## 6.1 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

**VOCABULARY:** Fill in the blanks.

1. ________ means "measurement of triangles."
2. An ________ is determined by rotating a ray about its endpoint.
3. Two angles that have the same initial and terminal sides are ________.
4. The angle measure that is equivalent to $\frac{1}{360}$ of a complete revolution about an angle's vertex is one ________.
5. Angles with measures between 0° and 90° are ________ angles, and angles with measures between 90° and 180° are ________ angles.
6. Two positive angles that have a sum of 90° are ________ angles, whereas two positive angles that have a sum of 180° are ________ angles.
7. One ________ is the measure of a central angle that intercepts an arc equal to the radius of the circle.
8. The ________ speed of a particle is the ratio of the arc length traveled to the time traveled.
9. The ________ speed of a particle is the ratio of the change in the central angle to time.
10. The area of a sector of a circle with radius $r$ and central angle $\theta$, where $\theta$ is measured in radians, is given by the formula ________.

### SKILLS AND APPLICATIONS

In Exercises 11–14, estimate the number of degrees in the angle.

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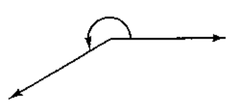

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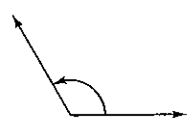

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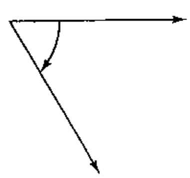

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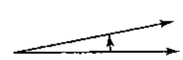


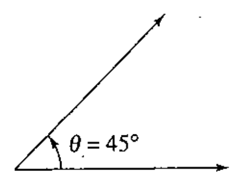
In Exercises 15–18, determine the quadrant in which each angle lies.

| | | |
|---|---|---|
| **15.** | (a) 130° | (b) 285° |
| **16.** | (a) 8.3° | (b) 257° 30′ |
| **17.** | (a) −132° 50′ | (b) −336° |
| **18.** | (a) −260° | (b) −3.4° |

In Exercises 19–22, sketch each angle in standard position.

| | | |
|---|---|---|
| **19.** | (a) 30° | (b) 150° |
| **20.** | (a) −270° | (b) −120° |
| **21.** | (a) 405° | (b) 480° |
| **22.** | (a) −750° | (b) −600° |

In Exercises 23–26, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in degrees.

**23.** (a)
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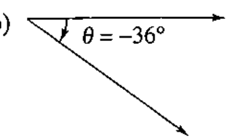
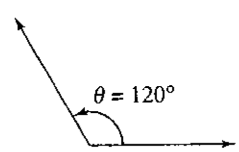



**24.** (a)
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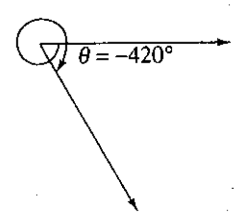



| | | |
|---|---|---|
| **25.** | (a) $\theta = 300°$ | (b) $\theta = 740°$ |
| **26.** | (a) $\theta = -520°$ | (b) $\theta = 230°$ |

In Exercises 27–30, convert each angle measure to decimal degree form.

| | | |
|---|---|---|
| **27.** | (a) 54° 45′ | (b) −128° 30′ |
| **28.** | (a) 245° 10′ | (b) 2° 12′ |
| **29.** | (a) 85° 18′ 30″ | (b) 330° 25″ |
| **30.** | (a) −135° 36″ | (b) −408° 16′ 20″ |

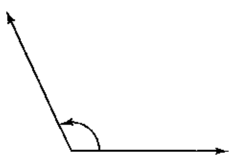
In Exercises 31–34, convert each angle measure to D° M′ S″ form.

| | | |
|---|---|---|
| **31.** | (a) 240.6° | (b) −145.8° |
| **32.** | (a) −345.12° | (b) 0.45° |
| **33.** | (a) 2.5° | (b) −3.58° |
| **34.** | (a) −0.355° | (b) 0.7865° |

In Exercises 35–38, find (if possible) the complement and supplement of each angle.

35. (a) $18^\circ$ (b) $85^\circ$
36. (a) $46^\circ$ (b) $93^\circ$
37. (a) $24^\circ$ (b) $126^\circ$
38. (a) $87^\circ$ (b) $166^\circ$

In Exercises 39–44, estimate the angle to the nearest one-half radian.

39.


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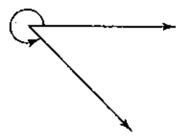

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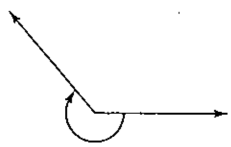

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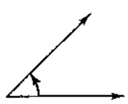

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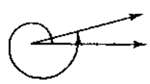


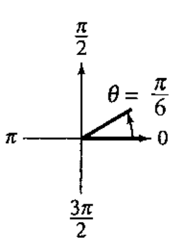
In Exercises 45–50, determine the quadrant in which each angle lies. (The angle measure is given in radians.)

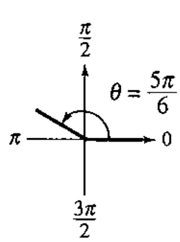
45. (a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{4}$
46. (a) $\frac{11\pi}{8}$ (b) $\frac{9\pi}{8}$
47. (a) $\frac{\pi}{5}$ (b) $\frac{7\pi}{5}$
48. (a) $-\frac{\pi}{12}$ (b) $-\frac{11\pi}{9}$
49. (a) $-1$ (b) $-2$
50. (a) 6.02 (b) 2.25

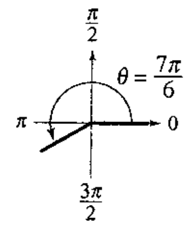
In Exercises 51–54, sketch each angle in standard position.

51. (a) $\frac{\pi}{3}$ (b) $-\frac{2\pi}{3}$
52. (a) $-\frac{7\pi}{4}$ (b) $\frac{5\pi}{2}$
53. (a) $\frac{11\pi}{6}$ (b) $-3$
54. (a) 4 (b) $7\pi$

In Exercises 55–60, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in radians.

55. (a)



(b)



56. (a)
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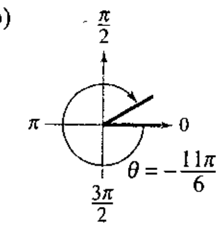



57. (a) $\theta = \frac{2\pi}{3}$ (b) $\theta = -\frac{\pi}{12}$
58. (a) $\theta = -\frac{3\pi}{4}$ (b) $\theta = -\frac{7\pi}{4}$
59. (a) $\theta = -\frac{9\pi}{4}$ (b) $\theta = -\frac{2\pi}{15}$
60. (a) $\theta = \frac{8\pi}{9}$ (b) $\theta = \frac{8\pi}{45}$

In Exercises 61 and 62, find (if possible) the complement and supplement of each angle.

61. (a) $\frac{\pi}{12}$ (b) $\frac{11\pi}{12}$
62. (a) $\frac{\pi}{6}$ (b) $\frac{3\pi}{4}$

In Exercises 63–66, rewrite each angle in radian measure as a multiple of $\pi$. (Do not use a calculator.)

63. (a) $30^\circ$ (b) $45^\circ$
64. (a) $315^\circ$ (b) $120^\circ$
65. (a) $-20^\circ$ (b) $-60^\circ$
66. (a) $-270^\circ$ (b) $144^\circ$

In Exercises 67–70, rewrite each angle in degree measure. (Do not use a calculator.)

67. (a) $\frac{3\pi}{2}$ (b) $\frac{7\pi}{6}$
68. (a) $-\frac{7\pi}{12}$ (b) $\frac{\pi}{9}$
69. (a) $\frac{5\pi}{4}$ (b) $-\frac{7\pi}{3}$
70. (a) $\frac{11\pi}{6}$ (b) $\frac{34\pi}{15}$

In Exercises 71–78, convert the angle measure from degrees to radians. Round to three decimal places.

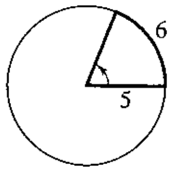
71. $45^\circ$
72. $87.4^\circ$
73. $-216.35^\circ$
74. $-48.27^\circ$
75. $532^\circ$
76. $345^\circ$
77. $-0.83^\circ$
78. $0.54^\circ$

In Exercises 79–84, convert the angle measure from radians to degrees. Round to three decimal places.

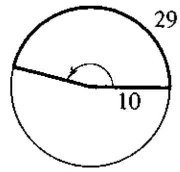
79. $\frac{\pi}{7}$
80. $\frac{5\pi}{11}$
81. $\frac{15\pi}{8}$
82. $\frac{13\pi}{2}$
83. $-2$
84. $-0.57$

In Exercises 85–88, find the angle in radians.

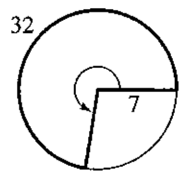
**85.**



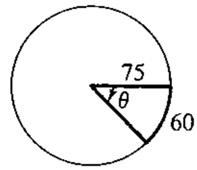

**86.**




**87.**




**88.**




In Exercises 89–92, find the radian measure of the central angle of a circle of radius $r$ that intercepts an arc of length $s$.

| | *Radius r* | *Arc Length s* |
|---|---|---|
| **89.** | 4 inches | 18 inches |
| **90.** | 14 feet | 8 feet |
| **91.** | 14.5 centimeters | 25 centimeters |
| **92.** | 80 kilometers | 160 kilometers |

In Exercises 93–96, find the length of the arc on a circle of radius $r$ intercepted by a central angle $\theta$.

| | *Radius r* | *Central Angle θ* |
|---|---|---|
| **93.** | 15 inches | 120° |
| **94.** | 9 feet | 60° |
| **95.** | 3 meters | 1 radian |
| **96.** | 20 centimeters | $\pi/4$ radian |

In Exercises 97–100, find the area of the sector of the circle with radius $r$ and central angle $\theta$.

| | *Radius r* | *Central Angle θ* |
|---|---|---|
| **97.** | 6 inches | $\pi/3$ radians |
| **98.** | 12 millimeters | $\pi/4$ radian |
| **99.** | 2.5 feet | 225° |
| **100.** | 1.4 miles | 330° |

**DISTANCE BETWEEN CITIES** In Exercises 101 and 102, find the distance between the cities. Assume that Earth is a sphere of radius 4000 miles and that the cities are on the same longitude (one city is due north of the other).

| | *City* | *Latitude* |
|---|---|---|
| **101.** | Dallas, Texas | 32° 47′ 9″ N |
| | Omaha, Nebraska | 41° 15′ 50″ N |
| **102.** | San Francisco, California | 37° 47′ 36″ N |
| | Seattle, Washington | 47° 37′ 18″ N |

**103. DIFFERENCE IN LATITUDES** Assuming that Earth is a sphere of radius 6378 kilometers, what is the difference in the latitudes of Syracuse, New York and Annapolis, Maryland, where Syracuse is 450 kilometers due north of Annapolis?

**104. DIFFERENCE IN LATITUDES** Assuming that Earth is a sphere of radius 6378 kilometers, what is the difference in the latitudes of Lynchburg, Virginia and Myrtle Beach, South Carolina, where Lynchburg is 400 kilometers due north of Myrtle Beach?

**105. INSTRUMENTATION** The pointer on a voltmeter is 6 centimeters in length (see figure). Find the angle through which the pointer rotates when it moves 2.5 centimeters on the scale.

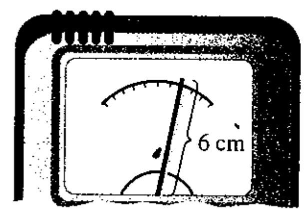



FIGURE FOR 105

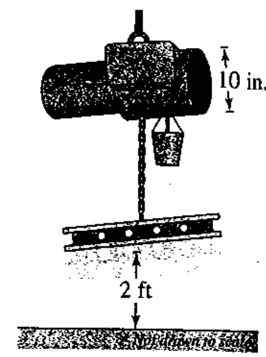



FIGURE FOR 106

**106. ELECTRIC HOIST** An electric hoist is being used to lift a beam (see figure). The diameter of the drum on the hoist is 10 inches, and the beam must be raised 2 feet. Find the number of degrees through which the drum must rotate.

**107. ANGULAR SPEED** A car is moving at a rate of 65 miles per hour, and the diameter of its wheels is 2.5 feet.

(a) Find the number of revolutions per minute the wheels are rotating.

(b) Find the angular speed of the wheels in radians per minute.

**108. ANGULAR SPEED** A two-inch-diameter pulley on an electric motor that runs at 1700 revolutions per minute is connected by a belt to a four-inch-diameter pulley on a saw arbor.

(a) Find the angular speed (in radians per minute) of each pulley.

(b) Find the revolutions per minute of the saw.

**109. LINEAR AND ANGULAR SPEED** A $7\frac{1}{4}$-inch circular power saw blade rotates at 5200 revolutions per minute.

(a) Find the angular speed of the saw blade in radians per minute.

(b) Find the linear speed (in feet per minute) of one of the 24 cutting teeth as they contact the wood being cut.

110. **LINEAR AND ANGULAR SPEED** A carousel with a 50-foot diameter makes 4 revolutions per minute.
   (a) Find the angular speed of the carousel in radians per minute.
   (b) Find the linear speed (in feet per minute) of the platform rim of the carousel.

111. **LINEAR AND ANGULAR SPEED** The diameter of a DVD is approximately 12 centimeters. The drive motor of the DVD player is controlled to rotate precisely between 200 and 500 revolutions per minute, depending on what track is being read.
   (a) Find an interval for the angular speed of a DVD as it rotates.
   (b) Find an interval for the linear speed of a point on the outermost track as the DVD rotates.

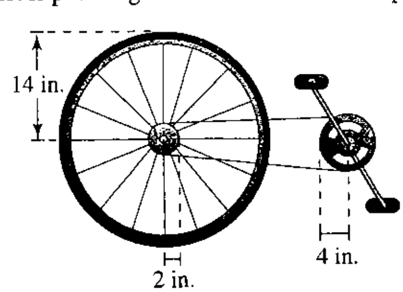
112. **ANGULAR SPEED** A computerized spin balance machine rotates a 25-inch-diameter tire at 480 revolutions per minute.
   (a) Find the road speed (in miles per hour) at which the tire is being balanced.
   (b) At what rate should the spin balance machine be set so that the tire is being tested for 55 miles per hour?

113. **AREA** A sprinkler on a golf green is set to spray water over a distance of 15 meters and to rotate through an angle of 140°. Draw a diagram that shows the region that can be irrigated with the sprinkler. Find the area of the region.

114. **AREA** A car's rear windshield wiper rotates 125°. The total length of the wiper mechanism is 25 inches and the length of the wiper blade is 14 inches. Find the area wiped by the wiper blade.

115. **AREA** A sprinkler system on a farm is set to spray water over a distance of 35 meters and rotates through an angle of 140°. Draw a diagram that shows the region that can be irrigated with the sprinkler. Find the area of the region.

116. **SPEED OF A BICYCLE** The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 4 inches, 2 inches, and 14 inches, respectively. A cyclist is pedaling at a rate of 1 revolution per second.




   (a) Find the speed of the bicycle in feet per second and miles per hour.
   (b) Use your result from part (a) to write a function for the distance $d$ (in miles) a cyclist travels in terms of the number $n$ of revolutions of the pedal sprocket.
   (c) Write a function for the distance $d$ (in miles) a cyclist travels in terms of the time $t$ (in seconds). Compare this function with the function from part (b).
   (d) Classify the types of functions you found in parts (b) and (c). Explain your reasoning.

## EXPLORATION

**TRUE OR FALSE?** In Exercises 117–119, determine whether the statement is true or false. Justify your answer.

117. A measurement of 4 radians corresponds to two complete revolutions from the initial side to the terminal side of an angle.

118. The difference between the measures of two coterminal angles is always a multiple of 360° if expressed in degrees and is always a multiple of $2\pi$ radians if expressed in radians.

119. An angle that measures $-1260°$ lies in Quadrant III.

120. **CAPSTONE** Write a short paragraph in your own words explaining the meaning of each of the following.
   (a) an angle in standard position
   (b) positive and negative angles
   (c) coterminal angles
   (d) angle measure in degrees and radians
   (e) obtuse and acute angles
   (f) complementary and supplementary angles

121. **THINK ABOUT IT** A fan motor turns at a given angular speed. How does the speed of the tips of the blades change if a fan of greater diameter is installed on the motor? Explain.

122. **THINK ABOUT IT** Is a degree or a radian the larger unit of measure? Explain.

123. **WRITING** If the radius of a circle is increasing and the magnitude of a central angle is held constant, how is the length of the intercepted arc changing? Explain your reasoning.

124. **PROOF** Prove that the area of a circular sector of radius $r$ with central angle $\theta$ is $A = \frac{1}{2}\theta r^2$, where $\theta$ is measured in radians.

## 6.2 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

### VOCABULARY

1. Match the trigonometric function with its right triangle definition.

(a) Sine (b) Cosine (c) Tangent (d) Cosecant (e) Secant (f) Cotangent

(i) $\frac{\text{hypotenuse}}{\text{adjacent}}$ (ii) $\frac{\text{adjacent}}{\text{opposite}}$ (iii) $\frac{\text{hypotenuse}}{\text{opposite}}$ (iv) $\frac{\text{adjacent}}{\text{hypotenuse}}$ (v) $\frac{\text{opposite}}{\text{hypotenuse}}$ (vi) $\frac{\text{opposite}}{\text{adjacent}}$

In Exercises 2–4, fill in the blanks.

2. Relative to the angle $\theta$, the three sides of a right triangle are the ________ side, the ________ side, and the ________.
3. Cofunctions of ________ angles are equal.
4. An angle that measures from the horizontal upward to an object is called the angle of ________, whereas an angle that measures from the horizontal downward to an object is called the angle of ________.

### SKILLS AND APPLICATIONS

In Exercises 5–8, find the exact values of the six trigonometric functions of the angle $\theta$ shown in the figure. (Use the Pythagorean Theorem to find the third side of the triangle.)

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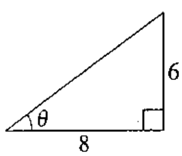


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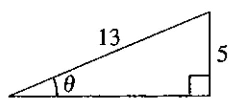


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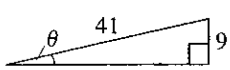


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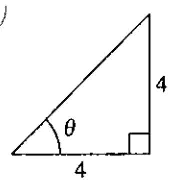


In Exercises 9–12, find the exact values of the six trigonometric functions of the angle $\theta$ for each of the two triangles. Explain why the function values are the same.

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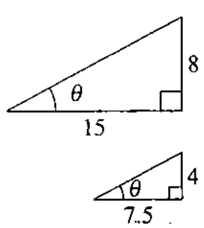


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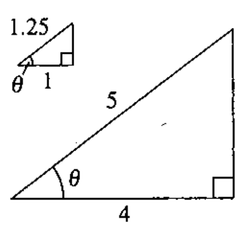


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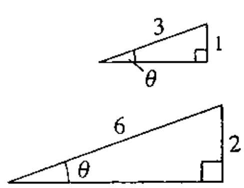


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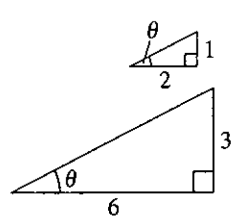



In Exercises 13–20, sketch a right triangle corresponding to the trigonometric function of the acute angle $\theta$. Use the Pythagorean Theorem to determine the third side and then find the other five trigonometric functions of $\theta$.

13. $\tan\theta = \frac{3}{4}$
14. $\cos\theta = \frac{5}{6}$
15. $\sec\theta = \frac{3}{2}$
16. $\tan\theta = \frac{4}{5}$
17. $\sin\theta = \frac{1}{5}$
18. $\sec\theta = \frac{17}{7}$
19. $\cot\theta = 3$
20. $\csc\theta = 9$

In Exercises 21–30, construct an appropriate triangle to complete the table. ($0° \le \theta \le 90°$, $0 \le \theta \le \pi/2$)

| | *Function* | *θ (deg)* | *θ (rad)* | *Function Value* |
|---|---|---|---|---|
| 21. | sin | 30° | | |
| 22. | cos | 45° | | |
| 23. | sec | | $\frac{\pi}{4}$ | |
| 24. | tan | | $\frac{\pi}{3}$ | |
| 25. | cot | | | $\frac{\sqrt{3}}{3}$ |
| 26. | csc | | | $\sqrt{2}$ |
| 27. | csc | | $\frac{\pi}{6}$ | |
| 28. | sin | | $\frac{\pi}{4}$ | |
| 29. | cot | | | 1 |
| 30. | tan | | | $\frac{\sqrt{3}}{3}$ |

In Exercises 31–36, use the given function value(s), and trigonometric identities (including the cofunction identities), to find the indicated trigonometric functions.

31. $\sin 60° = \dfrac{\sqrt{3}}{2}$, $\cos 60° = \dfrac{1}{2}$

(a) $\sin 30°$ (b) $\cos 30°$

(c) $\tan 60°$ (d) $\cot 60°$

32. $\sin 30° = \dfrac{1}{2}$, $\tan 30° = \dfrac{\sqrt{3}}{3}$

(a) $\csc 30°$ (b) $\cot 60°$

(c) $\cos 30°$ (d) $\cot 30°$

33. $\cos \theta = \frac{1}{3}$

(a) $\sin \theta$ (b) $\tan \theta$

(c) $\sec \theta$ (d) $\csc(90° - \theta)$

34. $\sec \theta = 5$

(a) $\cos \theta$ (b) $\cot \theta$

(c) $\cot(90° - \theta)$ (d) $\sin \theta$

35. $\cot \alpha = 5$

(a) $\tan \alpha$ (b) $\csc \alpha$

(c) $\cot(90° - \alpha)$ (d) $\cos \alpha$

36. $\cos \beta = \dfrac{\sqrt{7}}{4}$

(a) $\sec \beta$ (b) $\sin \beta$

(c) $\cot \beta$ (d) $\sin(90° - \beta)$

In Exercises 37–46, use trigonometric identities to transform the left side of the equation into the right side $(0 < \theta < \pi/2)$.

37. $\tan \theta \cot \theta = 1$

38. $\cos \theta \sec \theta = 1$

39. $\tan \alpha \cos \alpha = \sin \alpha$

40. $\cot \alpha \sin \alpha = \cos \alpha$

41. $(1 + \sin \theta)(1 - \sin \theta) = \cos^2 \theta$

42. $(1 + \cos \theta)(1 - \cos \theta) = \sin^2 \theta$

43. $(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$

44. $\sin^2 \theta - \cos^2 \theta = 2 \sin^2 \theta - 1$

45. $\dfrac{\sin \theta}{\cos \theta} + \dfrac{\cos \theta}{\sin \theta} = \csc \theta \sec \theta$

46. $\dfrac{\tan \beta + \cot \beta}{\tan \beta} = \csc^2 \beta$

In Exercises 47–54, use a calculator to evaluate each function. Round your answers to four decimal places. (Be sure the calculator is in the correct angle mode.)

47. (a) $\tan 23.5°$ (b) $\cot 66.5°$

48. (a) $\sin 16.35°$ (b) $\csc 16.35°$

49. (a) $\cos 16° 18'$ (b) $\sin 73° 56'$

50. (a) $\sec 42° 12'$ (b) $\csc 48° 7'$

51. (a) $\cot \dfrac{\pi}{16}$ (b) $\tan \dfrac{\pi}{16}$

52. (a) $\sec 0.75$ (b) $\cos 0.75$

53. (a) $\csc 1$ (b) $\tan \frac{1}{2}$

54. (a) $\sec\left(\dfrac{\pi}{2} - 1\right)$ (b) $\cot\left(\dfrac{\pi}{2} - \dfrac{1}{2}\right)$

In Exercises 55–60, find the values of $\theta$ in degrees $(0° < \theta < 90°)$ and radians $(0 < \theta < \pi/2)$ without the aid of a calculator.

55. (a) $\sin \theta = \frac{1}{2}$ (b) $\csc \theta = 2$

56. (a) $\cos \theta = \dfrac{\sqrt{2}}{2}$ (b) $\tan \theta = 1$

57. (a) $\sec \theta = 2$ (b) $\cot \theta = 1$

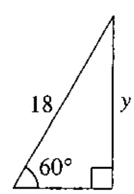
58. (a) $\tan \theta = \sqrt{3}$ (b) $\cos \theta = \frac{1}{2}$

59. (a) $\csc \theta = \dfrac{2\sqrt{3}}{3}$ (b) $\sin \theta = \dfrac{\sqrt{2}}{2}$

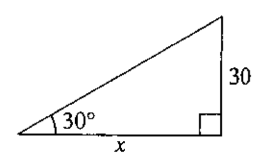
60. (a) $\cot \theta = \dfrac{\sqrt{3}}{3}$ (b) $\sec \theta = \sqrt{2}$

In Exercises 61–64, solve for $x$, $y$, or $r$ as indicated.

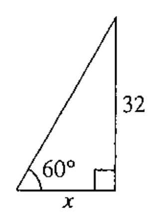
61. Solve for $y$.



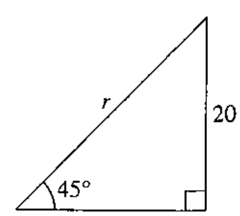

62. Solve for $x$.




63. Solve for $x$.




64. Solve for $r$.

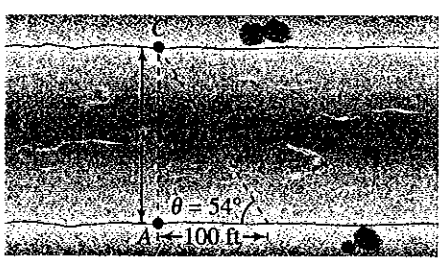



65. **EMPIRE STATE BUILDING** You are standing 45 meters from the base of the Empire State Building. You estimate that the angle of elevation to the top of the 86th floor (the observatory) is 82°. If the total height of the building is another 123 meters above the 86th floor, what is the approximate height of the building? One of your friends is on the 86th floor. What is the distance between you and your friend?

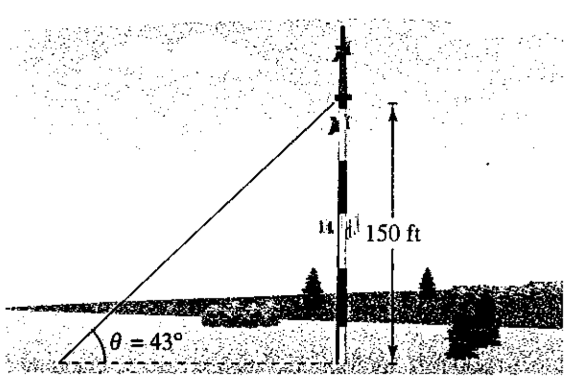
66. **HEIGHT** A six-foot person walks from the base of a broadcasting tower directly toward the tip of the shadow cast by the tower. When the person is 132 feet from the tower and 3 feet from the tip of the shadow, the person's shadow starts to appear beyond the tower's shadow.
    (a) Draw a right triangle that gives a visual representation of the problem. Show the known quantities of the triangle and use a variable to indicate the height of the tower.
    (b) Use a trigonometric function to write an equation involving the unknown quantity.
    (c) What is the height of the tower?

67. **ANGLE OF ELEVATION** You are skiing down a mountain with a vertical height of 1250 feet. The distance from the top of the mountain to the base is 2500 feet. What is the angle of elevation from the base to the top of the mountain?

68. **WIDTH OF A RIVER** A biologist wants to know the width $w$ of a river so that instruments for studying the pollutants in the water can be set properly. From point $A$, the biologist walks downstream 100 feet and sights to point $C$ (see figure). From this sighting, it is determined that $\theta = 54°$. How wide is the river?

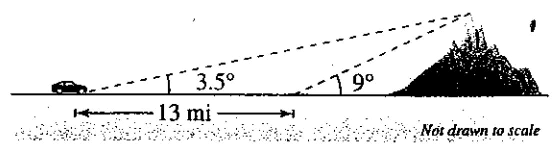



69. **LENGTH** A guy wire runs from the ground to a cell tower. The wire is attached to the cell tower 150 feet above the ground. The angle formed between the wire and the ground is 43° (see figure).




    (a) How long is the guy wire?
    (b) How far from the base of the tower is the guy wire anchored to the ground?

70. **HEIGHT OF A MOUNTAIN** In traveling across flat land, you notice a mountain directly in front of you. Its angle of elevation (to the peak) is 3.5°. After you drive 13 miles closer to the mountain, the angle of elevation is 9°. Approximate the height of the mountain.




71. **MACHINE SHOP CALCULATIONS** A steel plate has the form of one-fourth of a circle with a radius of 60 centimeters. Two two-centimeter holes are to be drilled in the plate positioned as shown in the figure. Find the coordinates of the center of each hole.

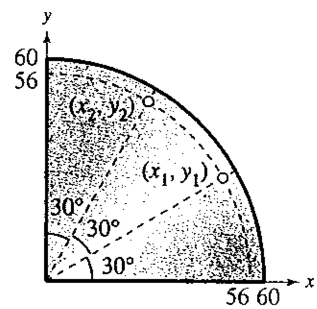



FIGURE FOR 71

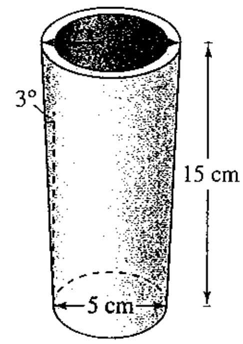
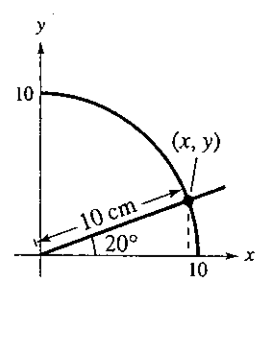



FIGURE FOR 72

72. **MACHINE SHOP CALCULATIONS** A tapered shaft has a diameter of 5 centimeters at the small end and is 15 centimeters long (see figure). The taper is 3°. Find the diameter $d$ of the large end of the shaft.

73. **GEOMETRY** Use a compass to sketch a quarter of a circle of radius 10 centimeters. Using a protractor, construct an angle of 20° in standard position (see figure). Drop a perpendicular line from the point of intersection of the terminal side of the angle and the arc of the circle. By actual measurement, calculate the coordinates $(x, y)$ of the point of intersection and use these measurements to approximate the six trigonometric functions of a 20° angle.



**74. HEIGHT** A 20-meter line is used to tether a helium-filled balloon. Because of a breeze, the line makes an angle of approximately 85° with the ground.

(a) Draw a right triangle that gives a visual representation of the problem. Show the known quantities of the triangle and use a variable to indicate the height of the balloon.

(b) Use a trigonometric function to write an equation involving the unknown quantity.

(c) What is the height of the balloon?

(d) The breeze becomes stronger and the angle the balloon makes with the ground decreases. How does this affect the triangle you drew in part (a)?

(e) Complete the table, which shows the heights (in meters) of the balloon for decreasing angle measures $\theta$.

| Angle, $\theta$ | 80° | 70° | 60° | 50° |
|---|---|---|---|---|
| Height | | | | |

| Angle, $\theta$ | 40° | 30° | 20° | 10° |
|---|---|---|---|---|
| Height | | | | |

(f) As the angle the balloon makes with the ground approaches 0°, how does this affect the height of the balloon? Draw a right triangle to explain your reasoning.

## EXPLORATION

**TRUE OR FALSE?** In Exercises 75–80, determine whether the statement is true or false. Justify your answer.

**75.** $\sin 60° \csc 60° = 1$

**76.** $\sec 30° = \csc 60°$

**77.** $\sin 45° + \cos 45° = 1$

**78.** $\cot^2 10° - \csc^2 10° = -1$

**79.** $\dfrac{\sin 60°}{\sin 30°} = \sin 2°$

**80.** $\tan[(5°)^2] = \tan^2 5°$

**81. WRITING** In right triangle trigonometry, explain why $\sin 30° = \frac{1}{2}$ regardless of the size of the triangle.

**82. THINK ABOUT IT** You are given only the value $\tan \theta$. Is it possible to find the value of $\sec \theta$ without finding the measure of $\theta$? Explain.

**83. THINK ABOUT IT**

(a) Complete the table.

| $\theta$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|
| $\sin \theta$ | | | | | |

(b) Is $\theta$ or $\sin \theta$ greater for $\theta$ in the interval (0, 0.5

(c) As $\theta$ approaches 0, how do $\theta$ and $\sin \theta$ comp Explain.

**84. THINK ABOUT IT**

(a) Complete the table.

| $\theta$ | 0 | 0.3 | 0.6 | 0.9 | 1.2 | 1.5 |
|---|---|---|---|---|---|---|
| $\sin \theta$ | | | | | | |
| $\cos \theta$ | | | | | | |

(b) Discuss the behavior of the sine function for $\theta$ i interval [0, 1.5].

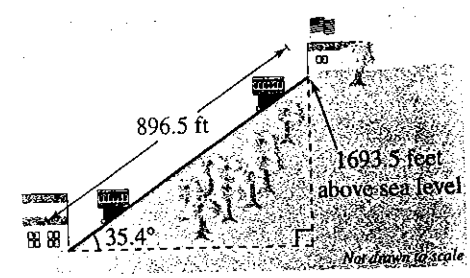
(c) Discuss the behavior of the cosine function for the interval [0, 1.5].

(d) Use the definitions of the sine and cosine func to explain the results of parts (b) and (c).

**85. THINK ABOUT IT** Use a graphing utility to com the table and make a conjecture about the relatio between $\cos \theta$ and $\sin(90° - \theta)$. What are the ang and $90° - \theta$ called?

| $\theta$ | 0° | 20° | 40° | 60° | 80° |
|---|---|---|---|---|---|
| $\cos \theta$ | | | | | |
| $\sin(90° - \theta)$ | | | | | |

**86. CAPSTONE** The Johnstown Inclined Plane Pennsylvania is one of the longest and steepest h in the world. The railway cars travel a distanc 896.5 feet at an angle of approximately 35.4°, risin a height of 1693.5 feet above sea level.




(a) Find the vertical rise of the inclined plane.

(b) Find the elevation of the lower end of the inc plane.

(c) The cars move up the mountain at a rat 300 feet per minute. Find the rate at which the vertically.

## 6.3 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

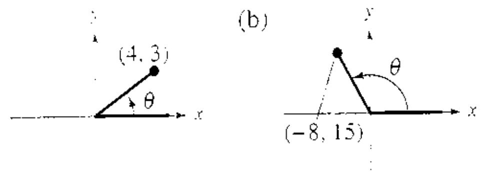
### VOCABULARY: Fill in the blanks.

1. The acute positive angle that is formed by the terminal side of the angle $\theta$ and the horizontal axis is called the ________ angle of $\theta$ and is denoted by $\theta'$.
2. A function $f$ is ________ if there exists a positive real number $c$ such that $f(t + c) = f(t)$ for all $t$ in the domain of $f$.
3. The smallest number $c$ for which a function $f$ is periodic is called the ________ of $f$.
4. The cosine and secant functions are ________ functions, and the sine, cosecant, tangent, and cotangent functions are ________ functions.

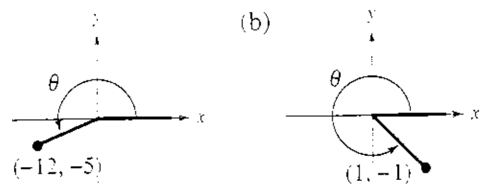
### SKILLS AND APPLICATIONS

In Exercises 5–8, determine the exact values of the six trigonometric functions of the angle $\theta$.

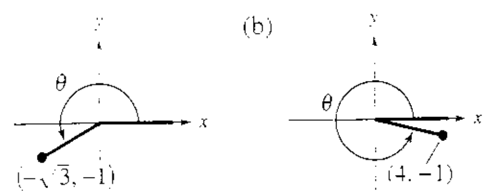
5. (a) (b)



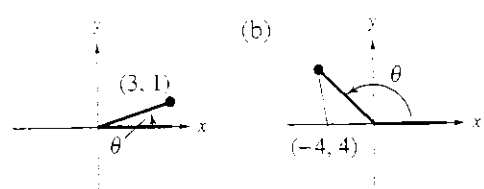

6. (a) (b)




7. (a) (b)




8. (a) (b)




In Exercises 9–14, the point is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.

9. $(5, 12)$
10. $(8, 15)$
11. $(-5, -2)$
12. $(-4, 10)$
13. $(-5.4, 7.2)$
14. $(3\frac{1}{2}, -7\frac{3}{4})$

In Exercises 15–18, state the quadrant in which $\theta$ lies.

15. $\sin\theta > 0$ and $\cos\theta > 0$
16. $\sin\theta < 0$ and $\cos\theta < 0$
17. $\sin\theta > 0$ and $\cos\theta < 0$
18. $\sec\theta > 0$ and $\cot\theta < 0$

In Exercises 19–28, find the values of the six trigonometric functions of $\theta$ with the given constraint.

| | *Function Value* | *Constraint* |
|---|---|---|
| 19. | $\tan\theta = -\frac{15}{8}$ | $\sin\theta > 0$ |
| 20. | $\cos\theta = \frac{8}{17}$ | $\tan\theta < 0$ |
| 21. | $\sin\theta = \frac{3}{5}$ | $\theta$ lies in Quadrant II. |
| 22. | $\cos\theta = -\frac{4}{5}$ | $\theta$ lies in Quadrant III. |
| 23. | $\cot\theta = -3$ | $\cos\theta > 0$ |
| 24. | $\csc\theta = 4$ | $\cot\theta < 0$ |
| 25. | $\sec\theta = -2$ | $\sin\theta < 0$ |
| 26. | $\sin\theta = 0$ | $\sec\theta = -1$ |
| 27. | $\cot\theta$ is undefined. | $\pi/2 \le \theta \le 3\pi/2$ |
| 28. | $\tan\theta$ is undefined. | $\pi \le \theta \le 2\pi$ |

In Exercises 29–32, the terminal side of $\theta$ lies on the given line in the specified quadrant. Find the values of the six trigonometric functions of $\theta$ by finding a point on the line.

| | *Line* | *Quadrant* |
|---|---|---|
| 29. | $y = -x$ | II |
| 30. | $y = \frac{1}{3}x$ | III |
| 31. | $2x - y = 0$ | III |
| 32. | $4x + 3y = 0$ | IV |

In Exercises 33–40, evaluate the trigonometric function of the quadrant angle.

**33.** $\sin \pi$ **34.** $\csc \frac{3\pi}{2}$

**35.** $\sec \frac{3\pi}{2}$ **36.** $\sec \pi$

**37.** $\sin \frac{\pi}{2}$ **38.** $\cot \pi$

**39.** $\csc \pi$ **40.** $\cot \frac{\pi}{2}$

In Exercises 41–48, find the reference angle $\theta'$, and sketch $\theta$ and $\theta'$ in standard position.

**41.** $\theta = 160°$ **42.** $\theta = 309°$

**43.** $\theta = -125°$ **44.** $\theta = -215°$

**45.** $\theta = \frac{2\pi}{3}$ **46.** $\theta = \frac{7\pi}{6}$

**47.** $\theta = 4.8$ **48.** $\theta = 11.6$

In Exercises 49–64, evaluate the sine, cosine, and tangent of the angle without using a calculator.

**49.** $225°$ **50.** $300°$

**51.** $750°$ **52.** $-405°$

**53.** $-150°$ **54.** $-840°$

**55.** $\frac{2\pi}{3}$ **56.** $\frac{3\pi}{4}$

**57.** $\frac{5\pi}{4}$ **58.** $\frac{7\pi}{6}$

**59.** $-\frac{\pi}{6}$ **60.** $-\frac{\pi}{2}$

**61.** $\frac{11\pi}{4}$ **62.** $\frac{10\pi}{3}$

**63.** $\frac{9\pi}{4}$ **64.** $-\frac{23\pi}{4}$

In Exercises 65–70, find the indicated trigonometric value in the specified quadrant.

| | *Function* | *Quadrant* | *Trigonometric Value* |
|---|---|---|---|
| **65.** | $\sin \theta = -\frac{3}{5}$ | IV | $\cos \theta$ |
| **66.** | $\cot \theta = -3$ | II | $\sin \theta$ |
| **67.** | $\tan \theta = \frac{3}{2}$ | III | $\sec \theta$ |
| **68.** | $\csc \theta = -2$ | IV | $\cot \theta$ |
| **69.** | $\cos \theta = \frac{5}{8}$ | I | $\sec \theta$ |
| **70.** | $\sec \theta = -\frac{9}{4}$ | III | $\tan \theta$ |

In Exercises 71–86, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is set in the correct angle mode.)

**71.** $\sin 10°$ **72.** $\sec 225°$

**73.** $\cos(-110°)$ **74.** $\csc(-330°)$

**75.** $\tan 304°$ **76.** $\cot 178°$

**77.** $\sec 72°$ **78.** $\tan(-188°)$

**79.** $\tan 4.5$ **80.** $\cot 1.35$

**81.** $\tan \frac{\pi}{9}$ **82.** $\tan\left(-\frac{\pi}{9}\right)$

**83.** $\sin(-0.65)$ **84.** $\sec 0.29$

**85.** $\cot\left(-\frac{11\pi}{8}\right)$ **86.** $\csc\left(-\frac{15\pi}{14}\right)$

In Exercises 87–92, find two solutions of the equation. Give your answers in degrees ($0° \le \theta < 360°$) and in radians ($0 \le \theta < 2\pi$). Do not use a calculator.

**87.** (a) $\sin \theta = \frac{1}{2}$ (b) $\sin \theta = -\frac{1}{2}$

**88.** (a) $\cos \theta = \frac{\sqrt{2}}{2}$ (b) $\cos \theta = -\frac{\sqrt{2}}{2}$

**89.** (a) $\csc \theta = \frac{2\sqrt{3}}{3}$ (b) $\cot \theta = -1$

**90.** (a) $\sec \theta = 2$ (b) $\sec \theta = -2$

**91.** (a) $\tan \theta = 1$ (b) $\cot \theta = -\sqrt{3}$

**92.** (a) $\sin \theta = \frac{\sqrt{3}}{2}$ (b) $\sin \theta = -\frac{\sqrt{3}}{2}$

In Exercises 93–100, find the point $(x, y)$ on the unit circle that corresponds to the real number $t$. Use the result to evaluate $\sin t$, $\cos t$, and $\tan t$.

**93.** $t = \frac{\pi}{4}$

**94.** $t = \frac{\pi}{3}$

**95.** $t = \frac{5\pi}{6}$

**96.** $t = \frac{3\pi}{4}$

**97.** $t = \frac{4\pi}{3}$

**98.** $t = \frac{5\pi}{3}$

**99.** $t = \frac{\pi}{2}$

**100.** $t = \pi$

**ESTIMATION** In Exercises 101 and 102, use the figure below and a straightedge to approximate the value of each trigonometric function. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

**101.** (a) $\sin 5$ (b) $\cos 2$

**102.** (a) $\sin 0.75$ (b) $\cos 2.5$

**ESTIMATION** In Exercises 103 and 104, use the figure below and a straightedge to approximate the solution of each equation, where $0 \le t < 2\pi$. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

**103.** (a) $\sin t = 0.25$ (b) $\cos t = -0.25$

**104.** (a) $\sin t = -0.75$ (b) $\cos t = 0.75$

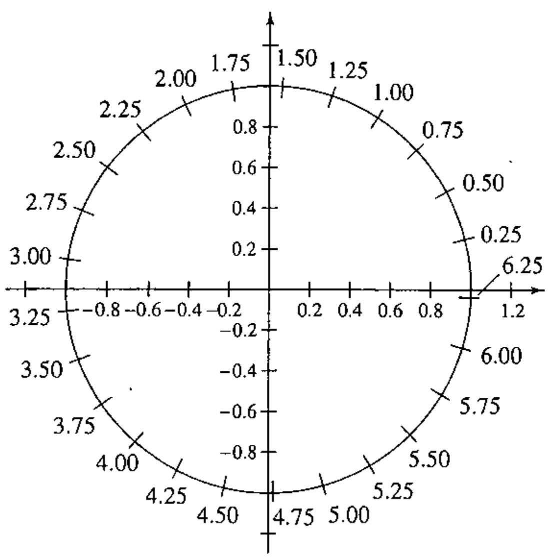



FIGURE FOR 101–104

**105. DATA ANALYSIS: METEOROLOGY** The table shows the monthly normal temperatures (in degrees Fahrenheit) for selected months in New York City ($N$) and Fairbanks, Alaska ($F$). (Source: National Climatic Data Center)

| Month | New York City, $N$ | Fairbanks, $F$ |
|---|---|---|
| January | 33 | −10 |
| April | 52 | 32 |
| July | 77 | 62 |
| October | 58 | 24 |
| December | 38 | −6 |

(a) Use the *regression* feature of a graphing utility to find a model of the form $y = a\sin(bt + c) + d$ for each city. Let $t$ represent the month, with $t = 1$ corresponding to January.

(b) Use the models from part (a) to find the monthly normal temperatures for the two cities in February, March, May, June, August, September, and November.

(c) Compare the models for the two cities.

**106. SALES** A company that produces snowboards forecasts monthly sales over the next 2 years to be

$$S = 23.1 + 0.442t + 4.3\cos\frac{\pi t}{6}$$

where $S$ is measured in thousands of units and $t$ is the time in months, with $t = 1$ representing January 2010. Predict sales for each of the following months.

(a) February 2010

(b) February 2011

(c) June 2010

(d) June 2011

**107. HARMONIC MOTION** The displacement from equilibrium of an oscillating weight suspended by a spring is given by

$$y(t) = 2\cos 6t$$

where $y$ is the displacement in centimeters and $t$ is the time in seconds (see figure). Find the displacement when (a) $t = 0$, (b) $t = \frac{1}{4}$, and (c) $t = \frac{1}{2}$.

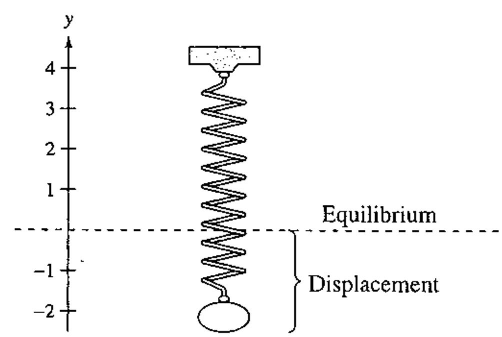



FIGURE FOR 107 AND 108

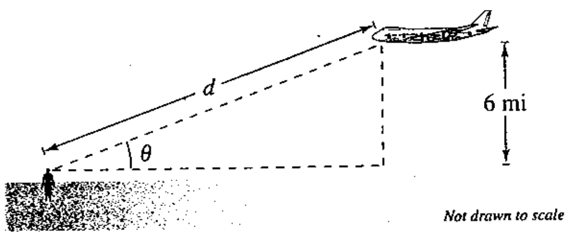
**108. HARMONIC MOTION** The displacement from equilibrium of an oscillating weight suspended by a spring and subject to the damping effect of friction is given by

$$y(t) = 2e^{-t}\cos 6t$$

where $y$ is the displacement in centimeters and $t$ is the time in seconds (see figure). Find the displacement when (a) $t = 0$, (b) $t = \frac{1}{4}$, and (c) $t = \frac{1}{2}$.

**109. ELECTRIC CIRCUITS** The current $I$ (in amperes) when 100 volts is applied to a circuit is given by $I = 5e^{-2t}\sin t$, where $t$ is the time (in seconds) after the voltage is applied. Approximate the current at $t = 0.7$ second after the voltage is applied.

110. **DISTANCE** An airplane, flying at an altitude of 6 miles, is on a flight path that passes directly over an observer (see figure). If $\theta$ is the angle of elevation from the observer to the plane, find the distance $d$ from the observer to the plane when (a) $\theta = 30°$, (b) $\theta = 90°$, and (c) $\theta = 120°$.




## EXPLORATION

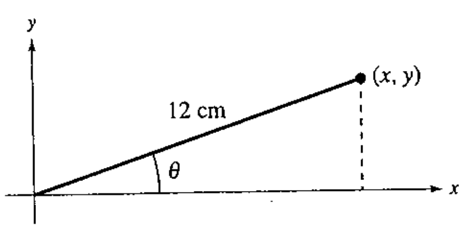
**TRUE OR FALSE?** In Exercises 111 and 112, determine whether the statement is true or false. Justify your answer.

111. In each of the four quadrants, the signs of the secant function and sine function will be the same.

112. To find the reference angle for an angle $\theta$ (given in degrees), find the integer $n$ such that $0 \le 360°n - \theta \le 360°$. The difference $360°n - \theta$ is the reference angle.

113. **THINK ABOUT IT** Because $f(t) = \sin t$ is an odd function and $g(t) = \cos t$ is an even function, what can be said about the function $h(t) = f(t)g(t)$?

114. **WRITING** Consider an angle in standard position with $r = 12$ centimeters, as shown in the figure. Write a short paragraph describing the changes in the values of $x$, $y$, $\sin\theta$, $\cos\theta$, and $\tan\theta$ as $\theta$ increases continuously from 0° to 90°.




115. **CONJECTURE**

(a) Use a graphing utility to complete the table.

| $\theta$ | 0° | 20° | 40° | 60° | 80° |
|---|---|---|---|---|---|
| $\sin\theta$ | | | | | |
| $\sin(180° - \theta)$ | | | | | |

(b) Make a conjecture about the relationship between $\sin\theta$ and $\sin(180° - \theta)$.

116. **CONJECTURE**

(a) Use a graphing utility to complete the table.

| $\theta$ | 0 | 0.3 | 0.6 | 0.9 | 1.2 | 1.5 |
|---|---|---|---|---|---|---|
| $\cos\left(\frac{3\pi}{2} - \theta\right)$ | | | | | | |
| $-\sin\theta$ | | | | | | |

(b) Make a conjecture about the relationship between $\cos\left(\frac{3\pi}{2} - \theta\right)$ and $-\sin\theta$.

117. **WRITING** Use a graphing utility to graph each of the six trigonometric functions. Determine the domain, range, period, and zeros of each function. Then determine whether each function is even or odd. Identify, and write a short paragraph describing, any inherent patterns in the trigonometric functions. What can you conclude?

118. **CAPSTONE** Write a short paper in your own words explaining to a classmate how to evaluate the six trigonometric functions of any angle $\theta$ in standard position. Include an explanation of reference angles and how to use them, the signs of the functions in each of the four quadrants, and the trigonometric values of common angles. Be sure to include figures or diagrams in your paper.

119. **THINK ABOUT IT** Let $(x_1, y_1)$ and $(x_2, y_2)$ be points on the unit circle corresponding to $t = t_1$ and $t = \pi - t_1$, respectively.

(a) Identify the symmetry of the points $(x_1, y_1)$ and $(x_2, y_2)$.

(b) Make a conjecture about any relationship between $\sin t_1$ and $\sin(\pi - t_1)$.

(c) Make a conjecture about any relationship between $\cos t_1$ and $\cos(\pi - t_1)$.

120. **GRAPHICAL ANALYSIS** With your graphing utility in *radian* and *parametric* modes, enter the equations $X_{1T} = \cos T$ and $Y_{1T} = \sin T$ and use the following settings.

Tmin = 0, Tmax = 6.3, Tstep = 0.1
Xmin = −1.5, Xmax = 1.5, Xscl = 1
Ymin = −1, Ymax = 1, Yscl = 1

(a) Graph the entered equations and describe the graph.

(b) Use the *trace* feature to move the cursor around the graph. What do the $t$-values represent? What do the $x$- and $y$-values represent?

(c) What are the least and greatest values of $x$ and $y$?

# 6.4 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exerc

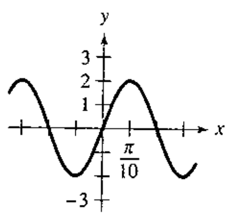
**VOCABULARY:** Fill in the blanks.

1. One period of a sine or cosine function is called one ________ of the sine or cosine curve.
2. The ________ of a sine or cosine curve represents half the distance between the maximum and minimum values of the function.
3. For the function given by $y = a\sin(bx - c)$, $\frac{c}{b}$ represents the ________ ________ of the graph of the function.
4. For the function given by $y = d + a\cos(bx - c)$, $d$ represents a ________ ________ of the graph of the function.

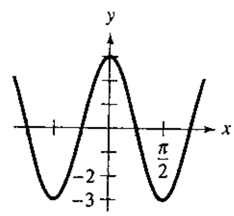
## SKILLS AND APPLICATIONS

In Exercises 5–18, find the period and amplitude.

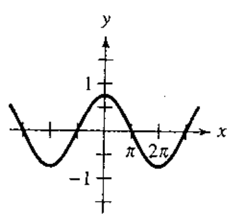
5. $y = 2\sin 5x$



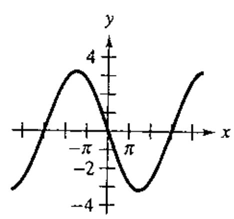

6. $y = 3\cos 2x$



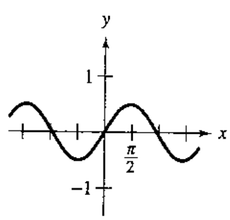

7. $y = \frac{3}{4}\cos\frac{x}{2}$



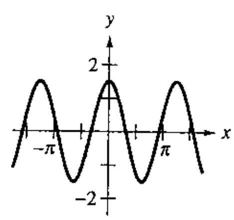

8. $y = -3\sin\frac{x}{3}$




9. $y = \frac{1}{2}\sin\frac{\pi x}{3}$




10. $y = \frac{3}{2}\cos\frac{\pi x}{2}$



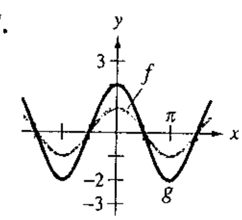

11. $y = -4\sin x$
12. $y = -\cos\frac{2x}{3}$
13. $y = 3\sin 10x$
14. $y = \frac{1}{5}\sin 6x$
15. $y = \frac{5}{3}\cos\frac{4x}{5}$
16. $y = \frac{5}{2}\cos\frac{x}{4}$
17. $y = \frac{1}{4}\sin 2\pi x$
18. $y = \frac{2}{3}\cos\frac{\pi x}{10}$

In Exercises 19–26, describe the relationship between 1 graphs of $f$ and $g$. Consider amplitude, period, and shifts.

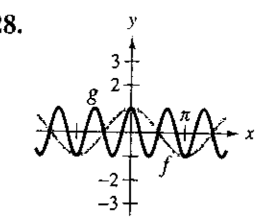
19. $f(x) = \sin x$
    $g(x) = \sin(x - \pi)$
20. $f(x) = \cos x$
    $g(x) = \cos(x + \pi)$
21. $f(x) = \cos 2x$
    $g(x) = -\cos 2x$
22. $f(x) = \sin 3x$
    $g(x) = \sin(-3x)$
23. $f(x) = \cos x$
    $g(x) = \cos 2x$
24. $f(x) = \sin x$
    $g(x) = \sin 3x$
25. $f(x) = \sin 2x$
    $g(x) = 3 + \sin 2x$
26. $f(x) = \cos 4x$
    $g(x) = -2 + \cos 4x$

In Exercises 27–30, describe the relationship between tl graphs of $f$ and $g$. Consider amplitude, period, and shifts.

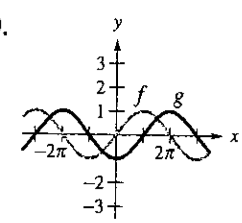
27.



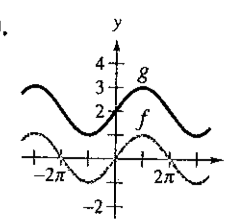

28.




29.




30.




In Exercises 31–38, graph $f$ and $g$ on the same set o coordinate axes. (Include two full periods.)

31. $f(x) = -2\sin x$
    $g(x) = 4\sin x$
32. $f(x) = \sin x$
    $g(x) = \sin\frac{x}{3}$
33. $f(x) = \cos x$
    $g(x) = 2 + \cos x$
34. $f(x) = 2\cos 2x$
    $g(x) = -\cos 4x$

35. $f(x) = -\frac{1}{2}\sin\frac{x}{2}$
$g(x) = 3 - \frac{1}{2}\sin\frac{x}{2}$

36. $f(x) = 4\sin\pi x$
$g(x) = 4\sin\pi x - 3$

37. $f(x) = 2\cos x$
$g(x) = 2\cos(x - \pi)$

38. $f(x) = -\cos x$
$g(x) = -\cos(x - \pi)$

In Exercises 39–60, sketch the graph of the function. (Include two full periods.)

39. $y = 5\sin x$
40. $y = \frac{1}{4}\sin x$
41. $y = \frac{1}{3}\cos x$
42. $y = 4\cos x$
43. $y = \cos\frac{x}{2}$
44. $y = \sin 4x$
45. $y = \cos 2\pi x$
46. $y = \sin\frac{\pi x}{4}$
47. $y = -\sin\frac{2\pi x}{3}$
48. $y = -10\cos\frac{\pi x}{6}$
49. $y = \sin\left(x - \frac{\pi}{2}\right)$
50. $y = \sin(x - 2\pi)$
51. $y = 3\cos(x - \pi)$
52. $y = 4\cos\left(x - \frac{\pi}{4}\right)$
53. $y = 2 - \sin\frac{2\pi x}{3}$
54. $y = -3 + 5\cos\frac{\pi t}{12}$
55. $y = 2 + \frac{1}{10}\cos 60\pi x$
56. $y = 2\cos x - 3$
57. $y = 3\cos(x - \pi) - 3$
58. $y = 4\cos\left(x + \frac{\pi}{4}\right) + 4$
59. $y = \frac{2}{3}\cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$
60. $y = -3\cos(6x + \pi)$

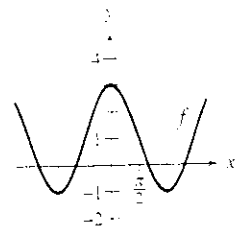
In Exercises 61–66, $g$ is related to a parent function $f(x) = \sin(x)$ or $f(x) = \cos(x)$. (a) Describe the sequence of transformations from $f$ to $g$. (b) Sketch the graph of $g$. (c) Use function notation to write $g$ in terms of $f$.

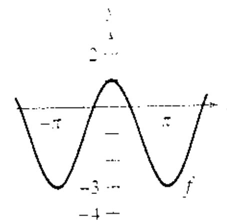
61. $g(x) = \sin(4x - \pi)$
62. $g(x) = \sin(2x - \pi)$
63. $g(x) = \cos(x - \pi) + 2$
64. $g(x) = 1 + \cos(x + \pi)$
65. $g(x) = 2\sin(4x - \pi) - 3$
66. $g(x) = 4 - \sin(2x + \pi)$

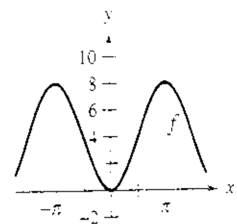
In Exercises 67–72, use a graphing utility to graph the function. Include two full periods. Be sure to choose an appropriate viewing window.

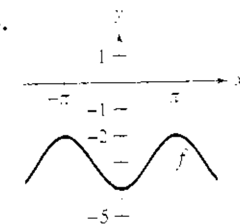
67. $y = -2\sin(4x - \pi)$
68. $y = -4\sin\left(\frac{2}{3}x - \frac{\pi}{3}\right)$
69. $y = \cos\left(2\pi x - \frac{\pi}{2}\right) + 1$
70. $y = 3\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) - 2$
71. $y = -0.1\sin\left(\frac{\pi x}{10} + \pi\right)$
72. $y = \frac{1}{100}\sin 120\pi t$

**GRAPHICAL REASONING** In Exercises 73–76, find $a$ and $d$ for the function $f(x) = a\cos x + d$ such that the graph of $f$ matches the figure.

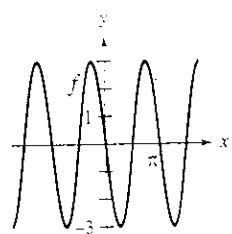
73.


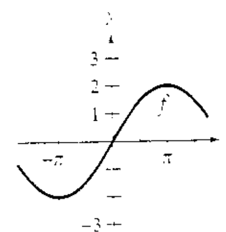
74.


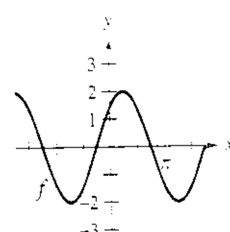
75.


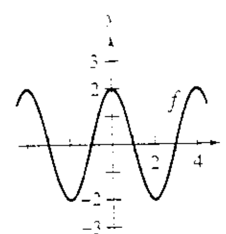
76.


**GRAPHICAL REASONING** In Exercises 77–80, find $a$, $b$, and $c$ for the function $f(x) = a\sin(bx - c)$ such that the graph of $f$ matches the figure.

77.


78.


79.


80.


In Exercises 81 and 82, use a graphing utility to graph $y_1$ and $y_2$ in the interval $[-2\pi, 2\pi]$. Use the graphs to find real numbers $x$ such that $y_1 = y_2$.

81. $y_1 = \sin x$
$y_2 = -\frac{1}{2}$

82. $y_1 = \cos x$
$y_2 = -1$

In Exercises 83–86, write an equation for the function that is described by the given characteristics.

83. A sine curve with a period of $\pi$, an amplitude of 2, a right phase shift of $\pi/2$, and a vertical translation up 1 unit

**84.** A sine curve with a period of $4\pi$, an amplitude of 3, a left phase shift of $\pi/4$, and a vertical translation down 1 unit

**85.** A cosine curve with a period of $\pi$, an amplitude of 1, a left phase shift of $\pi$, and a vertical translation down $\frac{3}{2}$ units

**86.** A cosine curve with a period of $4\pi$, an amplitude of 3, a right phase shift of $\pi/2$, and a vertical translation up 2 units

**87. RESPIRATORY CYCLE** For a person at rest, the velocity $v$ (in liters per second) of airflow during a respiratory cycle (the time from the beginning of one breath to the beginning of the next) is given by $v = 0.85 \sin \frac{\pi t}{3}$, where $t$ is the time (in seconds). (Inhalation occurs when $v > 0$, and exhalation occurs when $v < 0$.)

(a) Find the time for one full respiratory cycle.

(b) Find the number of cycles per minute.

(c) Sketch the graph of the velocity function.

**88. RESPIRATORY CYCLE** After exercising for a few minutes, a person has a respiratory cycle for which the velocity of airflow is approximated by $v = 1.75 \sin \frac{\pi t}{2}$, where $t$ is the time (in seconds). (Inhalation occurs when $v > 0$, and exhalation occurs when $v < 0$.)

(a) Find the time for one full respiratory cycle.

(b) Find the number of cycles per minute.

(c) Sketch the graph of the velocity function.

**89. DATA ANALYSIS: METEOROLOGY** The table shows the maximum daily high temperatures in Las Vegas $L$ and International Falls $I$ (in degrees Fahrenheit) for month $t$, with $t = 1$ corresponding to January. (Source: National Climatic Data Center)

| Month, $t$ | Las Vegas, $L$ | International Falls, $I$ |
|---|---|---|
| 1 | 57.1 | 13.8 |
| 2 | 63.0 | 22.4 |
| 3 | 69.5 | 34.9 |
| 4 | 78.1 | 51.5 |
| 5 | 87.8 | 66.6 |
| 6 | 98.9 | 74.2 |
| 7 | 104.1 | 78.6 |
| 8 | 101.8 | 76.3 |
| 9 | 93.8 | 64.7 |
| 10 | 80.8 | 51.7 |
| 11 | 66.0 | 32.5 |
| 12 | 57.3 | 18.1 |

(a) A model for the temperature in Las Vegas is given by

$$L(t) = 80.60 + 23.50 \cos\left(\frac{\pi t}{6} - 3.67\right).$$

Find a trigonometric model for International Falls.

(b) Use a graphing utility to graph the data points and the model for the temperatures in Las Vegas. How well does the model fit the data?

(c) Use a graphing utility to graph the data points and the model for the temperatures in International Falls. How well does the model fit the data?

(d) Use the models to estimate the average maximum temperature in each city. Which term of the models did you use? Explain.

(e) What is the period of each model? Are the periods what you expected? Explain.

(f) Which city has the greater variability in temperature throughout the year? Which factor of the models determines this variability? Explain.

**90. HEALTH** The function given by

$$P = 100 - 20 \cos \frac{5\pi t}{3}$$

approximates the blood pressure $P$ (in millimeters of mercury) at time $t$ (in seconds) for a person at rest.

(a) Find the period of the function.

(b) Find the number of heartbeats per minute.

**91. PIANO TUNING** When tuning a piano, a technician strikes a tuning fork for the A above middle C and sets up a wave motion that can be approximated by $y = 0.001 \sin 880\pi t$, where $t$ is the time (in seconds).

(a) What is the period of the function?

(b) The frequency $f$ is given by $f = 1/p$. What is the frequency of the note?

**92. DATA ANALYSIS: ASTRONOMY** The percents $y$ (in decimal form) of the moon's face that was illuminated on day $x$ in the year 2009, where $x = 1$ represents January 1, are shown in the table. (Source: U.S. Naval Observatory)

| $x$ | $y$ |
|---|---|
| 4 | 0.5 |
| 11 | 1.0 |
| 18 | 0.5 |
| 26 | 0.0 |
| 33 | 0.5 |
| 40 | 1.0 |

(a) Create a scatter plot of the data.

(b) Find a trigonometric model that fits the data.

(c) Add the graph of your model in part (b) to the scatter plot. How well does the model fit the data?

(d) What is the period of the model?

(e) Estimate the moon's percent illumination for March 12, 2009.

**93. FUEL CONSUMPTION** The daily consumption $C$ (in gallons) of diesel fuel on a farm is modeled by

$$C = 30.3 + 21.6\sin\left(\frac{2\pi t}{365} + 10.9\right)$$

where $t$ is the time (in days), with $t = 1$ corresponding to January 1.

(a) What is the period of the model? Is it what you expected? Explain.

(b) What is the average daily fuel consumption? Which term of the model did you use? Explain.

(c) Use a graphing utility to graph the model. Use the graph to approximate the time of the year when consumption exceeds 40 gallons per day.

**94. FERRIS WHEEL** A Ferris wheel is built such that the height $h$ (in feet) above ground of a seat on the wheel at time $t$ (in seconds) can be modeled by

$$h(t) = 53 + 50\sin\left(\frac{\pi}{10}t - \frac{\pi}{2}\right).$$

(a) Find the period of the model. What does the period tell you about the ride?

(b) Find the amplitude of the model. What does the amplitude tell you about the ride?

(c) Use a graphing utility to graph one cycle of the model.

## EXPLORATION

**TRUE OR FALSE?** In Exercises 95–97, determine whether the statement is true or false. Justify your answer.

**95.** The graph of the function given by $f(x) = \sin(x + 2\pi)$ translates the graph of $f(x) = \sin x$ exactly one period to the right so that the two graphs look identical.

**96.** The function given by $y = \frac{1}{2}\cos 2x$ has an amplitude that is twice that of the function given by $y = \cos x$.

**97.** The graph of $y = -\cos x$ is a reflection of the graph of $y = \sin(x + \pi/2)$ in the $x$-axis.

**98. WRITING** Sketch the graph of $y = \cos bx$ for $b = \frac{1}{2}$, 2, and 3. How does the value of $b$ affect the graph? How many complete cycles occur between 0 and $2\pi$ for each value of $b$?

**99. WRITING** Sketch the graph of $y = \sin(x - c)$ for $c = -\pi/4$, 0, and $\pi/4$. How does the value of $c$ affect the graph?

**100. CAPSTONE** Use a graphing utility to graph the function given by $y = d + a\sin(bx - c)$, for several different values of $a$, $b$, $c$, and $d$. Write a paragraph describing the changes in the graph corresponding to changes in each constant.

**CONJECTURE** In Exercises 101 and 102, graph $f$ and $g$ on the same set of coordinate axes. Include two full periods. Make a conjecture about the functions.

**101.** $f(x) = \sin x,\quad g(x) = \cos\left(x - \frac{\pi}{2}\right)$

**102.** $f(x) = \sin x,\quad g(x) = -\cos\left(x + \frac{\pi}{2}\right)$

**103.** Using calculus, it can be shown that the sine and cosine functions can be approximated by the polynomials

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} \quad \text{and} \quad \cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

where $x$ is in radians.

(a) Use a graphing utility to graph the sine function and its polynomial approximation in the same viewing window. How do the graphs compare?

(b) Use a graphing utility to graph the cosine function and its polynomial approximation in the same viewing window. How do the graphs compare?

(c) Study the patterns in the polynomial approximations of the sine and cosine functions and predict the next term in each. Then repeat parts (a) and (b). How did the accuracy of the approximations change when an additional term was added?

**104.** Use the polynomial approximations of the sine and cosine functions in Exercise 103 to approximate the following function values. Compare the results with those given by a calculator. Is the error in the approximation the same in each case? Explain.

(a) $\sin\frac{1}{2}$ (b) $\sin 1$ (c) $\sin\frac{\pi}{6}$

(d) $\cos(-0.5)$ (e) $\cos 1$ (f) $\cos\frac{\pi}{4}$

**PROJECT: METEOROLOGY** To work an extended application analyzing the mean monthly temperature and mean monthly precipitation in Honolulu, Hawaii, visit this text's website at *academic.cengage.com*.

## 6.5 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

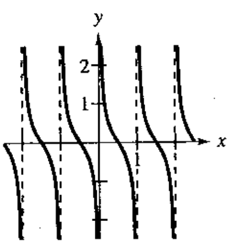
### VOCABULARY: Fill in the blanks.

1. The tangent, cotangent, and cosecant functions are ________ , so the graphs of these functions have symmetry with respect to the ________.
2. The graphs of the tangent, cotangent, secant, and cosecant functions all have ________ asymptotes.
3. To sketch the graph of a secant or cosecant function, first make a sketch of its corresponding ________ function.
4. For the functions given by $f(x) = g(x) \cdot \sin x$, $g(x)$ is called the ________ factor of the function $f(x)$.
5. The period of $y = \tan x$ is ________.
6. The domain of $y = \cot x$ is all real numbers such that ________.
7. The range of $y = \sec x$ is ________.
8. The period of $y = \csc x$ is ________.

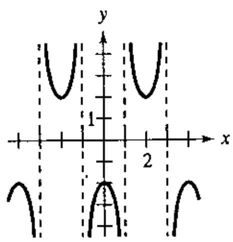
### SKILLS AND APPLICATIONS

In Exercises 9–14, match the function with its graph. State the period of the function. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

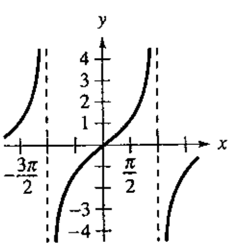
(a)



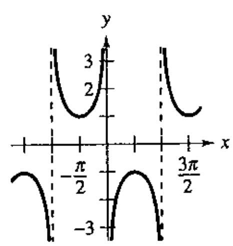
(b)



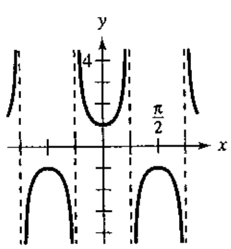
(c)



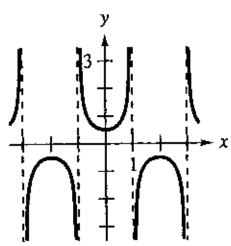
(d)



(e)



(f)



9. $y = \sec 2x$
10. $y = \tan \dfrac{x}{2}$
11. $y = \dfrac{1}{2} \cot \pi x$
12. $y = -\csc x$
13. $y = \dfrac{1}{2} \sec \dfrac{\pi x}{2}$
14. $y = -2 \sec \dfrac{\pi x}{2}$

In Exercises 15–38, sketch the graph of the function. Include two full periods.

15. $y = \dfrac{1}{3} \tan x$
16. $y = \tan 4x$
17. $y = -2 \tan 3x$
18. $y = -3 \tan \pi x$
19. $y = -\frac{1}{2} \sec x$
20. $y = \frac{1}{4} \sec x$
21. $y = \csc \pi x$
22. $y = 3 \csc 4x$
23. $y = \frac{1}{2} \sec \pi x$
24. $y = -2 \sec 4x + 2$
25. $y = \csc \dfrac{x}{2}$
26. $y = \csc \dfrac{x}{3}$
27. $y = 3 \cot 2x$
28. $y = 3 \cot \dfrac{\pi x}{2}$
29. $y = 2 \sec 3x$
30. $y = -\frac{1}{2} \tan x$
31. $y = \tan \dfrac{\pi x}{4}$
32. $y = \tan(x + \pi)$
33. $y = 2 \csc(x - \pi)$
34. $y = \csc(2x - \pi)$
35. $y = 2 \sec(x + \pi)$
36. $y = -\sec \pi x + 1$
37. $y = \dfrac{1}{4} \csc\left(x + \dfrac{\pi}{4}\right)$
38. $y = 2 \cot\left(x + \dfrac{\pi}{2}\right)$

In Exercises 39–48, use a graphing utility to graph the function. Include two full periods.

39. $y = \tan \dfrac{x}{3}$
40. $y = -\tan 2x$
41. $y = -2 \sec 4x$
42. $y = \sec \pi x$
43. $y = \tan\left(x - \dfrac{\pi}{4}\right)$
44. $y = \dfrac{1}{4} \cot\left(x - \dfrac{\pi}{2}\right)$
45. $y = -\csc(4x - \pi)$
46. $y = 2 \sec(2x - \pi)$
47. $y = 0.1 \tan\left(\dfrac{\pi x}{4} + \dfrac{\pi}{4}\right)$
48. $y = \dfrac{1}{3} \sec\left(\dfrac{\pi x}{2} + \dfrac{\pi}{2}\right)$

In Exercises 49–56, use a graph to solve the equation on the interval $[-2\pi, 2\pi]$.

**49.** $\tan x = 1$

**50.** $\tan x = \sqrt{3}$

**51.** $\cot x = -\dfrac{\sqrt{3}}{3}$

**52.** $\cot x = 1$

**53.** $\sec x = -2$

**54.** $\sec x = 2$

**55.** $\csc x = \sqrt{2}$

**56.** $\csc x = -\dfrac{2\sqrt{3}}{3}$

In Exercises 57–64, use the graph of the function to determine whether the function is even, odd, or neither. Verify your answer algebraically.

**57.** $f(x) = \sec x$

**58.** $f(x) = \tan x$

**59.** $g(x) = \cot x$

**60.** $g(x) = \csc x$

**61.** $f(x) = x + \tan x$

**62.** $f(x) = x^2 - \sec x$

**63.** $g(x) = x \csc x$

**64.** $g(x) = x^2 \cot x$

**65. GRAPHICAL REASONING** Consider the functions given by

$$f(x) = 2\sin x \quad \text{and} \quad g(x) = \frac{1}{2}\csc x$$

on the interval $(0, \pi)$.

(a) Graph $f$ and $g$ in the same coordinate plane.

(b) Approximate the interval in which $f > g$.

(c) Describe the behavior of each of the functions as $x$ approaches $\pi$. How is the behavior of $g$ related to the behavior of $f$ as $x$ approaches $\pi$?

**66. GRAPHICAL REASONING** Consider the functions given by

$$f(x) = \tan\frac{\pi x}{2} \quad \text{and} \quad g(x) = \frac{1}{2}\sec\frac{\pi x}{2}$$

on the interval $(-1, 1)$.

(a) Use a graphing utility to graph $f$ and $g$ in the same viewing window.

(b) Approximate the interval in which $f < g$.

(c) Approximate the interval in which $2f < 2g$. How does the result compare with that of part (b)? Explain.

In Exercises 67–72, use a graphing utility to graph the two equations in the same viewing window. Use the graphs to determine whether the expressions are equivalent. Verify the results algebraically.

**67.** $y_1 = \sin x \csc x, \quad y_2 = 1$

**68.** $y_1 = \sin x \sec x, \quad y_2 = \tan x$

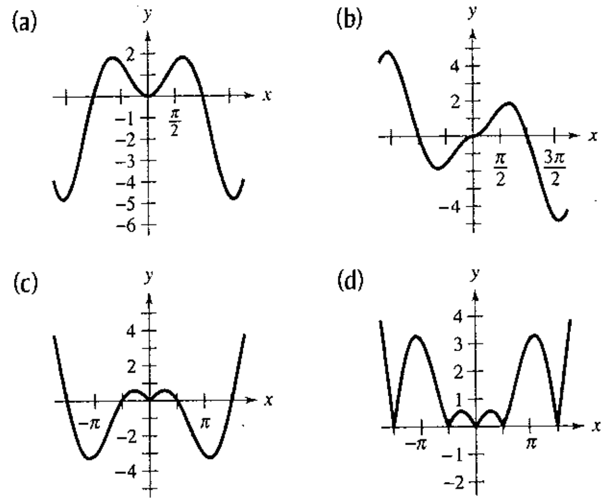
**69.** $y_1 = \dfrac{\cos x}{\sin x}, \quad y_2 = \cot x$

**70.** $y_1 = \tan x \cot^2 x, \quad y_2 = \cot x$

**71.** $y_1 = 1 + \cot^2 x, \quad y_2 = \csc^2 x$

**72.** $y_1 = \sec^2 x - 1, \quad y_2 = \tan^2 x$

In Exercises 73–76, match the function with its graph. Describe the behavior of the function as $x$ approaches zero. [The graphs are labeled (a), (b), (c), and (d).]




**73.** $f(x) = |x \cos x|$

**74.** $f(x) = x \sin x$

**75.** $g(x) = |x| \sin x$

**76.** $g(x) = |x| \cos x$

**CONJECTURE** In Exercises 77–80, graph the functions $f$ and $g$. Use the graphs to make a conjecture about the relationship between the functions.

**77.** $f(x) = \sin x + \cos\left(x + \dfrac{\pi}{2}\right), \quad g(x) = 0$

**78.** $f(x) = \sin x - \cos\left(x + \dfrac{\pi}{2}\right), \quad g(x) = 2\sin x$

**79.** $f(x) = \sin^2 x, \quad g(x) = \frac{1}{2}(1 - \cos 2x)$

**80.** $f(x) = \cos^2 \dfrac{\pi x}{2}, \quad g(x) = \dfrac{1}{2}(1 + \cos \pi x)$

In Exercises 81–84, use a graphing utility to graph the function and the damping factor of the function in the same viewing window. Describe the behavior of the function as $x$ increases without bound.

**81.** $g(x) = e^{-x^2/2} \sin x$

**82.** $f(x) = e^{-x} \cos x$

**83.** $f(x) = 2^{-x/4} \cos \pi x$

**84.** $h(x) = 2^{-x^2/4} \sin x$

In Exercises 85–90, use a graphing utility to graph the function. Describe the behavior of the function as $x$ approaches zero.

**85.** $y = \dfrac{6}{x} + \cos x, \quad x > 0$

**86.** $y = \dfrac{4}{x} + \sin 2x, \quad x > 0$

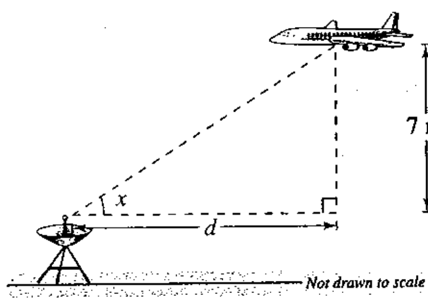
87. $g(x) = \dfrac{\sin x}{x}$

**88.** $f(x) = \dfrac{1 - \cos x}{x}$

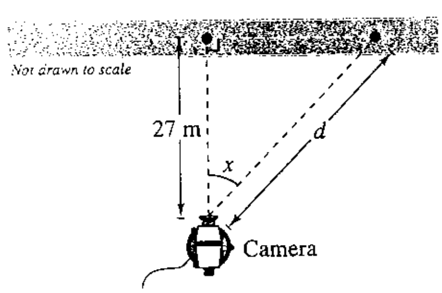
89. $f(x) = \sin \dfrac{1}{x}$

**90.** $h(x) = x \sin \dfrac{1}{x}$

91. **DISTANCE** A plane flying at an altitude of 7 miles above a radar antenna will pass directly over the radar antenna (see figure). Let $d$ be the ground distance from the antenna to the point directly under the plane and let $x$ be the angle of elevation to the plane from the antenna. ($d$ is positive as the plane approaches the antenna.) Write $d$ as a function of $x$ and graph the function over the interval $0 < x < \pi$.




92. **TELEVISION COVERAGE** A television camera is on a reviewing platform 27 meters from the street on which a parade will be passing from left to right (see figure). Write the distance $d$ from the camera to a particular unit in the parade as a function of the angle $x$, and graph the function over the interval $-\pi/2 < x < \pi/2$. (Consider $x$ as negative when a unit in the parade approaches from the left.)



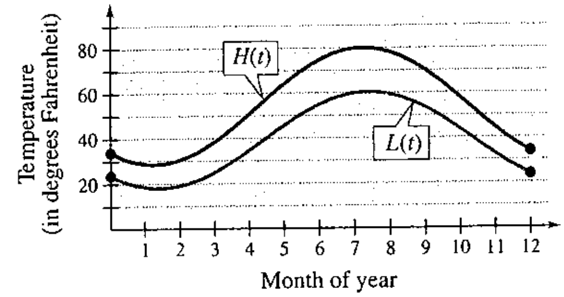

93. **METEOROLOGY** The normal monthly high temperatures $H$ (in degrees Fahrenheit) in Erie, Pennsylvania are approximated by

$$H(t) = 56.94 - 20.86 \cos(\pi t/6) - 11.58 \sin(\pi t/6)$$

and the normal monthly low temperatures $L$ are approximated by

$$L(t) = 41.80 - 17.13 \cos(\pi t/6) - 13.39 \sin(\pi t/6)$$

where $t$ is the time (in months), with $t = 1$ corresponding to January (see figure). (Source: National Climatic Data Center)



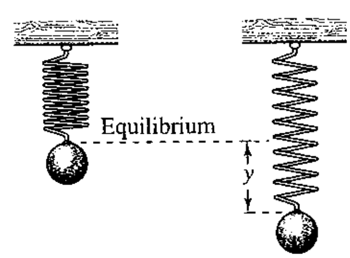

(a) What is the period of each function?

(b) During what part of the year is the difference between the normal high and normal low temperatures greatest? When is it smallest?

(c) The sun is northernmost in the sky around June 21, but the graph shows the warmest temperatures at a later date. Approximate the lag time of the temperatures relative to the position of the sun.

**94.** **SALES** The projected monthly sales $S$ (in thousands of units) of lawn mowers (a seasonal product) are modeled by $S = 74 + 3t - 40 \cos(\pi t/6)$, where $t$ is the time (in months), with $t = 1$ corresponding to January. Graph the sales function over 1 year.

**95.** **HARMONIC MOTION** An object weighing $W$ pounds is suspended from the ceiling by a steel spring (see figure). The weight is pulled downward (positive direction) from its equilibrium position and released. The resulting motion of the weight is described by the function $y = \frac{1}{2}e^{-t/4} \cos 4t,\ t > 0$, where $y$ is the distance (in feet) and $t$ is the time (in seconds).




(a) Use a graphing utility to graph the function.

(b) Describe the behavior of the displacement function for increasing values of time $t$.

## EXPLORATION

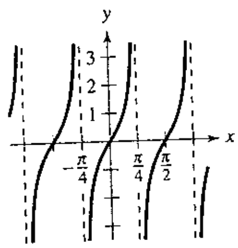
**TRUE OR FALSE?** In Exercises 96 and 97, determine whether the statement is true or false. Justify your answer.

**96.** The graph of $y = \csc x$ can be obtained on a calculator by graphing the reciprocal of $y = \sin x$.

**97.** The graph of $y = \sec x$ can be obtained on a calculator by graphing a translation of the reciprocal of $y = \sin x$.

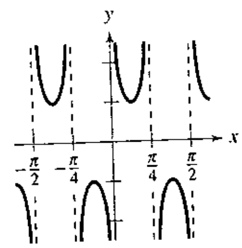
**98. CAPSTONE** Determine which function is represented by the graph. Do not use a calculator. Explain your reasoning.

(a)




(i) $f(x) = \tan 2x$
(ii) $f(x) = \tan(x/2)$
(iii) $f(x) = 2\tan x$
(iv) $f(x) = -\tan 2x$
(v) $f(x) = -\tan(x/2)$

(b)




(i) $f(x) = \sec 4x$
(ii) $f(x) = \csc 4x$
(iii) $f(x) = \csc(x/4)$
(iv) $f(x) = \sec(x/4)$
(v) $f(x) = \csc(4x - \pi)$

In Exercises 99 and 100, use a graphing utility to graph the function. Use the graph to determine the behavior of the function as $x \to c$.

(a) $x \to \frac{\pi^+}{2}$ $\left(\text{as } x \text{ approaches } \frac{\pi}{2} \text{ from the right}\right)$

(b) $x \to \frac{\pi^-}{2}$ $\left(\text{as } x \text{ approaches } \frac{\pi}{2} \text{ from the left}\right)$

(c) $x \to -\frac{\pi^+}{2}$ $\left(\text{as } x \text{ approaches } -\frac{\pi}{2} \text{ from the right}\right)$

(d) $x \to -\frac{\pi^-}{2}$ $\left(\text{as } x \text{ approaches } -\frac{\pi}{2} \text{ from the left}\right)$

**99.** $f(x) = \tan x$ **100.** $f(x) = \sec x$

In Exercises 101 and 102, use a graphing utility to graph the function. Use the graph to determine the behavior of the function as $x \to c$.

(a) As $x \to 0^+$, the value of $f(x) \to$ ___.
(b) As $x \to 0^-$, the value of $f(x) \to$ ___.
(c) As $x \to \pi^+$, the value of $f(x) \to$ ___.
(d) As $x \to \pi^-$, the value of $f(x) \to$ ___.

**101.** $f(x) = \cot x$ **102.** $f(x) = \csc x$

**103. THINK ABOUT IT** Consider the function given by $f(x) = x - \cos x$.

(a) Use a graphing utility to graph the function and verify that there exists a zero between 0 and 1. Use the graph to approximate the zero.

(b) Starting with $x_0 = 1$, generate a sequence $x_1, x_2, x_3, \ldots$, where $x_n = \cos(x_{n-1})$. For example,

$$x_0 = 1$$
$$x_1 = \cos(x_0)$$
$$x_2 = \cos(x_1)$$
$$x_3 = \cos(x_2)$$
$$\vdots$$

What value does the sequence approach?

**104. APPROXIMATION** Using calculus, it can be shown that the tangent function can be approximated by the polynomial

$$\tan x \approx x + \frac{2x^3}{3!} + \frac{16x^5}{5!}$$

where $x$ is in radians. Use a graphing utility to graph the tangent function and its polynomial approximation in the same viewing window. How do the graphs compare?

**105. APPROXIMATION** Using calculus, it can be shown that the secant function can be approximated by the polynomial

$$\sec x \approx 1 + \frac{x^2}{2!} + \frac{5x^4}{4!}$$

where $x$ is in radians. Use a graphing utility to graph the secant function and its polynomial approximation in the same viewing window. How do the graphs compare?

**106. PATTERN RECOGNITION**

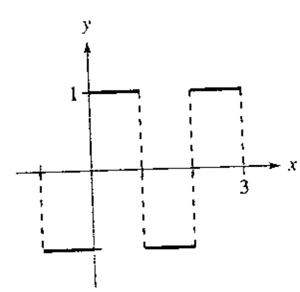
(a) Use a graphing utility to graph each function.

$$y_1 = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x\right)$$

$$y_2 = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x + \frac{1}{5}\sin 5\pi x\right)$$

(b) Identify the pattern started in part (a) and find a function $y_3$ that continues the pattern one more term. Use a graphing utility to graph $y_3$.

(c) The graphs in parts (a) and (b) approximate the periodic function in the figure. Find a function $y_4$ that is a better approximation.



# 6.6 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

## VOCABULARY: Fill in the blanks.

| | *Function* | *Alternative Notation* | *Domain* | *Range* |
|---|---|---|---|---|
| 1. | $y = \arcsin x$ | ________ | ________ | $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ |
| 2. | ________ | $y = \cos^{-1} x$ | $-1 \le x \le 1$ | ________ |
| 3. | $y = \arctan x$ | ________ | ________ | ________ |

4. Without restrictions, no trigonometric function has an ________ function.

## SKILLS AND APPLICATIONS

In Exercises 5–20, evaluate the expression without using a calculator.

5. $\arcsin \frac{1}{2}$
6. $\arcsin 0$
7. $\arccos \frac{1}{2}$
8. $\arccos 0$
9. $\arctan \dfrac{\sqrt{3}}{3}$
10. $\arctan(1)$
11. $\cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$
12. $\sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)$
13. $\arctan(-\sqrt{3})$
14. $\arctan \sqrt{3}$
15. $\arccos\left(-\dfrac{1}{2}\right)$
16. $\arcsin \dfrac{\sqrt{2}}{2}$
17. $\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$
18. $\tan^{-1}\left(-\dfrac{\sqrt{3}}{3}\right)$
19. $\tan^{-1} 0$
20. $\cos^{-1} 1$

In Exercises 21 and 22, use a graphing utility to graph $f$, $g$, and $y = x$ in the same viewing window to verify geometrically that $g$ is the inverse function of $f$. (Be sure to restrict the domain of $f$ properly.)

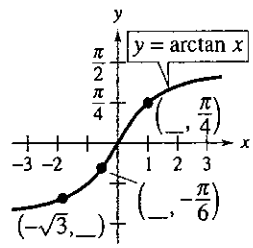
21. $f(x) = \sin x, \quad g(x) = \arcsin x$
22. $f(x) = \tan x, \quad g(x) = \arctan x$

In Exercises 23–40, use a calculator to evaluate the expression. Round your result to two decimal places.

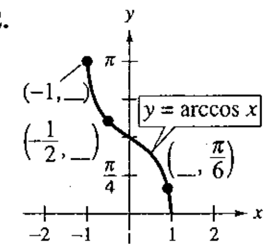
23. $\arccos 0.37$
24. $\arcsin 0.65$
25. $\arcsin(-0.75)$
26. $\arccos(-0.7)$
27. $\arctan(-3)$
28. $\arctan 25$
29. $\sin^{-1} 0.31$
30. $\cos^{-1} 0.26$
31. $\arccos(-0.41)$
32. $\arcsin(-0.125)$
33. $\arctan 0.92$
34. $\arctan 2.8$
35. $\arcsin \frac{7}{8}$
36. $\arccos\left(-\frac{1}{3}\right)$
37. $\tan^{-1} \frac{19}{4}$
38. $\tan^{-1}\left(-\frac{95}{7}\right)$
39. $\tan^{-1}(-\sqrt{372})$
40. $\tan^{-1}(-\sqrt{2165})$

In Exercises 41 and 42, determine the missing coordinates of the points on the graph of the function.

41.

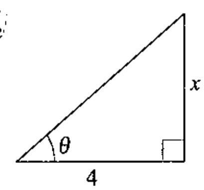



42.




In Exercises 43–48, use an inverse trigonometric function to write $\theta$ as a function of $x$.

43.

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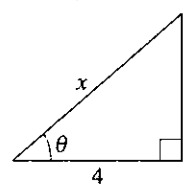


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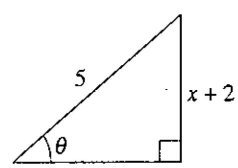


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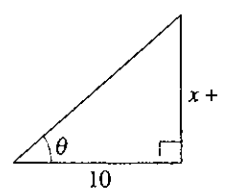


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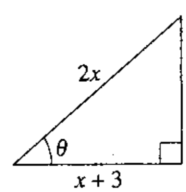


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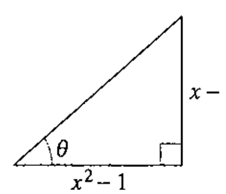



In Exercises 49–54, use the properties of inverse trigonometric functions to evaluate the expression.

49. $\sin(\arcsin 0.3)$
50. $\tan(\arctan 45)$
51. $\cos[\arccos(-0.1)]$
52. $\sin[\arcsin(-0.2)]$
53. $\arcsin(\sin 3\pi)$
54. $\arccos\left(\cos \dfrac{7\pi}{2}\right)$

In Exercises 55–66, find the exact value of the expression. (*Hint:* Sketch a right triangle.)

55. $\sin\left(\arctan \frac{3}{4}\right)$
56. $\sec\left(\arcsin \frac{4}{5}\right)$
57. $\cos(\tan^{-1} 2)$
58. $\sin\left(\cos^{-1} \frac{\sqrt{5}}{5}\right)$
59. $\cos\left(\arcsin \frac{5}{13}\right)$
60. $\csc\left[\arctan\left(-\frac{5}{12}\right)\right]$
61. $\sec\left[\arctan\left(-\frac{3}{5}\right)\right]$
62. $\tan\left[\arcsin\left(-\frac{3}{4}\right)\right]$
63. $\sin\left[\arccos\left(-\frac{2}{3}\right)\right]$
64. $\cot\left(\arctan \frac{5}{8}\right)$
65. $\csc\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$
66. $\sec\left[\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$

In Exercises 67–76, write an algebraic expression that is equivalent to the expression. (*Hint:* Sketch a right triangle, as demonstrated in Example 7.)

67. $\cot(\arctan x)$
68. $\sin(\arctan x)$
69. $\cos(\arcsin 2x)$
70. $\sec(\arctan 3x)$
71. $\sin(\arccos x)$
72. $\sec[\arcsin(x - 1)]$
73. $\tan\left(\arccos \frac{x}{3}\right)$
74. $\cot\left(\arctan \frac{1}{x}\right)$
75. $\csc\left(\arctan \frac{x}{\sqrt{2}}\right)$
76. $\cos\left(\arcsin \frac{x - h}{r}\right)$

In Exercises 77 and 78, use a graphing utility to graph $f$ and $g$ in the same viewing window to verify that the two functions are equal. Explain why they are equal. Identify any asymptotes of the graphs.

77. $f(x) = \sin(\arctan 2x), \quad g(x) = \frac{2x}{\sqrt{1 + 4x^2}}$
78. $f(x) = \tan\left(\arccos \frac{x}{2}\right), \quad g(x) = \frac{\sqrt{4 - x^2}}{x}$

In Exercises 79–82, fill in the blank.

79. $\arctan \frac{9}{x} = \arcsin(\quad), \quad x \neq 0$
80. $\arcsin \frac{\sqrt{36 - x^2}}{6} = \arccos(\quad), \quad 0 \leq x \leq 6$
81. $\arccos \frac{3}{\sqrt{x^2 - 2x + 10}} = \arcsin(\quad)$
82. $\arccos \frac{x - 2}{2} = \arctan(\quad), \quad |x - 2| \leq 2$

In Exercises 83 and 84, sketch a graph of the function and compare the graph of $g$ with the graph of $f(x) = \arcsin x$.

83. $g(x) = \arcsin(x - 1)$
84. $g(x) = \arcsin \frac{x}{2}$

In Exercises 85–90, sketch a graph of the function.

85. $y = 2 \arccos x$
86. $g(t) = \arccos(t + 2)$
87. $f(x) = \arctan 2x$
88. $f(x) = \frac{\pi}{2} + \arctan x$
89. $h(v) = \tan(\arccos v)$
90. $f(x) = \arccos \frac{x}{4}$

In Exercises 91–96, use a graphing utility to graph the function.

91. $f(x) = 2 \arccos(2x)$
92. $f(x) = \pi \arcsin(4x)$
93. $f(x) = \arctan(2x - 3)$
94. $f(x) = -3 + \arctan(\pi x)$
95. $f(x) = \pi - \sin^{-1}\left(\frac{2}{3}\right)$
96. $f(x) = \frac{\pi}{2} + \cos^{-1}\left(\frac{1}{\pi}\right)$

In Exercises 97 and 98, write the function in terms of the sine function by using the identity

$$A \cos \omega t + B \sin \omega t = \sqrt{A^2 + B^2} \sin\left(\omega t + \arctan \frac{A}{B}\right).$$

Use a graphing utility to graph both forms of the function. What does the graph imply?

97. $f(t) = 3 \cos 2t + 3 \sin 2t$
98. $f(t) = 4 \cos \pi t + 3 \sin \pi t$

In Exercises 99–104, fill in the blank. If not possible, state the reason. (*Note:* The notation $x \to c^+$ indicates that $x$ approaches $c$ from the right and $x \to c^-$ indicates that $x$ approaches $c$ from the left.)

99. As $x \to 1^-$, the value of $\arcsin x \to$ ___.
100. As $x \to 1^-$, the value of $\arccos x \to$ ___.

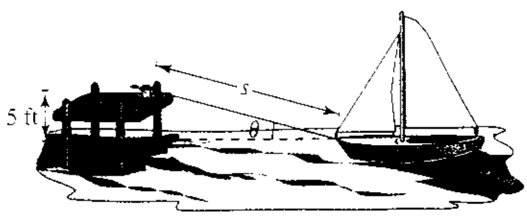
101. As $x \to \infty$, the value of $\arctan x \to$ ___ .

102. As $x \to -1^{+}$, the value of $\arcsin x \to$ ___ .

103. As $x \to -1^{+}$, the value of $\arccos x \to$ ___ .

104. As $x \to -\infty$, the value of $\arctan x \to$ ___ .

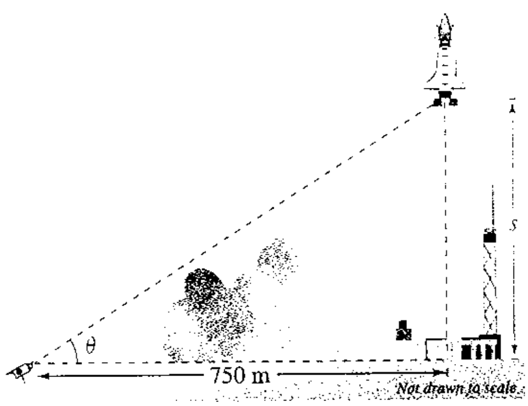
105. **DOCKING A BOAT** A boat is pulled in by means of a winch located on a dock 5 feet above the deck of the boat (see figure). Let $\theta$ be the angle of elevation from the boat to the winch and let $s$ be the length of the rope from the winch to the boat.




(a) Write $\theta$ as a function of $s$.

(b) Find $\theta$ when $s = 40$ feet and $s = 20$ feet.

106. **PHOTOGRAPHY** A television camera at ground level is filming the lift-off of a space shuttle at a point 750 meters from the launch pad (see figure). Let $\theta$ be the angle of elevation to the shuttle and let $s$ be the height of the shuttle.

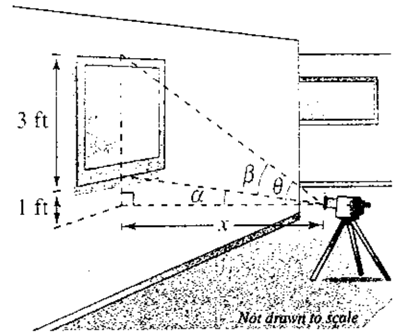



(a) Write $\theta$ as a function of $s$.

(b) Find $\theta$ when $s = 300$ meters and $s = 1200$ meters.

107. **PHOTOGRAPHY** A photographer is taking a picture of a three-foot-tall painting hung in an art gallery. The camera lens is 1 foot below the lower edge of the painting (see figure). The angle $\beta$ subtended by the camera lens $x$ feet from the painting is

$$\beta = \arctan\frac{3x}{x^2 + 4}, \quad x > 0.$$

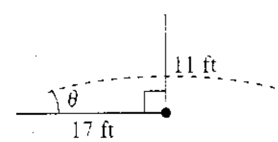



(a) Use a graphing utility to graph $\beta$ as a function of $x$.

(b) Move the cursor along the graph to approximate the distance from the picture when $\beta$ is maximum.

(c) Identify the asymptote of the graph and discuss its meaning in the context of the problem.

108. **GRANULAR ANGLE OF REPOSE** Different types of granular substances naturally settle at different angles when stored in cone-shaped piles. This angle $\theta$ is called the *angle of repose* (see figure). When rock salt is stored in a cone-shaped pile 11 feet high, the diameter of the pile's base is about 34 feet. Bulk-Store Structures, Inc.




(a) Find the angle of repose for rock salt.

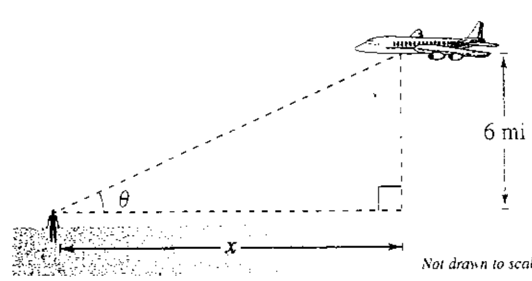
(b) How tall is a pile of rock salt that has a base diameter of 40 feet?

109. **GRANULAR ANGLE OF REPOSE** When whole corn is stored in a cone-shaped pile 20 feet high, the diameter of the pile's base is about 82 feet.

(a) Find the angle of repose for whole corn.

(b) How tall is a pile of corn that has a base diameter of 100 feet?

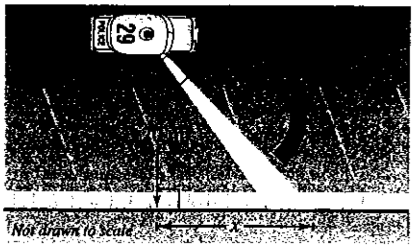
110. **ANGLE OF ELEVATION** An airplane flies at an altitude of 6 miles toward a point directly over an observer. Consider $\theta$ and $x$ as shown in the figure.




(a) Write $\theta$ as a function of $x$.

(b) Find $\theta$ when $x = 7$ miles and $x = 1$ mile.

**111. SECURITY PATROL** A security car with its spotlight on is parked 20 meters from a warehouse. Consider $\theta$ and $x$ as shown in the figure.




(a) Write $\theta$ as a function of $x$.

(b) Find $\theta$ when $x = 5$ meters and $x = 12$ meters.

## EXPLORATION

**TRUE OR FALSE?** In Exercises 112–114, determine whether the statement is true or false. Justify your answer.

**112.** $\sin \dfrac{5\pi}{6} = \dfrac{1}{2} \implies \arcsin \dfrac{1}{2} = \dfrac{5\pi}{6}$

**113.** $\tan \dfrac{5\pi}{4} = 1 \implies \arctan 1 = \dfrac{5\pi}{4}$

**114.** $\arctan x = \dfrac{\arcsin x}{\arccos x}$

**115.** Define the inverse cotangent function by restricting the domain of the cotangent function to the interval $(0, \pi)$, and sketch its graph.

**116.** Define the inverse secant function by restricting the domain of the secant function to the intervals $[0, \pi/2)$ and $(\pi/2, \pi]$, and sketch its graph.

**117.** Define the inverse cosecant function by restricting the domain of the cosecant function to the intervals $[-\pi/2, 0)$ and $(0, \pi/2]$, and sketch its graph.

**118. CAPSTONE** Use the results of Exercises 115–117 to explain how to graph (a) the inverse cotangent function, (b) the inverse secant function, and (c) the inverse cosecant function on a graphing utility.

**In Exercises 119–126, use the results of Exercises 115–117 to evaluate each expression without using a calculator.**

**119.** $\operatorname{arcsec} \sqrt{2}$ **120.** $\operatorname{arcsec} 1$

**121.** $\operatorname{arccot}(-1)$ **122.** $\operatorname{arccot}(-\sqrt{3})$

**123.** $\operatorname{arccsc} 2$ **124.** $\operatorname{arccsc}(-1)$

**125.** $\operatorname{arccsc}\left(\dfrac{2\sqrt{3}}{3}\right)$ **126.** $\operatorname{arcsec}\left(-\dfrac{2\sqrt{3}}{3}\right)$

**In Exercises 127–134, use the results of Exercises 115–117 and a calculator to approximate the value of the expression. Round your result to two decimal places.**

**127.** $\operatorname{arcsec} 2.54$ **128.** $\operatorname{arcsec}(-1.52)$

**129.** $\operatorname{arccot} 5.25$ **130.** $\operatorname{arccot}(-10)$

**131.** $\operatorname{arccot} \frac{5}{3}$ **132.** $\operatorname{arccot}\left(-\frac{16}{7}\right)$

**133.** $\operatorname{arccsc}\left(-\frac{25}{3}\right)$ **134.** $\operatorname{arccsc}(-12)$

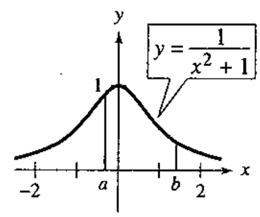
**135. AREA** In calculus, it is shown that the area of the region bounded by the graphs of $y = 0$, $y = 1/(x^2 + 1)$, $x = a$, and $x = b$ is given by

$$\text{Area} = \arctan b - \arctan a$$

(see figure). Find the area for the following values of $a$ and $b$.

(a) $a = 0, b = 1$ (b) $a = -1, b = 1$

(c) $a = 0, b = 3$ (d) $a = -1, b = 3$




**136. THINK ABOUT IT** Use a graphing utility to graph the functions

$$f(x) = \sqrt{x} \quad \text{and} \quad g(x) = 6 \arctan x.$$

For $x > 0$, it appears that $g > f$. Explain why you know that there exists a positive real number $a$ such that $g < f$ for $x > a$. Approximate the number $a$.

**137. THINK ABOUT IT** Consider the functions given by

$$f(x) = \sin x \quad \text{and} \quad f^{-1}(x) = \arcsin x.$$

(a) Use a graphing utility to graph the composite functions $f \circ f^{-1}$ and $f^{-1} \circ f$.

(b) Explain why the graphs in part (a) are not the graph of the line $y = x$. Why do the graphs of $f \circ f^{-1}$ and $f^{-1} \circ f$ differ?

**138. PROOF** Prove each identity.

(a) $\arcsin(-x) = -\arcsin x$

(b) $\arctan(-x) = -\arctan x$

(c) $\arctan x + \arctan \dfrac{1}{x} = \dfrac{\pi}{2}, \quad x > 0$

(d) $\arcsin x + \arccos x = \dfrac{\pi}{2}$

(e) $\arcsin x = \arctan \dfrac{x}{\sqrt{1 - x^2}}$

## 6.7 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

### VOCABULARY: Fill in the blanks.

1. A ________ measures the acute angle a path or line of sight makes with a fixed north-south line.
2. A point that moves on a coordinate line is said to be in simple ________ ________ if its distance $d$ from the origin at time $t$ is given by either $d = a \sin \omega t$ or $d = a \cos \omega t$.
3. The time for one complete cycle of a point in simple harmonic motion is its ________.
4. The number of cycles per second of a point in simple harmonic motion is its ________.

### SKILLS AND APPLICATIONS

In Exercises 5–14, solve the right triangle shown in the figure for all unknown sides and angles. Round your answers to two decimal places.

5. $A = 30°,\quad b = 3$
6. $B = 54°,\quad c = 15$
7. $B = 71°,\quad b = 24$
8. $A = 8.4°,\quad a = 40.5$
9. $a = 3,\quad b = 4$
10. $a = 25,\quad c = 35$
11. $b = 16,\quad c = 52$
12. $b = 1.32,\quad c = 9.45$
13. $A = 12°15',\quad c = 430.5$
14. $B = 65°12',\quad a = 14.2$

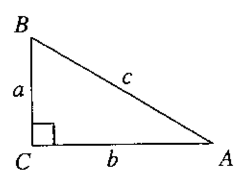



FIGURE FOR 5–14

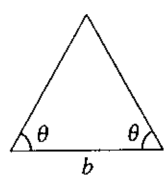
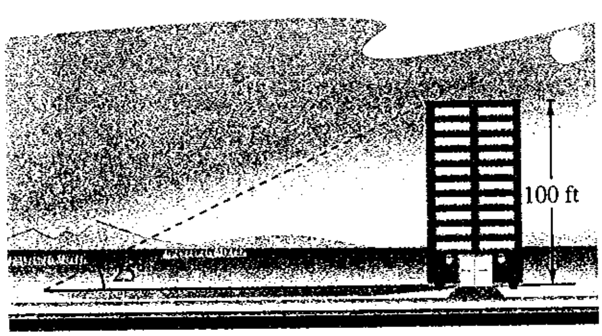



FIGURE FOR 15–18

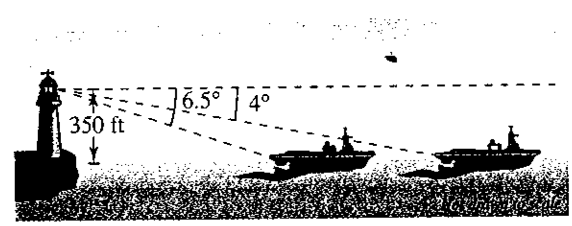
In Exercises 15–18, find the altitude of the isosceles triangle shown in the figure. Round your answers to two decimal places.

15. $\theta = 45°,\quad b = 6$
16. $\theta = 18°,\quad b = 10$
17. $\theta = 32°,\quad b = 8$
18. $\theta = 27°,\quad b = 11$

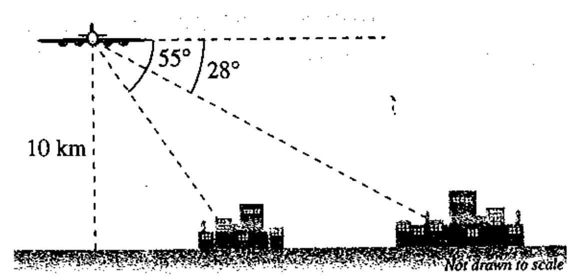
19. **LENGTH** The sun is 25° above the horizon. Find the length of a shadow cast by a building that is 100 feet tall (see figure).




20. **LENGTH** The sun is 20° above the horizon. Find the length of a shadow cast by a park statue that is 12 feet tall.
21. **HEIGHT** A ladder 20 feet long leans against the side of a house. Find the height from the top of the ladder to the ground if the angle of elevation of the ladder is 80°.
22. **HEIGHT** The length of a shadow of a tree is 125 feet when the angle of elevation of the sun is 33°. Approximate the height of the tree.
23. **HEIGHT** From a point 50 feet in front of a church, the angles of elevation to the base of the steeple and the top of the steeple are 35° and 47° 40′, respectively. Find the height of the steeple.
24. **DISTANCE** An observer in a lighthouse 350 feet above sea level observes two ships directly offshore. The angles of depression to the ships are 4° and 6.5° (see figure). How far apart are the ships?




25. **DISTANCE** A passenger in an airplane at an altitude of 10 kilometers sees two towns directly to the east of the plane. The angles of depression to the towns are 28° and 55° (see figure). How far apart are the towns?



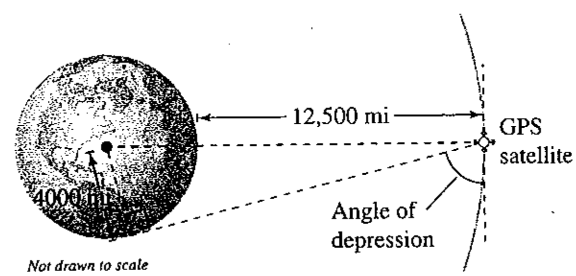
26. **ALTITUDE** You observe a plane approaching overhead and assume that its speed is 550 miles per hour. The angle of elevation of the plane is 16° at one time and 57° one minute later. Approximate the altitude of the plane.

27. **ANGLE OF ELEVATION** An engineer erects a 75-foot cellular telephone tower. Find the angle of elevation to the top of the tower at a point on level ground 50 feet from its base.

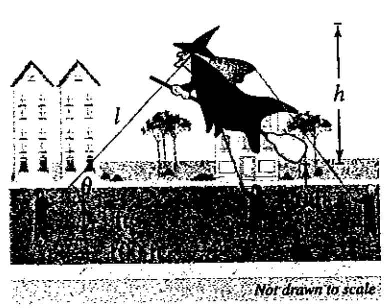
28. **ANGLE OF ELEVATION** The height of an outdoor basketball backboard is $12\frac{1}{2}$ feet, and the backboard casts a shadow $17\frac{1}{3}$ feet long.
   (a) Draw a right triangle that gives a visual representation of the problem. Label the known and unknown quantities.
   (b) Use a trigonometric function to write an equation involving the unknown quantity.
   (c) Find the angle of elevation of the sun.

29. **ANGLE OF DEPRESSION** A cellular telephone tower that is 150 feet tall is placed on top of a mountain that is 1200 feet above sea level. What is the angle of depression from the top of the tower to a cell phone user who is 5 horizontal miles away and 400 feet above sea level?

30. **ANGLE OF DEPRESSION** A Global Positioning System satellite orbits 12,500 miles above Earth's surface (see figure). Find the angle of depression from the satellite to the horizon. Assume the radius of Earth is 4000 miles.

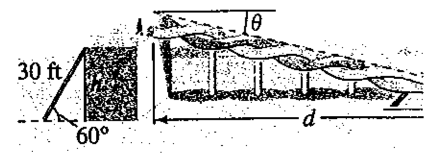



31. **HEIGHT** You are holding one of the tethers attached to the top of a giant character balloon in a parade. Before the start of the parade the balloon is upright and the bottom is floating approximately 20 feet above ground level. You are standing approximately 100 feet ahead of the balloon (see figure).



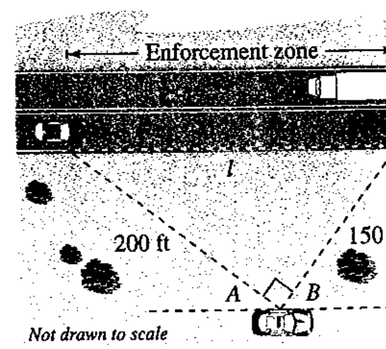

   (a) Find the length $l$ of the tether you a in terms of $h$, the height of the balloon f bottom.
   (b) Find an expression for the angle of eleva you to the top of the balloon.
   (c) Find the height $h$ of the balloon if th elevation to the top of the balloon is 35°.

32. **HEIGHT** The designers of a water park a a new slide and have sketched some p drawings. The length of the ladder is 30 f angle of elevation is 60° (see figure).




   (a) Find the height $h$ of the slide.
   (b) Find the angle of depression $\theta$ from the slide to the end of the slide at the ground the horizontal distance $d$ the rider travel
   (c) The angle of depression of the ride is l safety restrictions to be no less than 2 more than 30°. Find an interval for how travels horizontally.

33. **SPEED ENFORCEMENT** A police departn up a speed enforcement zone on a straigh highway. A patrol car is parked parallel t 200 feet from one end and 150 feet from th (see figure).




   (a) Find the length $l$ of the zone and the mea angles $A$ and $B$ (in degrees).
   (b) Find the minimum amount of time (in takes for a vehicle to pass through the z exceeding the posted speed limit of 35 m

34. **AIRPLANE ASCENT** During takeoff, an airplane's angle of ascent is 18° and its speed is 275 feet per second.
   (a) Find the plane's altitude after 1 minute.
   (b) How long will it take the plane to climb to an altitude of 10,000 feet?

35. **NAVIGATION** An airplane flying at 600 miles per hour has a bearing of 52°. After flying for 1.5 hours, how far north and how far east will the plane have traveled from its point of departure?

36. **NAVIGATION** A jet leaves Reno, Nevada and is headed toward Miami, Florida at a bearing of 100°. The distance between the two cities is approximately 2472 miles.
   (a) How far north and how far west is Reno relative to Miami?
   (b) If the jet is to return directly to Reno from Miami, at what bearing should it travel?

37. **NAVIGATION** A ship leaves port at noon and has a bearing of S 29° W. The ship sails at 20 knots.
   (a) How many nautical miles south and how many nautical miles west will the ship have traveled by 6:00 P.M.?
   (b) At 6:00 P.M., the ship changes course to due west. Find the ship's bearing and distance from the port of departure at 7:00 P.M.

38. **NAVIGATION** A privately owned yacht leaves a dock in Myrtle Beach, South Carolina and heads toward Freeport in the Bahamas at a bearing of S 1.4° E. The yacht averages a speed of 20 knots over the 428 nautical-mile trip.
   (a) How long will it take the yacht to make the trip?
   (b) How far east and south is the yacht after 12 hours?
   (c) If a plane leaves Myrtle Beach to fly to Freeport, what bearing should be taken?

39. **NAVIGATION** A ship is 45 miles east and 30 miles south of port. The captain wants to sail directly to port. What bearing should be taken?

40. **NAVIGATION** An airplane is 160 miles north and 85 miles east of an airport. The pilot wants to fly directly to the airport. What bearing should be taken?

41. **SURVEYING** A surveyor wants to find the distance across a swamp (see figure). The bearing from $A$ to $B$ is N 32° W. The surveyor walks 50 meters from $A$, and at the point $C$ the bearing to $B$ is N 68° W. Find (a) the bearing from $A$ to $C$ and (b) the distance from $A$ to $B$.

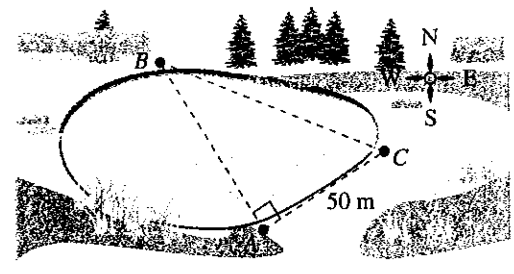
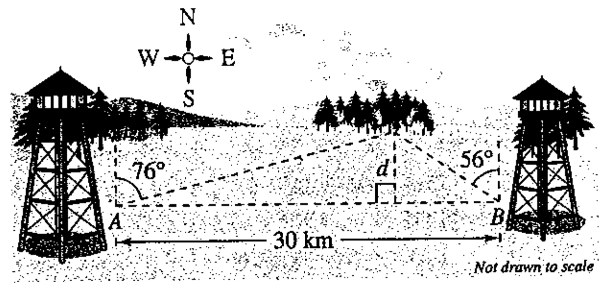



FIGURE FOR 41

42. **LOCATION OF A FIRE** Two fire towers are 30 kilometers apart, where tower $A$ is due west of tower $B$. A fire is spotted from the towers, and the bearings from $A$ and $B$ are N 76° E and N 56° W, respectively (see figure). Find the distance $d$ of the fire from the line segment $AB$.




**GEOMETRY** In Exercises 43 and 44, find the angle $\alpha$ between two nonvertical lines $L_1$ and $L_2$. The angle $\alpha$ satisfies the equation

$$\tan \alpha = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|$$

where $m_1$ and $m_2$ are the slopes of $L_1$ and $L_2$, respectively. (Assume that $m_1 m_2 \neq -1$.)

43. $L_1$: $3x - 2y = 5$
    $L_2$: $x + y = 1$

44. $L_1$: $2x - y = 8$
    $L_2$: $x - 5y = -4$

45. **GEOMETRY** Determine the angle between the diagonal of a cube and the diagonal of its base, as shown in the figure.

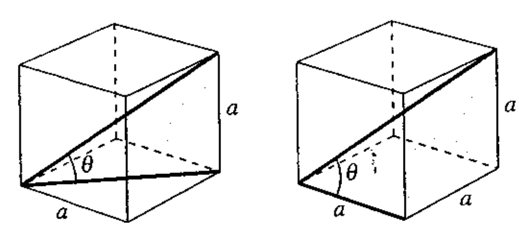



FIGURE FOR 45 FIGURE FOR 46

46. **GEOMETRY** Determine the angle between the diagonal of a cube and its edge, as shown in the figure.

47. **GEOMETRY** Find the length of the sides of a regular pentagon inscribed in a circle of radius 25 inches.

48. **GEOMETRY** Find the length of the sides of a regular hexagon inscribed in a circle of radius 25 inches.

49. **HARDWARE** Write the distance $y$ across the flat sides of a hexagonal nut as a function of $r$ (see figure).

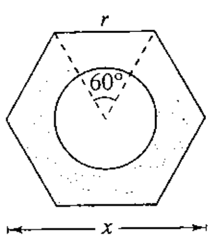



FIGURE FOR 49

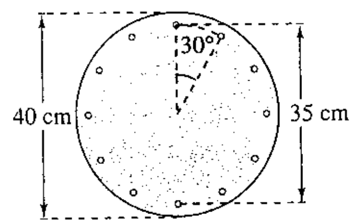
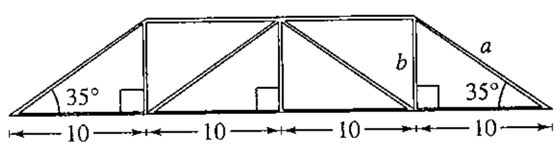



FIGURE FOR 50

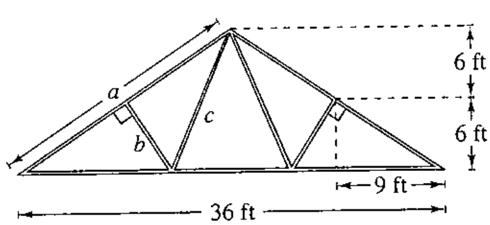
50. **BOLT HOLES** The figure shows a circular piece of sheet metal that has a diameter of 40 centimeters and contains 12 equally-spaced bolt holes. Determine the straight-line distance between the centers of consecutive bolt holes.

**TRUSSES** In Exercises 51 and 52, find the lengths of all the unknown members of the truss.

51.




52.




**HARMONIC MOTION** In Exercises 53–56, find a model for simple harmonic motion satisfying the specified conditions.

| | *Displacement* ($t = 0$) | *Amplitude* | *Period* |
|---|---|---|---|
| 53. | 0 | 4 centimeters | 2 seconds |
| 54. | 0 | 3 meters | 6 seconds |
| 55. | 3 inches | 3 inches | 1.5 seconds |
| 56. | 2 feet | 2 feet | 10 seconds |

**HARMONIC MOTION** In Exercises 57–60, for the simple harmonic motion described by the trigonometric function, find (a) the maximum displacement, (b) the frequency, (c) the value of $d$ when $t = 5$, and (d) the least positive value of $t$ for which $d = 0$. Use a graphing utility to verify your results.

57. $d = 9 \cos \frac{6\pi}{5} t$

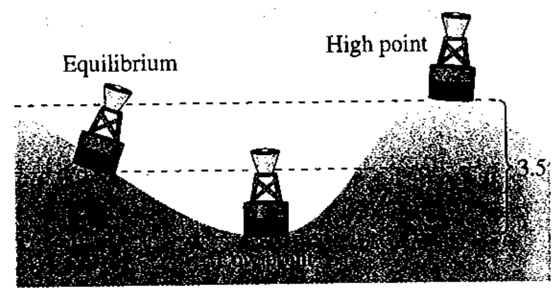
58. $d = \frac{1}{2} \cos 20\pi t$

59. $d = \frac{1}{4} \sin 6\pi t$

60. $d = \frac{1}{64} \sin 792\pi t$

61. **TUNING FORK** A point on the end of a tuning f moves in simple harmonic motion described $d = a \sin \omega t$. Find $\omega$ given that the tuning fork middle C has a frequency of 264 vibrations per seco

62. **WAVE MOTION** A buoy oscillates in sim harmonic motion as waves go past. It is noted that buoy moves a total of 3.5 feet from its low point to high point (see figure), and that it returns to its h point every 10 seconds. Write an equation that descri the motion of the buoy if its high point is at $t = 0$.



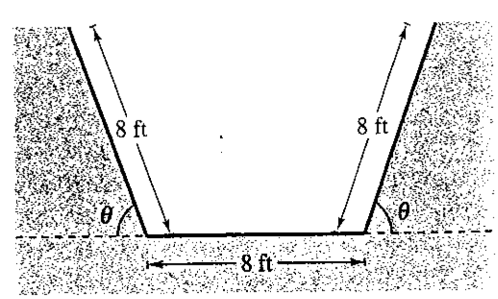

63. **OSCILLATION OF A SPRING** A ball that is bobt up and down on the end of a spring has a maxim displacement of 3 inches. Its motion (in ideal conditi is modeled by $y = \frac{1}{4} \cos 16t$ $(t > 0)$, where $y$ measured in feet and $t$ is the time in seconds.

(a) Graph the function.

(b) What is the period of the oscillations?

(c) Determine the first time the weight passes the p of equilibrium ($y = 0$).

64. **NUMERICAL AND GRAPHICAL ANALYSIS** cross section of an irrigation canal is an isosc trapezoid of which 3 of the sides are 8 feet long figure). The objective is to find the angle $\theta$ maximizes the area of the cross section. [*Hint:* The of a trapezoid is $(h/2)(b_1 + b_2)$.]



(a) Complete seven additional rows of the table.

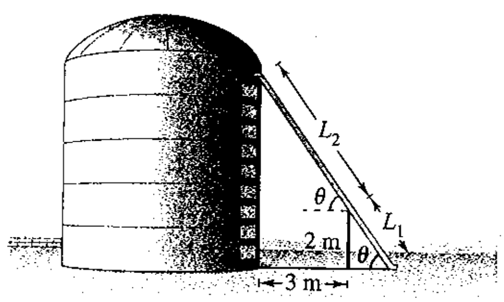
| Base 1 | Base 2 | Altitude | Area |
|---|---|---|---|
| 8 | $8 + 16 \cos 10°$ | $8 \sin 10°$ | 22.1 |
| 8 | $8 + 16 \cos 20°$ | $8 \sin 20°$ | 42.5 |

(b) Use a graphing utility to generate additional rows of the table. Use the table to estimate the maximum cross-sectional area.

(c) Write the area $A$ as a function of $\theta$.

(d) Use a graphing utility to graph the function. Use the graph to estimate the maximum cross-sectional area. How does your estimate compare with that of part (b)?

65. **NUMERICAL AND GRAPHICAL ANALYSIS** A 2-meter-high fence is 3 meters from the side of a grain storage bin. A grain elevator must reach from ground level outside the fence to the storage bin (see figure). The objective is to determine the shortest elevator that meets the constraints.




(a) Complete four rows of the table.

| $\theta$ | $L_1$ | $L_2$ | $L_1 + L_2$ |
|---|---|---|---|
| 0.1 | $\dfrac{2}{\sin 0.1}$ | $\dfrac{3}{\cos 0.1}$ | 23.0 |
| 0.2 | $\dfrac{2}{\sin 0.2}$ | $\dfrac{3}{\cos 0.2}$ | 13.1 |

(b) Use a graphing utility to generate additional rows of the table. Use the table to estimate the minimum length of the elevator.

(c) Write the length $L_1 + L_2$ as a function of $\theta$.

(d) Use a graphing utility to graph the function. Use the graph to estimate the minimum length. How does your estimate compare with that of part (b)?

66. **DATA ANALYSIS** The table shows the average sales $S$ (in millions of dollars) of an outerwear manufacturer for each month $t$, where $t = 1$ represents January.

| Time, $t$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Sales, $S$ | 13.46 | 11.15 | 8.00 | 4.85 | 2.54 | 1.70 |

| Time, $t$ | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|
| Sales, $S$ | 2.54 | 4.85 | 8.00 | 11.15 | 13.46 | 14.30 |

(a) Create a scatter plot of the data.

(b) Find a trigonometric model that fits the data. Graph the model with your scatter plot. How well does the model fit the data?

(c) What is the period of the model? Do you think it is reasonable given the context? Explain your reasoning.

(d) Interpret the meaning of the model's amplitude in the context of the problem.

67. **DATA ANALYSIS** The number of hours $H$ of daylight in Denver, Colorado on the 15th of each month are: 1(9.67), 2(10.72), 3(11.92), 4(13.25), 5(14.37), 6(14.97), 7(14.72), 8(13.77), 9(12.48), 10(11.18), 11(10.00), 12(9.38). The month is represented by $t$, with $t = 1$ corresponding to January. A model for the data is given by

$$H(t) = 12.13 + 2.77 \sin[(\pi t/6) - 1.60].$$

(a) Use a graphing utility to graph the data points and the model in the same viewing window.

(b) What is the period of the model? Is it what you expected? Explain.

(c) What is the amplitude of the model? What does it represent in the context of the problem? Explain.

## EXPLORATION

68. **CAPSTONE** While walking across flat land, you notice a wind turbine tower of height $h$ feet directly in front of you. The angle of elevation to the top of the tower is $A$ degrees. After you walk $d$ feet closer to the tower, the angle of elevation increases to $B$ degrees.

(a) Draw a diagram to represent the situation.

(b) Write an expression for the height $h$ of the tower in terms of the angles $A$ and $B$ and the distance $d$.

**TRUE OR FALSE?** In Exercises 69 and 70, determine whether the statement is true or false. Justify your answer.

69. The Leaning Tower of Pisa is not vertical, but if you know the angle of elevation $\theta$ to the top of the tower when you stand $d$ feet away from it, you can find its height $h$ using the formula $h = d \tan \theta$.

70. N 24° E means 24 degrees north of east.