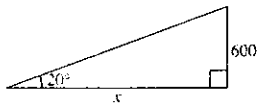


$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{5}{13}$ $\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{13}{5}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{12}{13}$ $\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{13}{12}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{5}{12}$ $\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{12}{5}$

7. adj $= \sqrt{41^2 - 9^2} = \sqrt{1681 - 81} = \sqrt{1600} = 40$

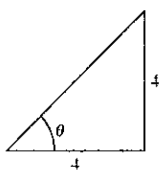


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{9}{41} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{41}{9}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{40}{41} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{41}{40}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{9}{40} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{40}{9}$$

8. hyp $= \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$

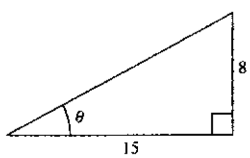


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{4\sqrt{2}}{4} = \sqrt{2}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{4\sqrt{2}}{4} = \sqrt{2}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{4} = 1 \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{4} = 1$$

9.

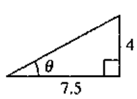


$$\text{hyp} = \sqrt{15^2 + 8^2} = \sqrt{289} = 17$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{8}{17} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{17}{8}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{15}{17} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{17}{15}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{8}{15} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{15}{8}$$



$$\text{hyp} = \sqrt{7.5^2 + 4^2} = \frac{17}{2}$$

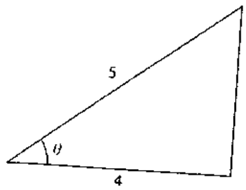
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{(17/2)} = \frac{8}{17} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{(17/2)}{4} = \frac{17}{8}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{7.5}{(17/2)} = \frac{15}{17} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{(17/2)}{7.5} = \frac{17}{15}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{7.5} = \frac{8}{15} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{7.5}{4} = \frac{15}{8}$$

The function values are the same because the triangles are similar, and corresponding sides are proportional.

10. $\text{opp} = \sqrt{5^2 - 4^2} = 3$

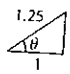


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{3}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{3}$$

$\text{opp} = \sqrt{1.25^2 - 1^2} = 0.75$



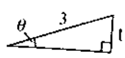
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{0.75}{1.25} = \frac{3}{5} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{1.25}{0.75} = \frac{5}{3}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{1}{1.25} = \frac{4}{5} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{1.25}{1} = \frac{5}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{0.75}{1} = \frac{3}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{0.75} = \frac{4}{3}$$

The function values are the same since the triangles are similar and the corresponding sides are proportional.

11. $\text{adj} = \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$

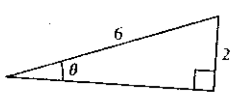


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{3} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = 3$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2\sqrt{2}}{3} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = 2\sqrt{2}$$

$\text{adj} = \sqrt{6^2 - 2^2} = \sqrt{32} = 4\sqrt{2}$

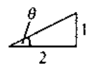


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2}{6} = \frac{1}{3} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{6}{2} = 3$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{6}{4\sqrt{2}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{5\sqrt{2}}{4} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

The function values are the same since the triangles are similar and the corresponding sides are proportional.

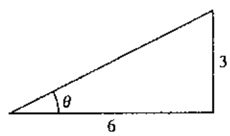
12. 

$$\text{hyp} = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{5}}{1} = \sqrt{5}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{5}}{2}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{2} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2}{1} = 2$$



$$\text{hyp} = \sqrt{3^2 + 6^2} = 3\sqrt{5}$$

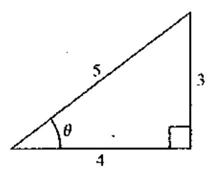
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{3\sqrt{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{3\sqrt{5}}{3} = \sqrt{5}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{6}{3\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{3\sqrt{5}}{6} = \frac{\sqrt{5}}{2}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{6} = \frac{1}{2} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{6}{3} = 2$$

The function values are the same because the triangles are similar, and corresponding sides are proportional.

13. Given: $\tan\theta = \frac{3}{4} = \frac{\text{opp}}{\text{adj}}$



$$3^2 + 4^2 = (\text{hyp})^2$$
$$\text{hyp} = 5$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}$$

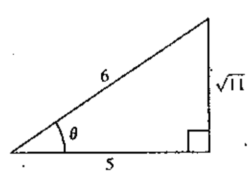
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{3}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{4}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{3}$$

14. Given: $\cos\theta = \frac{5}{6} = \frac{\text{adj}}{\text{hyp}}$



$$(\text{opp})^2 + 5^2 = 6^2$$
$$\text{opp} = \sqrt{11}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{11}}{6}$$

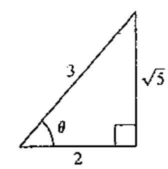
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{11}}{5}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{6\sqrt{11}}{11}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{6}{5}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{5\sqrt{11}}{11}$$

15. Given: $\sec\theta = \frac{3}{2} = \frac{\text{hyp}}{\text{adj}}$



$$(\text{opp})^2 + 2^2 = 3^2$$
$$\text{opp} = \sqrt{5}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{5}}{3}$$

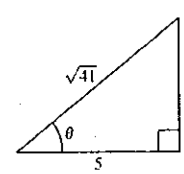
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{3}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{5}}{2}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{3\sqrt{5}}{5}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2\sqrt{5}}{5}$$

16. Given: $\tan\theta = \frac{4}{5} = \frac{\text{opp}}{\text{adj}}$



$$4^2 + 5^2 = (\text{hyp})^2$$
$$\text{hyp} = \sqrt{41}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4\sqrt{41}}{41}$$

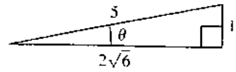
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5\sqrt{41}}{41}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{41}}{4}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{41}}{5}$$

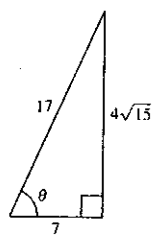
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{5}{4}$$

17. Given: $\sin\theta = \frac{1}{5} = \frac{\text{opp}}{\text{hyp}}$



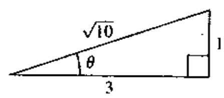
$$1^2 + (\text{adj})^2 = 5^2$$
$$\text{adj} = \sqrt{24} = 2\sqrt{6}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2\sqrt{6}}{5}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{6}}{12}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = 5$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5\sqrt{6}}{12}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = 2\sqrt{6}$$

18. Given: $\sec\theta = \frac{17}{7} = \frac{\text{hyp}}{\text{adj}}$



$$(\text{opp})^2 + 7^2 = 17^2$$
$$\text{opp} = \sqrt{240} = 4\sqrt{15}$$
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4\sqrt{15}}{17}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{7}{17}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4\sqrt{15}}{7}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{17\sqrt{15}}{60}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{7\sqrt{15}}{60}$$

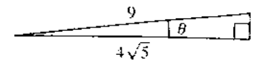
19. Given: $\cot\theta = 3 = \frac{3}{1} = \frac{\text{adj}}{\text{opp}}$



$$1^2 + 3^2 = (\text{hyp})^2$$
$$\text{hyp} = \sqrt{10}$$
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{10}}{10}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3\sqrt{10}}{10}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{3}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \sqrt{10}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{10}}{3}$$

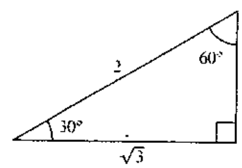
20. Given: $\csc\theta = 9 = \frac{9}{1} = \frac{\text{hyp}}{\text{opp}}$

$$1^2 + (\text{adj})^2 = 9^2$$
$$\text{adj} = \sqrt{80} = 4\sqrt{5}$$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{9}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4\sqrt{5}}{9}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{5}}{20}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{9\sqrt{5}}{20}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = 4\sqrt{5}$$

21.

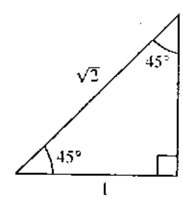


$$30° = 30°\left(\frac{\pi}{180°}\right) = \frac{\pi}{6} \text{ radian}$$
$$\sin 30° = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2}$$

22.

degree	radian	value
$\cos 45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$

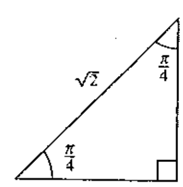
$$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$



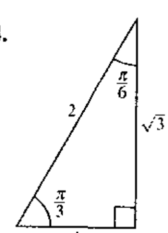
23.

degree	radian	value
$\sec 45°$	$\frac{\pi}{4}$	$\sqrt{2}$

$$\sec\frac{\pi}{4} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

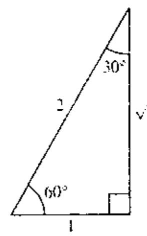


24.



$$\frac{\pi}{3} = \frac{\pi}{3}\left(\frac{180°}{\pi}\right) = 60°$$
$$\tan\frac{\pi}{3} = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

25.



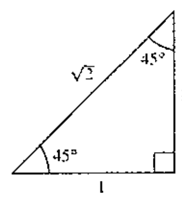
$$\cot\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} = \frac{\text{adj}}{\text{opp}}$$

$$\theta = 60° = \frac{\pi}{3} \text{ radian}$$

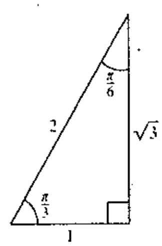
26.

degree	radian	value
csc 45°	$\frac{\pi}{4}$	$\sqrt{2}$

$$\csc 45° = \frac{\sqrt{2}}{1}$$



27.



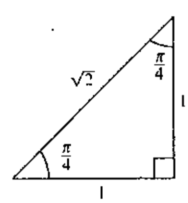
$$\frac{\pi}{6} = \frac{\pi}{6}\left(\frac{180°}{\pi}\right) = 30°$$

$$\csc\frac{\pi}{6} = \frac{\text{hyp}}{\text{opp}} = 2$$

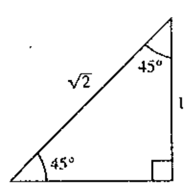
28.

degree	radian	value
sin 45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$

$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$



29.



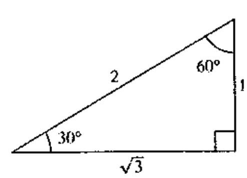
$$\cot\theta = 1 = \frac{1}{1} = \frac{\text{adj}}{\text{opp}}$$

$$\theta = 45° = 45°\left(\frac{\pi}{180°}\right) = \frac{\pi}{4}$$

30.

degree	radian	value
tan 30°	$\frac{\pi}{6}$	$\frac{1}{\sqrt{3}}$

$$\tan 30° = \frac{1}{\sqrt{3}}$$



31. $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$

(a) $\sin 30° = \cos 60° = \frac{1}{2}$

(b) $\cos 30° = \sin 60° = \frac{\sqrt{3}}{2}$

(c) $\tan 60° = \frac{\sin 60°}{\cos 60°} = \sqrt{3}$

(d) $\cot 60° = \frac{\cos 60°}{\sin 60°} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

32. $\sin 30° = \frac{1}{2}$, $\tan 30° = \frac{\sqrt{3}}{3}$

(a) $\csc 30° = \frac{1}{\sin 30°} = 2$

(b) $\cot 60° = \tan(90° - 60°) = \tan 30° = \frac{\sqrt{3}}{3}$

(c) $\cos 30° = \frac{\sin 30°}{\tan 30°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

(d) $\cot 30° = \frac{1}{\tan 30°} = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

33. $\cos\theta = \frac{1}{3}$

(a) $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta + \left(\frac{1}{3}\right)^2 = 1$$

$$\sin^2\theta = \frac{8}{9}$$

$$\sin\theta = \frac{2\sqrt{2}}{3}$$

(b) $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = 2\sqrt{2}$

(c) $\sec\theta = \frac{1}{\cos\theta} = 3$

(d) $\csc(90° - \theta) = \sec\theta = 3$

34. $\sec\theta = 5$

(a) $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{5}$

(b) $$\begin{aligned} 1 + \tan^2\theta &= \sec^2\theta \\ 1 + \tan^2\theta &= 5^2 \\ \tan^2\theta &= 24 \\ \tan\theta &= 2\sqrt{6} \end{aligned}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$$

(c) $\cot(90^\circ - \theta) = \tan\theta = 2\sqrt{6}$

(d) $$\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \sin^2\theta + \left(\frac{1}{5}\right)^2 &= 1 \\ \sin^2\theta &= \frac{24}{25} \\ \sin\theta &= \frac{2\sqrt{6}}{5} \end{aligned}$$

35. $\cot\alpha = 5$

(a) $\tan\alpha = \dfrac{1}{\cot\alpha} = \dfrac{1}{5}$

(b) $$\begin{aligned} \csc^2\alpha &= 1 + \cot^2\alpha \\ \csc^2\alpha &= 1 + 5^2 \\ \csc^2\alpha &= 26 \\ \csc\alpha &= \sqrt{26} \end{aligned}$$

(c) $\cot(90^\circ - \alpha) = \tan\alpha = \dfrac{1}{5}$

(d) $$\begin{aligned} \sec^2\alpha &= 1 + \tan^2\alpha \\ \sec^2\alpha &= 1 + \left(\frac{1}{5}\right)^2 \\ \sec^2\alpha &= \frac{26}{25} \\ \sec\alpha &= \frac{\sqrt{26}}{5} \\ \cos\alpha &= \frac{1}{\sec\alpha} = \frac{5\sqrt{26}}{26} \end{aligned}$$

36. $\cos\beta = \dfrac{\sqrt{7}}{4}$

(a) $\sec\beta = \dfrac{1}{\cos\beta} = \dfrac{4\sqrt{7}}{7}$

(b) $$\begin{aligned} \sin^2\beta + \cos^2\beta &= 1 \\ \sin^2\beta + \left(\frac{\sqrt{7}}{4}\right)^2 &= 1 \\ \sin^2\beta &= \frac{9}{16} \\ \sin\beta &= \frac{3}{4} \end{aligned}$$

(c) $\cot\beta = \dfrac{\cos\beta}{\sin\beta} = \dfrac{\frac{\sqrt{7}}{4}}{\frac{3}{4}} = \dfrac{\sqrt{7}}{3}$

(d) $\sin(90^\circ - \beta) = \cos\beta = \dfrac{\sqrt{7}}{4}$

37. $\tan\theta\cot\theta = \tan\theta\left(\dfrac{1}{\tan\theta}\right) = 1$

38. $\cos\theta\sec\theta = \cos\theta\dfrac{1}{\cos\theta} = 1$

39. $\tan\alpha\cos\alpha = \left(\dfrac{\sin\alpha}{\cos\alpha}\right)\cos\alpha = \sin\alpha$

40. $\cot\alpha\sin\alpha = \dfrac{\cos\alpha}{\sin\alpha}\sin\alpha = \cos\alpha$

41. $(1 + \sin\theta)(1 - \sin\theta) = 1 - \sin^2\theta = \cos^2\theta$

42. $$\begin{aligned} (1 + \cos\theta)(1 - \cos\theta) &= 1 - \cos^2\theta \\ &= (\sin^2\theta + \cos^2\theta) - \cos^2\theta \\ &= \sin^2\theta \end{aligned}$$

43. $$\begin{aligned} (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) &= \sec^2\theta - \tan^2\theta \\ &= (1 + \tan^2\theta) - \tan^2\theta \\ &= 1 \end{aligned}$$

44. $$\begin{aligned} \sin^2\theta - \cos^2\theta &= \sin^2\theta - (1 - \sin^2\theta) \\ &= \sin^2\theta - 1 + \sin^2\theta \\ &= 2\sin^2\theta - 1 \end{aligned}$$

45. $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta}\cdot\dfrac{1}{\cos\theta}$

$= \csc\theta\sec\theta$

46. $\dfrac{\tan\beta + \cot\beta}{\tan\beta} = \dfrac{\tan\beta}{\tan\beta} + \dfrac{\cot\beta}{\tan\beta}$

$= 1 + \dfrac{\cot\beta}{\dfrac{1}{\cot\beta}}$

$= 1 + \cot^2\beta = \csc^2\beta$

47. (a) $\tan 23.5° \approx 0.4348$

(b) $\cot 66.5° = \dfrac{1}{\tan 66.5°} \approx 0.4348$

48. (a) $\sin 16.35° \approx 0.2815$

(b) $\csc 16.35° = \dfrac{1}{\sin 16.35°} \approx 3.5523$

49. (a) $\cos 16°18' = \cos\left(16 + \dfrac{18}{60}\right)^{\circ} \approx 0.9598$

(b) $\sin 73°56' = \sin\left(73 + \dfrac{56}{60}\right)^{\circ} \approx 0.9609$

50. (a) $\sec 42°12' = \sec 42.2° = \dfrac{1}{\cos 42.2°} \approx 1.3499$

(b) $\csc 48°7' = \dfrac{1}{\sin\left(48 + \dfrac{7}{60}\right)^{\circ}} \approx 1.3432$

51. Make sure that your calculator is in radian mode.

(a) $\cot\dfrac{\pi}{16} = \dfrac{1}{\tan\dfrac{\pi}{16}} \approx 5.0273$

(b) $\tan\dfrac{\pi}{16} \approx 0.1989$

52. (a) $\sec 0.75 = \dfrac{1}{\cos 0.75} \approx 1.3667$

(**Note:** 0.75 is in radians)

(b) $\cos 0.75 \approx 0.7317$

53. Make sure that your calculator is in radian mode.

(a) $\csc 1 = \dfrac{1}{\sin 1} \approx 1.1884$

(b) $\tan\dfrac{1}{2} \approx 0.5463$

54. (a) $\sec\left(\dfrac{\pi}{2} - 1\right) = \dfrac{1}{\cos\left(\dfrac{\pi}{2} - 1\right)} \approx 1.1884$

(b) $\cot\left(\dfrac{\pi}{2} - \dfrac{1}{2}\right) = \dfrac{1}{\tan\left(\dfrac{\pi}{2} - \dfrac{1}{2}\right)} \approx 0.5463$

55. (a) $\sin\theta = \dfrac{1}{2} \Rightarrow \theta = 30° = \dfrac{\pi}{6}$

(b) $\csc\theta = 2 \Rightarrow \theta = 30° = \dfrac{\pi}{6}$

56. (a) $\cos\theta = \dfrac{\sqrt{2}}{2} \Rightarrow \theta = 45° = \dfrac{\pi}{4}$

(b) $\tan\theta = 1 \Rightarrow \theta = 45° = \dfrac{\pi}{4}$

57. (a) $\sec\theta = 2 \Rightarrow \theta = 60° = \dfrac{\pi}{3}$

(b) $\cot\theta = 1 \Rightarrow \theta = 45° = \dfrac{\pi}{4}$

58. (a) $\tan\theta = \sqrt{3} \Rightarrow \theta = 60° = \dfrac{\pi}{3}$

(b) $\cos\theta = \dfrac{1}{2} \Rightarrow \theta = 60° = \dfrac{\pi}{3}$

59. (a) $\csc\theta = \dfrac{2\sqrt{3}}{3} \Rightarrow \theta = 60° = \dfrac{\pi}{3}$

(b) $\sin\theta = \dfrac{\sqrt{2}}{2} \Rightarrow \theta = 45° = \dfrac{\pi}{4}$

60. (a) $\cot\theta = \dfrac{\sqrt{3}}{3}$

$\tan\theta = \dfrac{3}{\sqrt{3}} = \sqrt{3} \Rightarrow \theta = 60° = \dfrac{\pi}{3}$

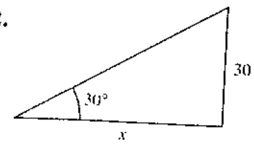
(b) $\sec\theta = \sqrt{2}$

$\cos\theta = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} \Rightarrow \theta = 45° = \dfrac{\pi}{4}$

61. $\sin 60° = \dfrac{y}{18}$

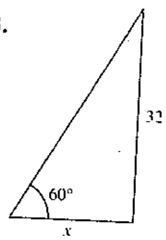
$y = 18\sin 60° = 18\dfrac{\sqrt{3}}{2} = 9\sqrt{3}$

62.



$$\tan 30^\circ = \frac{30}{x}$$
$$\frac{1}{\sqrt{3}} = \frac{30}{x}$$
$$x = 30\sqrt{3}$$

63.

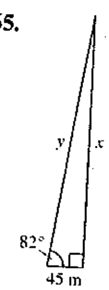


$$\tan 60^\circ = \frac{32}{x}$$
$$\sqrt{3} = \frac{32}{x}$$
$$\sqrt{3}x = 32$$
$$x = \frac{32}{\sqrt{3}} = \frac{32\sqrt{3}}{3}$$

64. $\sin 45^\circ = \dfrac{20}{r}$

$$r = \frac{20}{\sin 45^\circ} = \frac{20}{\frac{\sqrt{2}}{2}} = 20\sqrt{2}$$

65.



$$\tan 82^\circ = \frac{x}{45}$$
$$x = 45 \tan 82^\circ$$

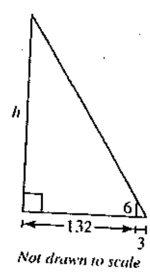
Height of the building:

$123 + 45 \tan 82^\circ \approx 443.2$ meters

Distance between friends:

$$\cos 82^\circ = \frac{45}{y} \Rightarrow y = \frac{45}{\cos 82^\circ}$$
$$\approx 323.34 \text{ meters}$$

66. (a)

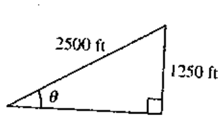


(b) $\tan \theta = \dfrac{6}{3} = \dfrac{h}{135}$

(c) $2(135) = h$

$h = 270$ feet

67.



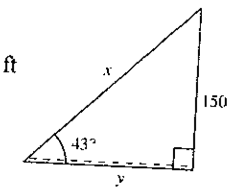
$$\sin \theta = \frac{1250}{2500} = \frac{1}{2}$$
$$\theta = 30^\circ = \frac{\pi}{6}$$

68. $\tan \theta = \dfrac{\text{opp}}{\text{adj}}$

$$\tan 54^\circ = \frac{w}{100}$$
$$w = 100 \tan 54^\circ \approx 137.6 \text{ feet}$$

69. (a) $\sin 43^\circ = \dfrac{150}{x}$

$$x = \frac{150}{\sin 43^\circ} \approx 219.9 \text{ ft}$$



(b) $\tan 43^\circ = \dfrac{150}{y}$

$$y = \frac{150}{\tan 43^\circ} \approx 160.9 \text{ ft}$$

70. Let h = the height of the mountain.

Let x = the horizontal distance from where the 9° angle of elevation is sighted to the point at that level directly below the mountain peak.

Then $\tan 3.5^\circ = \dfrac{h}{x + 13}$ and $\tan 9^\circ = \dfrac{h}{x}$.

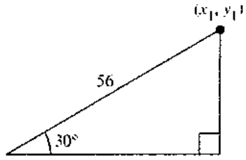
$$\tan 9^\circ = \frac{h}{x} \Rightarrow x = \frac{h}{\tan 9^\circ}$$

Substitute $x = \dfrac{h}{\tan 9^\circ}$ into the expression for $\tan 3.5^\circ$.

$$\tan 3.5^\circ = \frac{h}{\frac{h}{\tan 9^\circ} + 13}$$
$$\tan 3.5^\circ = \frac{h \tan 9^\circ}{h + 13 \tan 9^\circ}$$
$$h \tan 3.5^\circ + 13 \tan 9^\circ \tan 3.5^\circ = h \tan 9^\circ$$
$$13 \tan 9^\circ \tan 3.5^\circ = h(\tan 9^\circ - \tan 3.5^\circ)$$
$$\frac{13 \tan 9^\circ \tan 3.5^\circ}{\tan 9^\circ - \tan 3.5^\circ} = h$$
$$1.2953 \approx h$$

The mountain is about 1.3 miles high.

71.



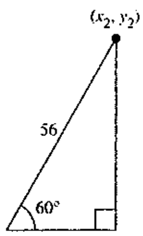
$$\sin 30° = \frac{y_1}{56}$$

$$y_1 = (\sin 30°)(56) = \left(\frac{1}{2}\right)(56) = 28$$

$$\cos 30° = \frac{x_1}{56}$$

$$x_1 = \cos 30°(56) = \frac{\sqrt{3}}{2}(56) = 28\sqrt{3}$$

$$(x_1, y_1) = \left(28\sqrt{3}, 28\right)$$



$$\sin 60° = \frac{y_2}{56}$$

$$y_2 = \sin 60°(56) = \left(\frac{\sqrt{3}}{2}\right)(56) = 28\sqrt{3}$$

$$\cos 60° = \frac{x_2}{56}$$

$$x_2 = (\cos 60°)(56) = \left(\frac{1}{2}\right)(56) = 28$$

$$(x_2, y_2) = \left(28, 28\sqrt{3}\right)$$

72. $\tan 3° = \dfrac{x}{15}$

$$x = 15 \tan 3°$$

$$d = 5 + 2x = 5 + 2(15 \tan 3°) \approx 6.57 \text{ centimeters}$$

73. $x \approx 9.397$, $y \approx 3.420$

$$\sin 20° = \frac{y}{10} \approx 0.34$$

$$\cos 20° = \frac{x}{10} \approx 0.94$$

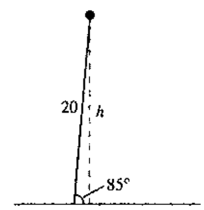
$$\tan 20° = \frac{y}{x} \approx 0.36$$

$$\cot 20° = \frac{x}{y} \approx 2.75$$

$$\sec 20° = \frac{10}{x} \approx 1.06$$

$$\csc 20° = \frac{10}{y} \approx 2.92$$

74. (a)



(b) $\sin 85° = \dfrac{h}{20}$

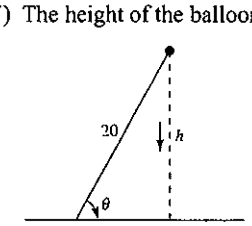
(c) $h = 20 \sin 85° \approx 19.9$ meters

(d) The side of the triangle labeled h will become shorter.

(e)

Angle, θ	Height (in meters)
80°	19.7
70°	18.8
60°	17.3
50°	15.3
40°	12.9
30°	10.0
20°	6.8
10°	3.5

(f) The height of the balloon decreases.



75. $\sin 60° \csc 60° = 1$

True, $\csc x = \dfrac{1}{\sin x} \Rightarrow \sin 60° \csc 60° = \sin 60°\left(\dfrac{1}{\sin 60°}\right) = 1$

76. True, because $\sec(90° - \theta) = \csc \theta$.

77. $\sin 45° + \sin 45° = 1$

False, $\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} = \sqrt{2} \neq 1$

78. True, because

$$\begin{aligned} 1 + \cot^2 \theta &= \csc^2 \theta \\ \cot^2 \theta &= \csc^2 \theta - 1 \\ \cot^2 \theta - \csc^2 \theta &= -1. \end{aligned}$$

79. $\dfrac{\sin 60°}{\sin 30°} = \sin 2°$

False, $\dfrac{\sin 60°}{\sin 30°} = \dfrac{\cos 30°}{\sin 30°} = \cot 30° \approx 1.7321$

$\sin 2° \approx 0.0349$

80. $\tan\left[(5°)^2\right] = \tan^2 5°$

False.

$\tan\left[(5°)^2\right] = \tan 25° \approx 0.466$

$\tan^2 5° = (\tan 5°)^2 \approx 0.008$

81. This is true because the corresponding sides of similar triangles are proportional.

82. Yes. Given $\tan \theta$, $\sec \theta$ can be found from the identity $1 + \tan^2 \theta = \sec^2 \theta$.

83. (a)

θ	0.1	0.2	0.3	0.4	0.5
$\sin \theta$	0.0998	0.1987	0.2955	0.3894	0.4794

(b) In the interval $(0, 0.5]$, $\theta > \sin \theta$.

(c) As $\theta \to 0$, $\sin \theta \to 0$, and $\dfrac{\theta}{\sin \theta} \to 1$.

84. (a)

θ	0	0.3	0.6	0.9	1.2	1.5
$\sin \theta$	0	0.2955	0.5646	0.7833	0.9320	0.9975
$\cos \theta$	1	0.9553	0.8253	0.6216	0.3624	0.0707

(b) On $[0, 1.5]$, $\sin \theta$ is an increasing function.

(c) On $[0, 1.5]$, $\cos \theta$ is a decreasing function.

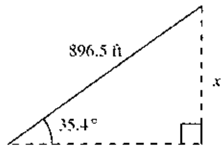
(d) As the angle increases the length of the side opposite the angle increases relative to the length of the hypotenuse and the length of the side adjacent to the angle decreases relative to the length of the hypotenuse. Thus the sine increases and the cosine decreases.

85.

θ	0°	20°	40°	60°	80°
$\cos \theta$	1	0.94	0.77	0.50	0.17
$\sin(90° - \theta)$	1	0.94	0.77	0.50	0.17

$\cos \theta$ and $\sin(90° - \theta)$ are equal. θ and $90° - \theta$ are complementary angles because $\theta + (90° - \theta) = 90°$.

86. (a)



$$\sin 35.4° = \frac{x}{896.5}$$

$$x = 896.5 \sin 35.4° \approx 519.33 \text{ feet}$$

(b) Because the top of the incline is 1693.5 feet above sea level and the vertical rise of the inclined plane is 519.33 feet, the elevation of the lower end of the inclined plan is about $1693.5 - 519.33 = 1174.17$ feet.

(c) Ascent time: $d = rt$

$$896.5 = 300t$$

$$3 \approx t$$

It takes about 3 minutes for the cars to get from the bottom to the top.

Vertical rate: $d = rt$

$$519.33 = r(3)$$

$$r = 173.11 \text{ ft/min}$$

Section 6.3 Trigonometric Functions of Any Angle

- Know the Definitions of Trigonometric Functions of Any Angle.

  If θ is in standard position, (x, y) a point on the terminal side and $r = \sqrt{x^2 + y^2} \neq 0$, then

  $\sin \theta = \dfrac{y}{r}$ $\qquad \csc \theta = \dfrac{r}{y}, y \neq 0$

  $\cos \theta = \dfrac{x}{r}$ $\qquad \sec \theta = \dfrac{r}{x}, x \neq 0$

  $\tan \theta = \dfrac{y}{x}, x \neq 0$ $\qquad \cot \theta = \dfrac{x}{y}, y \neq 0$

- You should know the signs of the trigonometric functions in each quadrant.
- You should know the trigonometric function values of the quadrant angles $0, \frac{\pi}{2}, \pi,$ and $\frac{3\pi}{2}$.
- You should be able to find reference angles.
- You should be able to evaluate trigonometric functions of any angle. (Use reference angles.)
- You should know that the period of sine and cosine is 2π.
- You should know which trigonometric functions are odd and even.

  Even: $\cos x$ and $\sec x$

  Odd: $\sin x$, $\tan x$, $\cot x$, $\csc x$

1. reference

2. periodic

3. period

4. even; odd

5. (a) $(x, y) = (4, 3)$

$r = \sqrt{16 + 9} = 5$

$\sin\theta = \frac{y}{r} = \frac{3}{5}$ $\quad \csc\theta = \frac{r}{y} = \frac{5}{3}$

$\cos\theta = \frac{x}{r} = \frac{4}{5}$ $\quad \sec\theta = \frac{r}{x} = \frac{5}{4}$

$\tan\theta = \frac{y}{x} = \frac{3}{4}$ $\quad \cot\theta = \frac{x}{y} = \frac{4}{3}$

(b) $(x, y) = (-8, 15)$

$r = \sqrt{64 + 225} = 17$

$\sin\theta = \frac{y}{r} = \frac{15}{17}$ $\quad \csc\theta = \frac{r}{y} = \frac{17}{15}$

$\cos\theta = \frac{x}{r} = -\frac{8}{17}$ $\quad \sec\theta = \frac{r}{x} = -\frac{17}{8}$

$\tan\theta = \frac{y}{x} = -\frac{15}{8}$ $\quad \cot\theta = \frac{x}{y} = -\frac{8}{15}$

6. (a) $x = -12, y = -5$

$r = \sqrt{(-12)^2 + (-5)^2} = 13$

$\sin\theta = \frac{y}{r} = -\frac{5}{13}$ $\quad \csc\theta = \frac{r}{y} = -\frac{13}{5}$

$\cos\theta = \frac{x}{r} = -\frac{12}{13}$ $\quad \sec\theta = \frac{r}{x} = -\frac{13}{12}$

$\tan\theta = \frac{y}{x} = \frac{-5}{-12} = \frac{5}{12}$ $\quad \cot\theta = \frac{x}{y} = \frac{-12}{-5} = \frac{12}{5}$

(b) $(x, y) = (1, -1)$

$r = \sqrt{1 + 1} = \sqrt{2}$

$\sin\theta = \frac{y}{r} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\quad \csc\theta = \frac{r}{y} = -\sqrt{2}$

$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \sec\theta = \frac{r}{x} = \sqrt{2}$

$\tan\theta = \frac{y}{x} = -1$ $\quad \cot\theta = \frac{x}{y} = -1$

7. (a) $(x, y) = \left(-\sqrt{3}, -1\right)$

$r = \sqrt{3 + 1} = 2$

$\sin\theta = \frac{y}{r} = -\frac{1}{2}$ $\quad \csc\theta = \frac{r}{y} = -2$

$\cos\theta = \frac{x}{r} = -\frac{\sqrt{3}}{2}$ $\quad \sec\theta = \frac{r}{x} = -\frac{2\sqrt{3}}{3}$

$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{3}$ $\quad \cot\theta = \frac{x}{y} = \sqrt{3}$

(b) $(x, y) = (4, -1)$

$r = \sqrt{16 + 1} = \sqrt{17}$

$\sin\theta = \frac{y}{r} = -\frac{1}{\sqrt{17}} = -\frac{\sqrt{17}}{17}$ $\quad \csc\theta = \frac{r}{y} = -\sqrt{17}$

$\cos\theta = \frac{x}{r} = \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$ $\quad \sec\theta = \frac{r}{x} = \frac{\sqrt{17}}{4}$

$\tan\theta = \frac{y}{x} = -\frac{1}{4}$ $\quad \cot\theta = \frac{x}{y} = -4$

8. (a) $x = 3, y = 1$

$$r = \sqrt{3^2 + 1^2} = \sqrt{10}$$

$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\cos\theta = \frac{x}{r} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\tan\theta = \frac{y}{x} = \frac{1}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{1} = \sqrt{10}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{3}{1} = 3$$

(b) $(x, y) = (-4, 4)$

$$r = \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$$

$$\sin\theta = \frac{y}{r} = \frac{4}{4\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \csc\theta = \frac{r}{y} = \frac{4\sqrt{2}}{4} = \sqrt{2}$$

$$\cos\theta = \frac{x}{r} = -\frac{4}{4\sqrt{2}} = -\frac{\sqrt{2}}{2} \qquad \sec\theta = \frac{r}{x} = -\frac{4\sqrt{2}}{4} = -\sqrt{2}$$

$$\tan\theta = \frac{y}{x} = -1 \qquad \cot\theta = \frac{x}{y} = -1$$

9. $(x, y) = (5, 12)$

$$r = \sqrt{25 + 144} = 13$$

$$\sin\theta = \frac{y}{r} = \frac{12}{13} \qquad \csc\theta = \frac{r}{y} = \frac{13}{12}$$

$$\cos\theta = \frac{x}{r} = \frac{5}{13} \qquad \sec\theta = \frac{r}{x} = \frac{13}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{12}{5} \qquad \cot\theta = \frac{x}{y} = \frac{5}{12}$$

10. $x = 8, y = 15$

$$r = \sqrt{8^2 + 15^2} = 17$$

$$\sin\theta = \frac{y}{r} = \frac{15}{17} \qquad \csc\theta = \frac{r}{y} = \frac{17}{15}$$

$$\cos\theta = \frac{x}{r} = \frac{8}{17} \qquad \sec\theta = \frac{r}{x} = \frac{17}{8}$$

$$\tan\theta = \frac{y}{x} = \frac{15}{8} \qquad \cot\theta = \frac{x}{y} = \frac{8}{15}$$

11. $x = -5, y = -2$

$$r = \sqrt{(-5)^2 + (-2)^2} = \sqrt{29}$$

$$\sin\theta = \frac{y}{r} = \frac{-2}{\sqrt{29}} = -\frac{2\sqrt{29}}{29}$$

$$\cos\theta = \frac{x}{r} = \frac{-5}{\sqrt{29}} = -\frac{5\sqrt{29}}{29}$$

$$\tan\theta = \frac{y}{x} = \frac{-2}{-5} = \frac{2}{5}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{-2} = -\frac{\sqrt{29}}{2}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{29}}{-5} = -\frac{\sqrt{29}}{5}$$

$$\cot\theta = \frac{x}{y} = \frac{-5}{-2} = \frac{5}{2}$$

12. $(x, y) = (-4, 10)$

$r = \sqrt{16 + 100} = 2\sqrt{29}$

$\sin\theta = \frac{y}{r} = \frac{5\sqrt{29}}{29}$

$\cos\theta = \frac{x}{r} = -\frac{2\sqrt{29}}{29}$

$\tan\theta = \frac{y}{x} = -\frac{5}{2}$

$\csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{5}$

$\sec\theta = \frac{r}{x} = -\frac{\sqrt{29}}{2}$

$\cot\theta = \frac{x}{y} = -\frac{2}{5}$

13. $(x, y) = (-5.4, 7.2)$

$r = \sqrt{29.16 + 51.84} = 9$

$\sin\theta = \frac{y}{r} = \frac{7.2}{9} = \frac{4}{5}$ $\qquad \csc\theta = \frac{r}{y} = \frac{9}{7.2} = \frac{5}{4}$

$\cos\theta = \frac{x}{r} = -\frac{5.4}{9} = -\frac{3}{5}$ $\qquad \sec\theta = \frac{r}{x} = -\frac{9}{5.4} = -\frac{5}{3}$

$\tan\theta = \frac{y}{x} = -\frac{7.2}{5.4} = -\frac{4}{3}$ $\qquad \tan\theta = \frac{x}{y} = -\frac{5.4}{7.2} = -\frac{3}{4}$

14. $x = 3\frac{1}{2} = \frac{7}{2}, y = -7\frac{3}{4} = -\frac{31}{4}$

$r = \sqrt{\left(\frac{7}{2}\right)^2 + \left(-\frac{31}{4}\right)^2} = \frac{\sqrt{1157}}{4}$

$\sin\theta = \frac{y}{r} = \frac{-\frac{31}{4}}{\frac{\sqrt{1157}}{4}} = -\frac{31\sqrt{1157}}{1157}$ $\qquad \csc\theta = \frac{r}{y} = \frac{\frac{\sqrt{1157}}{4}}{-\frac{31}{4}} = -\frac{\sqrt{1157}}{31}$

$\cos\theta = \frac{x}{r} = \frac{\frac{7}{2}}{\frac{\sqrt{1157}}{4}} = \frac{14\sqrt{1157}}{1157}$ $\qquad \sec\theta = \frac{r}{x} = \frac{\frac{\sqrt{1157}}{4}}{\frac{7}{2}} = \frac{\sqrt{1157}}{14}$

$\tan\theta = \frac{y}{x} = \frac{-\frac{31}{4}}{\frac{7}{2}} = -\frac{31}{14} = -2\frac{3}{14}$ $\qquad \cot\theta = \frac{x}{y} = \frac{\frac{7}{2}}{-\frac{31}{4}} = -\frac{14}{31}$

15. $\sin\theta > 0 \Rightarrow \theta$ lies in Quadrant I or in Quadrant II.

$\cos\theta > 0 \Rightarrow \theta$ lies in Quadrant I or in Quadrant IV.

$\sin\theta > 0$ and $\cos\theta > 0 \Rightarrow \theta$ lies in Quadrant I.

16. $\sin\theta < 0 \Rightarrow \theta$ lies in Quadrant III or in Quadrant IV.

$\cos\theta < 0 \Rightarrow \theta$ lies in Quadrant II or in Quadrant III.

$\sin\theta < 0$ and $\cos\theta < 0 \Rightarrow \theta$ lies in Quadrant III.

17. $\sin\theta > 0 \Rightarrow \theta$ lies in Quadrant I or in Quadrant II.

$\cos\theta < 0 \Rightarrow \theta$ lies in Quadrant II or in Quadrant III.

$\sin\theta > 0$ and $\cos\theta < 0 \Rightarrow \theta$ lies in Quadrant II.

18. $\sec\theta > 0 \Rightarrow \theta$ lies in Quadrant I or in Quadrant IV.

$\cot\theta < 0 \Rightarrow \theta$ lies in Quadrant II or in Quadrant IV.

$\sec\theta > 0$ and $\cot\theta < 0 \Rightarrow \theta$ lies in Quadrant IV.

19. $\tan\theta < 0$ and $\sin\theta > 0 \Rightarrow \theta$ is in Quadrant II $\Rightarrow x < 0$ and $y > 0$.

$\tan\theta = \frac{y}{x} = \frac{15}{-8} \Rightarrow r = 17$

$\sin\theta = \frac{y}{r} = \frac{15}{17}$ $\qquad \csc\theta = \frac{r}{y} = \frac{17}{15}$

$\cos\theta = \frac{x}{r} = -\frac{8}{17}$ $\qquad \sec\theta = \frac{r}{x} = -\frac{17}{8}$

$\tan\theta = \frac{y}{x} = -\frac{15}{8}$ $\qquad \cot\theta = \frac{x}{y} = -\frac{8}{15}$

20. $\cos\theta = \frac{x}{r} = \frac{8}{17} \Rightarrow y = |15|$

$\tan\theta < 0 \Rightarrow y = -15$

$\sin\theta = \frac{y}{r} = \frac{-15}{17} = -\frac{15}{17}$ $\qquad \csc\theta = -\frac{17}{15}$

$\cos\theta = \frac{x}{r} = \frac{8}{17}$ $\qquad \sec\theta = \frac{17}{8}$

$\tan\theta = \frac{y}{x} = \frac{-15}{8} = -\frac{15}{8}$ $\qquad \cot\theta = -\frac{8}{15}$

21. $\sin\theta = \frac{y}{r} = \frac{3}{5} \Rightarrow x^2 = 25 - 9 = 16$

θ in Quadrant II $\Rightarrow x = -4$

$\sin\theta = \frac{y}{r} = \frac{3}{5}$ $\qquad \csc\theta = \frac{r}{y} = \frac{5}{3}$

$\cos\theta = \frac{x}{r} = -\frac{4}{5}$ $\qquad \sec\theta = \frac{r}{x} = -\frac{5}{4}$

$\tan\theta = \frac{y}{x} = -\frac{3}{4}$ $\qquad \cot\theta = \frac{x}{y} = -\frac{4}{3}$

22. $\cos\theta = \frac{x}{r} = \frac{-4}{5} \Rightarrow y = |3|$

θ in Quadrant III $\Rightarrow y = -3$

$\sin\theta = \frac{y}{r} = -\frac{3}{5}$ $\qquad \csc\theta = -\frac{5}{3}$

$\cos\theta = \frac{x}{r} = -\frac{4}{5}$ $\qquad \sec\theta = -\frac{5}{4}$

$\tan\theta = \frac{y}{x} = \frac{3}{4}$ $\qquad \cot\theta = \frac{4}{3}$

23. $\cot\theta = \frac{x}{y} = -\frac{3}{1} = \frac{3}{-1}$

$\cos\theta > 0 \Rightarrow \theta$ is in Quadrant IV $\Rightarrow x$ is positive;

$x = 3, y = -1, r = \sqrt{10}$

$\sin\theta = \frac{y}{r} = -\frac{\sqrt{10}}{10}$ $\qquad \csc\theta = \frac{r}{y} = -\sqrt{10}$

$\cos\theta = \frac{x}{r} = \frac{3\sqrt{10}}{10}$ $\qquad \sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{3}$

$\tan\theta = \frac{y}{x} = -\frac{1}{3}$ $\qquad \cot\theta = \frac{x}{y} = -3$

24. $\csc\theta = \frac{r}{y} = \frac{4}{1} \Rightarrow x = |\sqrt{15}|$

$\cot\theta < 0 \Rightarrow x = -\sqrt{15}$

$\sin\theta = \frac{y}{r} = \frac{1}{4}$ $\qquad \csc\theta = 4$

$\cos\theta = \frac{x}{r} = -\frac{\sqrt{15}}{4}$ $\qquad \sec\theta = -\frac{4\sqrt{15}}{15}$

$\tan\theta = \frac{y}{x} = -\frac{\sqrt{15}}{15}$ $\qquad \cot\theta = -\sqrt{15}$

25. $\sec\theta = \frac{r}{x} = \frac{2}{-1} \Rightarrow y^2 = 4 - 1 = 3$

$\sin\theta < 0 \Rightarrow \theta$ is in Quadrant III $\Rightarrow y = -\sqrt{3}$

$\sin\theta = \frac{y}{r} = -\frac{\sqrt{3}}{2}$ $\qquad \csc\theta = \frac{r}{y} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{x}{r} = -\frac{1}{2}$ $\qquad \sec\theta = \frac{r}{x} = -2$

$\tan\theta = \frac{y}{x} = \sqrt{3}$ $\qquad \cot\theta = \frac{x}{y} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

26. $\sin\theta = 0 \Rightarrow \theta = 0 + \pi n$

$\sec\theta = -1 \Rightarrow \theta = \pi + 2\pi n$

$y = 0, x = -r$

$\sin\theta = 0$ $\qquad \csc\theta$ is undefined.

$\cos\theta = \frac{x}{r} = -\frac{r}{r} = -1$ $\qquad \sec\theta \frac{r}{x} = \frac{r}{-r} = -1$

$\tan\theta \frac{y}{r} = \frac{\theta}{r} = 0$ $\qquad \cot\theta$ is undefined.

27. $\cot\theta$ is undefined,

$\frac{\pi}{2} \le \theta \le \frac{3\pi}{2} \Rightarrow y = 0 \Rightarrow \theta = \pi$

$\sin\theta = 0$ $\qquad \csc\theta$ is undefined.

$\cos\theta = -1$ $\qquad \sec\theta = -1$

$\tan\theta = 0$ $\qquad \cot\theta$ is undefined.

28. $\tan\theta$ is undefined $\Rightarrow \theta = n\pi + \frac{\pi}{2}$

$\pi \le \theta \le 2\pi \Rightarrow \theta = \frac{3\pi}{2}, x = 0, y = -r$

$\sin\theta = \frac{y}{r} = \frac{-r}{r} = -1$ $\quad$ $\csc\theta = \frac{r}{y} = -1$

$\cos\theta = \frac{x}{r} = \frac{0}{r} = 0$ $\quad$ $\sec\theta$ is undefined.

$\tan\theta$ is undefined. $\quad$ $\cot\theta = \frac{x}{y} = \frac{0}{y} = 0$

29. To find a point on the terminal side of θ, use any point on the line $y = -x$ that lies in Quadrant II. $(-1, 1)$ is one such point.

$x = -1, y = 1, r = \sqrt{2}$

$\sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad$ $\csc\theta = \sqrt{2}$

$\cos\theta = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\quad$ $\sec\theta = -\sqrt{2}$

$\tan\theta = -1$ $\quad$ $\cot\theta = -1$

30. Let $x > 0$.

$\left(-x, -\frac{1}{3}x\right)$, Quadrant III

$r = \sqrt{x^2 + \frac{1}{9}x^2} = \frac{\sqrt{10}x}{3}$

$\sin\theta = \frac{y}{r} = \frac{\left(-\frac{1}{3}\right)x}{\frac{\sqrt{10}x}{3}} = -\frac{\sqrt{10}}{10}$ $\quad$ $\csc\theta = \frac{r}{y} = \frac{\frac{\sqrt{10}x}{3}}{\left(-\frac{1}{3}\right)x} = -\sqrt{10}$

$\cos\theta = \frac{x}{r} = \frac{-x}{\frac{\sqrt{10}x}{3}} = -\frac{3\sqrt{10}}{10}$ $\quad$ $\sec\theta = \frac{r}{x} = \frac{\frac{\sqrt{10}x}{3}}{-x} = -\frac{\sqrt{10}}{3}$

$\tan\theta = \frac{y}{x} = \frac{\left(-\frac{1}{3}\right)x}{-x} = \frac{1}{3}$ $\quad$ $\cot\theta = \frac{x}{y} = \frac{-x}{\left(-\frac{1}{3}\right)x} = 3$

31. To find a point on the terminal side of θ, use any point on the line $y = 2x$ that lies in Quadrant III. $(-1, -2)$ is one such point.

$x = -1, y = -2, y = \sqrt{5}$

$\sin\theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$ $\quad$ $\csc\theta = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2}$

$\cos\theta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$ $\quad$ $\sec\theta = \frac{\sqrt{5}}{-1} = -\sqrt{5}$

$\tan\theta = \frac{-2}{-1} = 2$ $\quad$ $\cot\theta = \frac{-1}{-2} = \frac{1}{2}$

32. Let $x > 0$.

$4x + 3y = 0 \Rightarrow y = -\frac{4}{3}x$

$\left(x, -\frac{4}{3}x\right)$, Quadrant IV

$r = \sqrt{x^2 + \frac{16}{9}x^2} = \frac{5}{3}x$

$\sin\theta = \frac{y}{r} = \frac{\left(-\frac{4}{3}\right)x}{\frac{5}{3}x} = -\frac{4}{5}$ $\quad$ $\csc\theta = -\frac{5}{4}$

$\cos\theta = \frac{x}{r} = \frac{x}{\frac{5}{3}x} = \frac{3}{5}$ $\quad$ $\sec\theta = \frac{5}{3}$

$\tan\theta = \frac{y}{x} = \frac{\left(-\frac{4}{3}\right)x}{x} = -\frac{4}{3}$ $\quad$ $\cot\theta = -\frac{3}{4}$

33. $(x, y) = (-1, 0), r = 1$

$\sin \pi = \dfrac{y}{r} = \dfrac{0}{1} = 0$

34. $(x, y) = (0, -1), r = 1$

$\csc \dfrac{3\pi}{2} = \dfrac{1}{-1} = -1$

35. $(x, y) = (0, -1), r = 1$

$\sec \dfrac{3\pi}{2} = \dfrac{r}{x} = \dfrac{1}{0} \Rightarrow$ undefined

36. $(x, y) = (-1, 0), r = 1$

$\sec \pi = \dfrac{r}{x} = \dfrac{1}{-1} = -1$

37. $(x, y) = (0, 1), r = 1$

$\sin \dfrac{\pi}{2} = \dfrac{y}{r} = \dfrac{1}{1} = 1$

38. $(x, y) = (-1, 0), r = 1$

$\cot \pi = \dfrac{-1}{0}$ undefined.

39. $(x, y) = (-1, 0), r = 1$

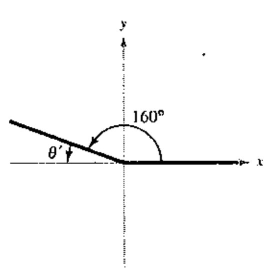
$\csc \pi = \dfrac{r}{y} = \dfrac{1}{0} \Rightarrow$ undefined

40. $(x, y) = (0, 1)$

$\cot \dfrac{\pi}{2} = \dfrac{x}{y} = \dfrac{0}{1} = 0$

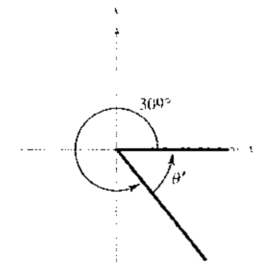
41. $\theta = 160°$

$\theta' = 180° - 160° = 20°$



42. $\theta = 309°$

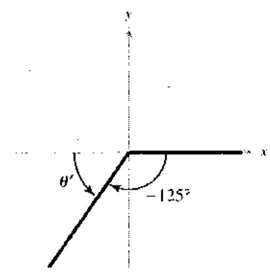
$\theta' = 360° - 309° = 51°$



43. $\theta = -125°$

$360° - 125° = 235°$ (coterminal angle)

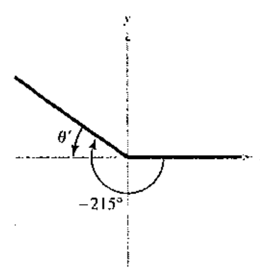
$\theta' = 235° - 180° = 55°$



44. $\theta = -215°$

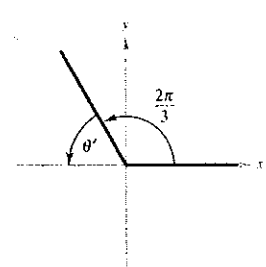
$360° - 215° = 145°$ (coterminal angle)

$\theta' = 180° - 145° = 35°$



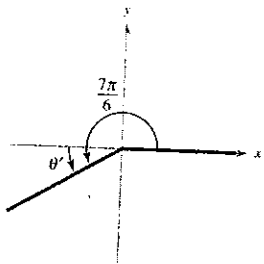
45. $\theta = \dfrac{2\pi}{3}$

$\theta' = \pi - \dfrac{2\pi}{3} = \dfrac{\pi}{3}$



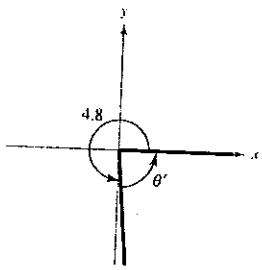
46. $\theta = \dfrac{7\pi}{6}$

$\theta' = \dfrac{7\pi}{6} - \pi = \dfrac{\pi}{6}$



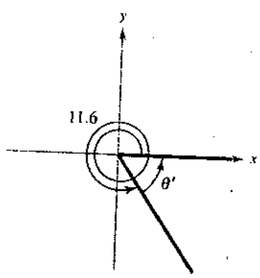
47. $\theta = 4.8$

$\theta' = 2\pi - 4.8\pi \approx 1.4832$



48. $\theta = 11.6$

$\theta' = 4\pi - 11.6 \approx 0.9664$



49. $\theta = 225°, \theta' = 45°$, Quadrant III

$\sin 225° = -\sin 45° = -\dfrac{\sqrt{2}}{2}$

$\cos 225° = -\cos 45° = -\dfrac{\sqrt{2}}{2}$

$\tan 225° = \tan 45° = 1$

50. $\theta = 300°, \theta' = 360° - 300° = 60°$ in Quadrant IV.

$\sin 300° = -\sin 60° = -\dfrac{\sqrt{3}}{2}$

$\cos 300° = \cos 60° = \dfrac{1}{2}$

$\tan 300° = -\tan 60° = -\sqrt{3}$

51. $\theta = 750°, \theta' = 30°$, Quadrant I

$\sin 750° = \sin 30° = \dfrac{1}{2}$

$\cos 750° = \cos 30° = \dfrac{\sqrt{3}}{2}$

$\tan 750° = \tan 30° = \dfrac{\sqrt{3}}{3}$

52. $\theta = -405°, \theta' = 405° - 360° = 45°$ in Quadrant IV.

$\sin(-405°) = -\sin 45° = -\dfrac{\sqrt{2}}{2}$

$\cos(-405°) = \cos 45° = \dfrac{\sqrt{2}}{2}$

$\tan(-405°) = -\tan 45° = -1$

53. $\theta = -150°, \theta' = 30°$, Quadrant III

$\sin(-150°) = -\sin 30° = -\dfrac{1}{2}$

$\cos(-150°) = -\cos 30° = -\dfrac{\sqrt{3}}{2}$

$\tan(-150°) = \tan 30° = \dfrac{\sqrt{3}}{3}$

54. $\theta = -840°$ is coterminal with $240°$.

$\theta' = 240° - 180° = 60°$ in Quadrant III.

$\sin(-840°) = -\sin 60° = -\dfrac{\sqrt{3}}{2}$

$\cos(-840°) = -\cos 60° = -\dfrac{1}{2}$

$\tan(-840°) = \tan 60° = \sqrt{3}$

55. $\theta = \dfrac{2\pi}{3}, \theta' = \dfrac{\pi}{3}$ in Quadrant II

$\sin \dfrac{2\pi}{3} = \sin \dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

$\cos \dfrac{2\pi}{3} = -\cos \dfrac{\pi}{3} = -\dfrac{1}{2}$

$\tan \dfrac{2\pi}{3} = -\tan \dfrac{\pi}{3} = -\sqrt{3}$

56. $\theta = \dfrac{3\pi}{4}, \theta' = \dfrac{\pi}{4}$ in Quadrant II

$\sin \dfrac{3\pi}{4} = \sin \dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

$\cos \dfrac{3\pi}{4} = -\cos \dfrac{\pi}{4} = -\dfrac{\sqrt{2}}{2}$

$\tan \dfrac{3\pi}{4} = -\tan \dfrac{\pi}{4} = -1$

57. $\theta = \frac{5\pi}{4}, \theta' = \frac{\pi}{4}$ in Quadrant III

$$\sin\frac{5\pi}{4} = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$
$$\cos\frac{5\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$
$$\tan\frac{5\pi}{4} = \tan\frac{\pi}{4} = 1$$

58. $\theta = \frac{7\pi}{6}, \theta' = \frac{\pi}{6}$ in Quadrant III

$$\sin\frac{7\pi}{6} = -\sin\frac{\pi}{6} = -\frac{1}{2}$$
$$\cos\frac{7\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$
$$\tan\frac{7\pi}{6} = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

59. $\theta = -\frac{\pi}{6}, \theta' = \frac{\pi}{6}$, Quadrant IV

$$\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$$
$$\cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$
$$\tan\left(-\frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$$

60. $\theta = -\frac{\pi}{2}$ is coterminal with $\frac{3\pi}{2}$.

$$\sin\left(-\frac{\pi}{2}\right) = \sin\frac{3\pi}{2} = -1$$
$$\cos\left(-\frac{\pi}{2}\right) = \cos\frac{3\pi}{2} = 0$$
$$\tan\left(-\frac{\pi}{2}\right) = \tan\frac{3\pi}{2} \text{ is undefined.}$$

61. $\theta = \frac{11\pi}{4}, \theta' = \frac{\pi}{4}$, Quadrant II

$$\sin\frac{11\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$
$$\cos\frac{11\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$
$$\tan\frac{11\pi}{4} = -\tan\frac{\pi}{4} = -1$$

62. $\theta = \frac{10\pi}{3}$ is coterminal with $\frac{4\pi}{3}$.

$\theta' = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$ in Quadrant III.

$$\sin\frac{10\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$
$$\cos\frac{10\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$$
$$\tan\frac{10\pi}{3} = \tan\frac{\pi}{3} = \sqrt{3}$$

63. $\theta = \frac{9\pi}{4}, \theta' = \frac{\pi}{4}$ in Quadrant I

$$\sin\frac{9\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$
$$\cos\frac{9\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$
$$\tan\frac{9\pi}{4} = \tan\frac{\pi}{4} = 1$$

64. $\theta = -\frac{23\pi}{4}, \theta' = \frac{\pi}{4}$ in Quadrant I

$$\sin\left(-\frac{23\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$
$$\cos\left(-\frac{23\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$
$$\tan\left(-\frac{23\pi}{4}\right) = \tan\frac{\pi}{4} = 1$$

65.
$$\sin\theta = -\frac{3}{5}$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \sin^2\theta$$
$$\cos^2\theta = 1 - \left(-\frac{3}{5}\right)^2$$
$$\cos^2\theta = 1 - \frac{9}{25}$$
$$\cos^2\theta = \frac{16}{25}$$

$\cos\theta > 0$ in Quadrant IV.

$$\cos\theta = \frac{4}{5}$$

66.
$$\cot\theta = -3$$
$$1 + \cot^2\theta = \csc^2\theta$$
$$1 + (-3)^2 = \csc^2\theta$$
$$10 = \csc^2\theta$$
$\csc\theta > 0$ in Quadrant II.
$$\sqrt{10} = \csc\theta$$
$$\csc\theta = \frac{1}{\sin\theta}$$
$$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

67. $\tan\theta = \dfrac{3}{2}$
$$\sec^2\theta = 1 + \tan^2\theta$$
$$\sec^2\theta = 1 + \left(\frac{3}{2}\right)^2$$
$$\sec^2\theta = 1 + \frac{9}{4}$$
$$\sec^2\theta = \frac{13}{4}$$
$\sec\theta < 0$ in Quadrant III.
$$\sec\theta = -\frac{\sqrt{13}}{2}$$

68.
$$\csc\theta = -2$$
$$1 + \cot^2\theta = \csc^2\theta$$
$$\cot^2\theta = \csc^2\theta - 1$$
$$\cot^2\theta = (-2)^2 - 1$$
$$\cot^2\theta = 3$$
$\cot\theta < 0$ in Quadrant IV.
$$\cot\theta = -\sqrt{3}$$

69. $\cos\theta = \dfrac{5}{8}$
$$\cos\theta = \frac{1}{\sec\theta} \Rightarrow \sec\theta = \frac{1}{\cos\theta}$$
$$\sec\theta = \frac{1}{\frac{5}{8}} = \frac{8}{5}$$

70.
$$\sec\theta = -\frac{9}{4}$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$\tan^2\theta = \sec^2\theta - 1$$
$$\tan^2\theta = \left(-\frac{9}{4}\right)^2 - 1$$
$$\tan^2\theta = \frac{65}{16}$$
$\tan\theta > 0$ in Quadrant III.
$$\tan\theta = \frac{\sqrt{65}}{4}$$

71. $\sin 10^\circ \approx 0.1736$

72. $\sec 225^\circ = \dfrac{1}{\cos 225^\circ} \approx -1.4142$

73. $\cos(-110^\circ) \approx -0.3420$

74. $\csc(-330^\circ) = \dfrac{1}{\sin(-330^\circ)} = 2.0000$

75. $\tan 304^\circ \approx -1.4826$

76. $\cot 178^\circ \approx -28.6363$

77. $\sec 72^\circ = \dfrac{1}{\cos 72^\circ} \approx 3.2361$

78. $\tan(-188^\circ) = -0.1405$

79. $\tan 4.5 \approx 4.6373$

80. $\cot 1.35 = \dfrac{1}{\tan 1.35} \approx 0.2245$

81. $\tan\left(\dfrac{\pi}{9}\right) \approx 0.3640$

82. $\tan\left(-\dfrac{\pi}{9}\right) \approx -0.3640$

83. $\sin(-0.65) \approx -0.6052$

84. $\sec 0.29 = \dfrac{1}{\cos 0.29} \approx 1.0436$

85. $\cot\left(-\dfrac{11\pi}{8}\right) = \dfrac{1}{\tan(-11\pi/8)} \approx -0.4142$

86. $\csc\left(-\frac{15\pi}{14}\right) = \frac{1}{\sin(-15\pi/14)} \approx 4.4940$

87. (a) $\sin\theta = \frac{1}{2} \Rightarrow$ reference angle is $30°$ or $\frac{\pi}{6}$ and θ is in Quadrant I or Quadrant II.

Values in degrees: $30°, 150°$

Values in radian: $\frac{\pi}{6}, \frac{5\pi}{6}$

(b) $\sin\theta = \frac{1}{2} \Rightarrow$ reference angle is $30°$ or $\frac{\pi}{6}$ and θ is in Quadrant III or Quadrant IV.

Values in degrees: $210°, 330°$

Values in radians: $\frac{7\pi}{6}, \frac{11\pi}{6}$

88. (a) $\cos\theta = \frac{\sqrt{2}}{2} \Rightarrow$ reference angle is $45°$ or $\frac{\pi}{4}$ and θ is in Quadrant I or IV.

Values in degrees: $45°, 315°$

Values in radians: $\frac{\pi}{4}, \frac{7\pi}{4}$

(b) $\cos\theta = -\frac{\sqrt{2}}{2} \Rightarrow$ reference angle is $45°$ or $\frac{\pi}{4}$ and θ is in Quadrant II or III.

Values in degrees: $135°, 225°$

Values in radians: $\frac{3\pi}{4}; \frac{5\pi}{4}$

89. (a) $\csc\theta = \frac{2\sqrt{3}}{3} \Rightarrow$ reference angle is $60°$ or $\frac{\pi}{3}$ and θ is in Quadrant I or Quadrant II.

Values in degrees: $60°, 120°$

Values in radians: $\frac{\pi}{3}, \frac{2\pi}{3}$

(b) $\cot\theta = -1 \Rightarrow$ reference angle is $45°$ or $\frac{\pi}{4}$ and θ is in Quadrant II or Quadrant IV.

Values in degrees: $135°, 315°$

Values in radians: $\frac{3\pi}{4}, \frac{7\pi}{4}$

90. (a) $\sec\theta = 2 \Rightarrow$ reference angle is $60°$ or $\frac{\pi}{3}$ and θ is in Quadrant I or IV.

Values in degrees: $60°, 300°$

Values in radians: $\frac{\pi}{3}, \frac{5\pi}{3}$

(b) $\sec\theta = -2 \Rightarrow$ reference angle is $60°$ or $\frac{\pi}{3}$ and θ is in Quadrant II or III.

Values in degrees: $120°, 240°$

Values in radians: $\frac{2\pi}{3}, \frac{4\pi}{3}$

91. (a) $\tan\theta = 1 \Rightarrow$ reference angle is $45°$ or $\frac{\pi}{4}$ and θ is in Quadrant I or Quadrant III.

Values in degrees: $45°, 225°$

Values in radians: $\frac{\pi}{4}, \frac{5\pi}{4}$

(b) $\cot\theta = -\sqrt{3} \Rightarrow$ reference angle is $30°$ or $\frac{\pi}{6}$ and θ is in Quadrant II or Quadrant IV.

Values in degrees: $150°, 330°$

Values in radians: $\frac{5\pi}{6}, \frac{11\pi}{6}$

92. (a) $\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow$ reference angle is $60°$ or $\frac{\pi}{3}$ and θ is in Quadrant I or II.

Values in degrees: $60°, 120°$

Values in radians: $\frac{\pi}{3}, \frac{2\pi}{3}$

(b) $\sin\theta = -\frac{\sqrt{3}}{2} \Rightarrow$ reference angle is $60°$ or $\frac{\pi}{3}$ and θ is in Quadrant III or IV.

Values in degrees: $240°, 300°$

Values in radians: $\frac{4\pi}{3}, \frac{5\pi}{3}$

93. $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ corresponds to $t = \frac{\pi}{4}$ on the unit circle.

$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ since $\sin t = y$.

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ since $\cos t = x$.

$\tan\frac{\pi}{4} = 1$ since $\tan t = \frac{y}{x}$.

94. $t = \frac{\pi}{3}, (x, y) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ since $\sin t = y$.

$\cos\frac{\pi}{3} = \frac{1}{2}$ since $\cos t = x$.

$\tan\frac{\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$ since $\tan t = \frac{y}{x}$.

95. $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ corresponds to $t = \frac{5\pi}{6}$ on the unit circle.

$\sin\frac{5\pi}{6} = \frac{1}{2}$ since $\sin t = y$.

$\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$ since $\cos t = x$.

$\tan\frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$ since $\tan t = \frac{y}{x}$.

96. $t = \frac{3\pi}{4}, (x, y) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ because $\sin t = y$.

$\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$ because $\cos t = x$.

$\tan\frac{3\pi}{4} = -1$ because $\tan t = \frac{y}{x}$.

97. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ corresponds to $t = \frac{4\pi}{3}$ on the unit circle.

$\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$ since $\sin t = y$.

$\cos\frac{4\pi}{3} = -\frac{1}{2}$ since $\cos t = x$.

$\tan\frac{4\pi}{3} = \sqrt{3}$ since $\tan t = \frac{y}{x}$.

98. $t = \frac{5\pi}{3}, (x, y) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$ because $\sin t = y$.

$\cos\frac{5\pi}{3} = \frac{1}{2}$ because $\cos t = x$.

$\tan\frac{5\pi}{3} = -\sqrt{3}$ because $\tan t = \frac{y}{x}$.

99. $t = \frac{\pi}{2}, (x, y) = (0, 1)$

$\sin\frac{\pi}{2} = 1$ because $\sin t = y$.

$\cos\frac{\pi}{2} = 0$ because $\cos t = x$.

$\tan\frac{\pi}{2}$ is undefined because $\tan t = \frac{y}{x} = \frac{1}{0}$.

100. $t = \pi, (x, y) = (-1, 0)$

$\sin\pi = 0$ since $\sin t = y$.

$\cos\pi = -1$ since $\cos t = x$.

$\tan\pi = \frac{0}{-1} = 0$ since $\tan t = \frac{y}{x}$.

101. (a) $\sin 5 \approx -1$

(b) $\cos 2 \approx -0.4$

102. (a) $\sin 0.75 = y \approx 0.7$

(b) $\cos 2.5 = x \approx -0.8$

103. (a) $\sin t = 0.25$

$t \approx 0.25$ or 2.89

(b) $\cos t = -0.25$

$t \approx 1.82$ or 4.46

104. (a) $\sin t = -0.75$

$t \approx 4.0$ or $t \approx 5.4$

(b) $\cos t = 0.75$

$t \approx 0.72$ or $t \approx 5.56$

105. (a) New York City:

$N \approx 22.1\sin(0.52t - 2.22) + 55.01$

Fairbanks: $F \approx 36.6\sin(0.50t - 1.83) + 25.61$

(b)

Month	New York City	Fairbanks
February	35°	−1°
March	41°	14°
May	63°	48°
June	72°	59°
August	76°	56°
September	69°	42°
November	47°	7°

(c) The periods are about the same for both models, approximately 12 months.

106. $S = 23.1 + 0.442t + 4.3 \cos \dfrac{\pi t}{6}$

(a) $t = 2; S = 23.1 + 0.442(2) + 4.3 \cos \dfrac{\pi(2)}{6} = 26{,}134$ snowboards

(b) $t = 14; S = 23.1 + 0.442(14) + 4.3 \cos \dfrac{\pi(14)}{6} \approx 31{,}438$ snowboards

(c) $t = 6; S = 23.1 + 0.442(6) + 4.3 \cos \dfrac{\pi(6)}{6} \approx 21{,}452$ snowboards

(d) $t = 18; S = 23.1 + 0.442(18) + 4.3 \cos \dfrac{\pi(18)}{6} \approx 26{,}756$ snowboards

107. $y(t) = 2 \cos 6t$

(a) $y(0) = 2 \cos 0 = 2$ centimeters

(b) $y\left(\frac{1}{4}\right) = 2 \cos\left(\frac{3}{2}\right) \approx 0.14$ centimeter

(c) $y\left(\frac{1}{2}\right) = 2 \cos 3 \approx -1.98$ centimeters

108. $y(t) = 2e^{-t} \cos 6t$

(a) $t = 0$

$y(0) = 2e^{-0} \cos 0 = 2$ centimeters

(b) $t = \frac{1}{4}$

$y\left(\frac{1}{4}\right) = 2e^{-1/4} \cos\left(6 \cdot \frac{1}{4}\right) \approx 0.11$ centimeter

(c) $t = \frac{1}{2}$

$y\left(\frac{1}{2}\right) = 2e^{-1/2} \cos\left(6 \cdot \frac{1}{2}\right) \approx -1.2$ centimeters

109. $I = 5e^{-2t} \sin t$

$I(0.7) = 5e^{-1.4} \sin 0.7 \approx 0.79$

110. $\sin \theta = \dfrac{6}{d} \Rightarrow d = \dfrac{6}{\sin \theta}$

(a) $\theta = 30°$

$$d = \frac{6}{\sin 30°} = \frac{6}{1/2} = 12 \text{ miles}$$

(b) $\theta = 90°$

$$d = \frac{6}{\sin 90°} = \frac{6}{1} = 6 \text{ miles}$$

(c) $\theta = 120°$

$$d = \frac{6}{\sin 120°} = \frac{6}{\sqrt{3}/2} \approx 6.9 \text{ miles}$$

111. False. In each of the four quadrants, the sign of the secant function and the cosine function will be the same since they are reciprocals of each other.

112. False. The reference angle is always acute by definition. For θ in Quadrant II, $\theta' = 180° - \theta$. For θ in Quadrant III, $\theta' = \theta - 180°$. For θ in Quadrant IV, $\theta' = 360° - \theta$.

113.
$$\begin{aligned} h(t) &= f(t)g(t) \\ h(-t) &= f(-t)g(-t) \\ &= -f(t)g(t) \\ &= -h(t) \end{aligned}$$

Therefore, $h(t)$ is odd.

114. As θ increases from $0°$ to $90°$, x decreases from 12 cm to 0 cm, y increases from 0 cm to 12 cm, $\sin \theta = \dfrac{y}{12}$ increases from 0 to 1, $\cos \theta = \dfrac{y}{12}$ decreases from 1 to 0, and $\tan \theta = \dfrac{y}{x}$ increases without bound (and is undefined at $\theta = 90°$).

115. (a)

θ	$0°$	$20°$	$40°$	$60°$	$80°$
$\sin\theta$	0	0.342	0.643	0.866	0.985
$\sin(180° - \theta)$	0	0.342	0.643	0.866	0.985

(b) Conjecture: $\sin\theta = \sin(180° - \theta)$

116. (a)

θ	0	0.3	0.6	0.9	1.2	1.5
$\cos\left(\frac{3\pi}{2} - \theta\right)$	0	-0.2955	-0.5646	-0.7833	-0.9320	-0.9975
$-\sin\theta$	0	-0.2955	-0.5646	-0.7833	-0.9320	-0.9975

(b) $\cos\left(\frac{3\pi}{2} - \theta\right) = -\sin\theta$

117. $y = \sin x$

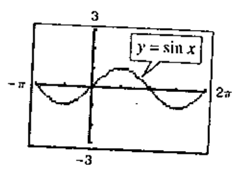
Domain: All real numbers x

Range: $[-1, 1]$

Period: 2π

Zeros: $n\pi$

The function is odd.



$y = \cos x$

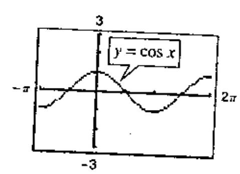
Domain: All real numbers x

Range: $[-1, 1]$

Period: 2π

Zeros: $n\pi + \frac{\pi}{2}$

The function is even.



$y = \tan x$

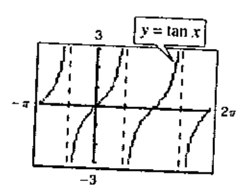
Domain: All real numbers x except $x = n\pi + \frac{\pi}{2}$

Range: $(-\infty, \infty)$

Period: π

Zeros: $n\pi$

The function is odd.



$y = \csc x$

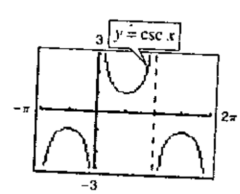
Domain: All real numbers x except $x = n\pi$

Range: $(-\infty, -1] \cup [1, \infty)$

Period: 2π

Zeros: none

The function is odd.



$y = \sec x$

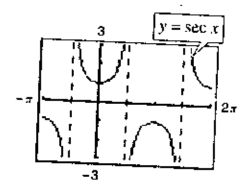
Domain: All real numbers x except $x = n\pi + \frac{\pi}{2}$

Range: $(-\infty, -1] \cup [1, \infty)$

Period: 2π

Zeros: none

The function is even.



$y = \cot x$

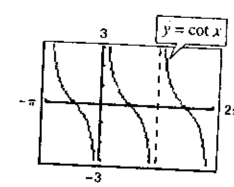
Domain: All real numbers x except $x = n\pi$

Range: $(-\infty, \infty)$

Period: π

Zeros: $n\pi + \frac{\pi}{2}$

The function is odd.



The secant function is similar to the tangent function because they both have vertical asymptotes at $x = n\pi + \frac{\pi}{2}$.

The cotangent function and the cosecant function both have vertical asymptotes at $x = n\pi$. A maximum point on the sine curve corresponds to a relative minimum on the cosecant curve. The maximum points of sine and cosine are interchanged with the minimum points of cosecant and secant. The x-intercepts of the sine and cosine functions become vertical asymptotes of the cosecant and secant functions.

The graphs of sine and cosine may be translated left or right (respectively) to $\pi/2$ to coincide with each other.

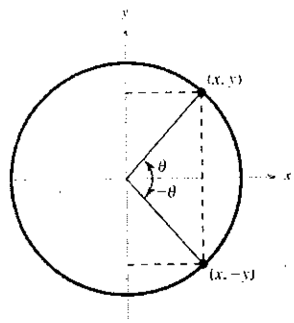
118. Answers will vary.

119. (a) The points have y-axis symmetry.

(b) $\sin t_1 = \sin(\pi - t_1)$ because they have the same y-value.

(c) $\cos(\pi - t_1) = -\cos t_1$ because the x-values have opposite signs.

120. (a)



The graph is a circle of radius 1 centered at $(0, 0)$.

(b) The t-values represent the central angle in radians.

The x- and y-values represent the location in the coordinate plane.

(c) $-1 \le x \le 1$

$-1 \le y \le 1$

Section 6.4 Graphs of Sine and Cosine Functions

- You should be able to graph $y = a\sin(bx - c)$ and $y = a\cos(bx - c)$. (Assume $b > 0$.)
- Amplitude: $|a|$
- Period: $\dfrac{2\pi}{b}$
- Shift: Solve $bx - c = 0$ and $bx - c = 2\pi$.
- Key increments: $\frac{1}{4}$ (period)

1. cycle

2. amplitude

3. phase shift

4. vertical translation

5. $y = 2\sin 5x$

Period: $\dfrac{2\pi}{5}$

Amplitude: $|2| = 2$

6. $y = 3\cos 2x$

Period: $\dfrac{2\pi}{2} = \pi$

Amplitude: $|3| = 3$

7. $y = \dfrac{3}{4}\cos\dfrac{x}{2}$

Period: $\dfrac{2\pi}{1/2} = 4\pi$

Amplitude: $\left|\dfrac{3}{4}\right| = \dfrac{3}{4}$