7.3 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. When solving a trigonometric equation, the preliminary goal is to ________ the trigonometric function involved in the equation.
2. The equation $2 \sin \theta + 1 = 0$ has the solutions $\theta = \frac{7\pi}{6} + 2n\pi$ and $\theta = \frac{11\pi}{6} + 2n\pi$, which are called ________ solutions.
3. The equation $2 \tan^2 x - 3 \tan x + 1 = 0$ is a trigonometric equation that is of ________ type.
4. A solution of an equation that does not satisfy the original equation is called an ________ solution.

SKILLS AND APPLICATIONS

In Exercises 5–10, verify that the x-values are solutions of the equation.

5. $2 \cos x - 1 = 0$
   (a) $x = \frac{\pi}{3}$ (b) $x = \frac{5\pi}{3}$
6. $\sec x - 2 = 0$
   (a) $x = \frac{\pi}{3}$ (b) $x = \frac{5\pi}{3}$
7. $3 \tan^2 2x - 1 = 0$
   (a) $x = \frac{\pi}{12}$ (b) $x = \frac{5\pi}{12}$
8. $2 \cos^2 4x - 1 = 0$
   (a) $x = \frac{\pi}{16}$ (b) $x = \frac{3\pi}{16}$
9. $2 \sin^2 x - \sin x - 1 = 0$
   (a) $x = \frac{\pi}{2}$ (b) $x = \frac{7\pi}{6}$
10. $\csc^4 x - 4 \csc^2 x = 0$
   (a) $x = \frac{\pi}{6}$ (b) $x = \frac{5\pi}{6}$

In Exercises 11–24, solve the equation.

11. $2 \cos x + 1 = 0$
12. $2 \sin x + 1 = 0$
13. $\sqrt{3} \csc x - 2 = 0$
14. $\tan x + \sqrt{3} = 0$
15. $3 \sec^2 x - 4 = 0$
16. $3 \cot^2 x - 1 = 0$
17. $\sin x(\sin x + 1) = 0$
18. $(3 \tan^2 x - 1)(\tan^2 x - 3) = 0$
19. $4 \cos^2 x - 1 = 0$
20. $\sin^2 x = 3 \cos^2 x$
21. $2 \sin^2 2x = 1$
22. $\tan^2 3x = 3$
23. $\tan 3x(\tan x - 1) = 0$
24. $\cos 2x(2 \cos x + 1) = 0$

In Exercises 25–38, find all solutions of the equation in the interval $[0, 2\pi)$.

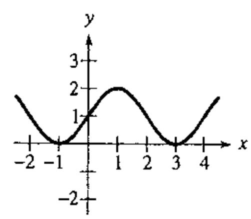
25. $\cos^3 x = \cos x$
26. $\sec^2 x - 1 = 0$
27. $3 \tan^3 x = \tan x$
28. $2 \sin^2 x = 2 + \cos x$
29. $\sec^2 x - \sec x = 2$
30. $\sec x \csc x = 2 \csc x$
31. $2 \sin x + \csc x = 0$
32. $\sec x + \tan x = 1$
33. $2 \cos^2 x + \cos x - 1 = 0$
34. $2 \sin^2 x + 3 \sin x + 1 = 0$
35. $2 \sec^2 x + \tan^2 x - 3 = 0$
36. $\cos x + \sin x \tan x = 2$
37. $\csc x + \cot x = 1$
38. $\sin x - 2 = \cos x - 2$

In Exercises 39–44, solve the multiple-angle equation.

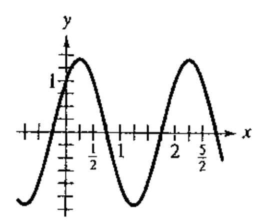
39. $\cos 2x = \frac{1}{2}$
40. $\sin 2x = -\frac{\sqrt{3}}{2}$
41. $\tan 3x = 1$
42. $\sec 4x = 2$
43. $\cos \frac{x}{2} = \frac{\sqrt{2}}{2}$
44. $\sin \frac{x}{2} = -\frac{\sqrt{3}}{2}$

In Exercises 45–48, find the x-intercepts of the graph.

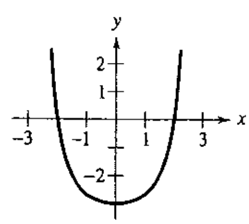
45. $y = \sin \frac{\pi x}{2} + 1$



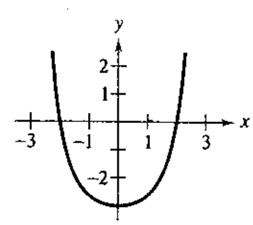
46. $y = \sin \pi x + \cos \pi x$



47. $y = \tan^2\left(\frac{\pi x}{6}\right) - 3$



48. $y = \sec^4\left(\frac{\pi x}{8}\right) - 4$



In Exercises 49–58, use a graphing utility to approximate the solutions (to three decimal places) of the equation in the interval $[0, 2\pi)$.

49. $2 \sin x - \cos x = 0$

50. $4 \sin^3 x + 2 \sin^2 x - 2 \sin x - 1 = 0$

51. $\dfrac{1 - \sin x}{\cos x} + \dfrac{\cos x}{1 - \sin x} = 4$

52. $\dfrac{\cos x \cot x}{1 - \sin x} = 3$

53. $x \tan x - 1 = 0$

54. $x \cos x - 1 = 0$

55. $\sec^2 x + 0.5 \tan x - 1 = 0$

56. $\csc^2 x + 0.5 \cot x - 5 = 0$

57. $2 \tan^2 x + 7 \tan x - 15 = 0$

58. $6 \sin^2 x - 7 \sin x + 2 = 0$

In Exercises 59–62, use the Quadratic Formula to solve the equation in the interval $[0, 2\pi)$. Then use a graphing utility to approximate the angle x.

59. $12 \sin^2 x - 13 \sin x + 3 = 0$

60. $3 \tan^2 x + 4 \tan x - 4 = 0$

61. $\tan^2 x + 3 \tan x + 1 = 0$

62. $4 \cos^2 x - 4 \cos x - 1 = 0$

In Exercises 63–74, use inverse functions where needed to find all solutions of the equation in the interval $[0, 2\pi)$.

63. $\tan^2 x + \tan x - 12 = 0$

64. $\tan^2 x - \tan x - 2 = 0$

65. $\tan^2 x - 6 \tan x + 5 = 0$

66. $\sec^2 x + \tan x - 3 = 0$

67. $2 \cos^2 x - 5 \cos x + 2 = 0$

68. $2 \sin^2 x - 7 \sin x + 3 = 0$

69. $\cot^2 x - 9 = 0$

70. $\cot^2 x - 6 \cot x + 5 = 0$

71. $\sec^2 x - 4 \sec x = 0$

72. $\sec^2 x + 2 \sec x - 8 = 0$

73. $\csc^2 x + 3 \csc x - 4 = 0$

74. $\csc^2 x - 5 \csc x = 0$

In Exercises 75–78, use a graphing utility to approximate the solutions (to three decimal places) of the equation in the given interval.

75. $3 \tan^2 x + 5 \tan x - 4 = 0, \quad \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

76. $\cos^2 x - 2 \cos x - 1 = 0, \quad [0, \pi]$

77. $4 \cos^2 x - 2 \sin x + 1 = 0, \quad \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

78. $2 \sec^2 x + \tan x - 6 = 0, \quad \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

In Exercises 79–84, (a) use a graphing utility to graph the function and approximate the maximum and minimum points on the graph in the interval $[0, 2\pi)$, and (b) solve the trigonometric equation and demonstrate that its solutions are the x-coordinates of the maximum and minimum points of f. (Calculus is required to find the trigonometric equation.)

	Function	*Trigonometric Equation*
79.	$f(x) = \sin^2 x + \cos x$	$2 \sin x \cos x - \sin x = 0$
80.	$f(x) = \cos^2 x - \sin x$	$-2 \sin x \cos x - \cos x = 0$
81.	$f(x) = \sin x + \cos x$	$\cos x - \sin x = 0$
82.	$f(x) = 2 \sin x + \cos 2x$	$2 \cos x - 4 \sin x \cos x = 0$
83.	$f(x) = \sin x \cos x$	$-\sin^2 x + \cos^2 x = 0$
84.	$f(x) = \sec x - \tan x - x$	$\sec x \tan x - \sec^2 x - 1 = 0$

FIXED POINT In Exercises 85 and 86, find the smallest positive fixed point of the function f. [A *fixed point* of a function f is a real number c such that $f(c) = c$.]

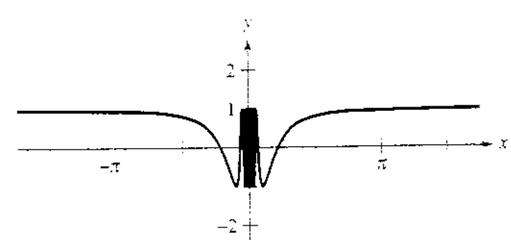
85. $f(x) = \tan \dfrac{\pi x}{4}$

86. $f(x) = \cos x$

87. **GRAPHICAL REASONING** Consider the function given by

$$f(x) = \cos \frac{1}{x}$$

and its graph shown in the figure.



(a) What is the domain of the function?

(b) Identify any symmetry and any asymptotes of the graph.

(c) Describe the behavior of the function as $x \to 0$.

(d) How many solutions does the equation

$$\cos \frac{1}{x} = 0$$

have in the interval $[-1, 1]$? Find the solutions.

(e) Does the equation $\cos(1/x) = 0$ have a greatest solution? If so, approximate the solution. If not, explain why.

See www.CalcChat.com for worked-out solutions to odd-numbered exercise

7.4 EXERCISES

VOCABULARY: Fill in the blank.

1. $\sin(u - v) =$ ______
2. $\cos(u + v) =$ ______
3. $\tan(u + v) =$ ______
4. $\sin(u + v) =$ ______
5. $\cos(u - v) =$ ______
6. $\tan(u - v) =$ ______

SKILLS AND APPLICATIONS

In Exercises 7–12, find the exact value of each expression.

7. (a) $\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$ (b) $\cos\frac{\pi}{4} + \cos\frac{\pi}{3}$
8. (a) $\sin\left(\frac{3\pi}{4} + \frac{5\pi}{6}\right)$ (b) $\sin\frac{3\pi}{4} + \sin\frac{5\pi}{6}$
9. (a) $\sin\left(\frac{7\pi}{6} - \frac{\pi}{3}\right)$ (b) $\sin\frac{7\pi}{6} - \sin\frac{\pi}{3}$
10. (a) $\cos(120° + 45°)$ (b) $\cos 120° + \cos 45°$
11. (a) $\sin(135° - 30°)$ (b) $\sin 135° - \cos 30°$
12. (a) $\sin(315° - 60°)$ (b) $\sin 315° - \sin 60°$

In Exercises 13–28, find the exact values of the sine, cosine, and tangent of the angle.

13. $\frac{11\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$
14. $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$
15. $\frac{17\pi}{12} = \frac{9\pi}{4} - \frac{5\pi}{6}$
16. $-\frac{\pi}{12} = \frac{\pi}{6} - \frac{\pi}{4}$
17. $105° = 60° + 45°$
18. $165° = 135° + 30°$
19. $195° = 225° - 30°$
20. $255° = 300° - 45°$
21. $\frac{13\pi}{12}$
22. $-\frac{7\pi}{12}$
23. $-\frac{13\pi}{12}$
24. $\frac{5\pi}{12}$
25. $285°$
26. $-105°$
27. $-165°$
28. $15°$

In Exercises 29–36, write the expression as the sine, cosine, or tangent of an angle.

29. $\sin 3 \cos 1.2 - \cos 3 \sin 1.2$
30. $\cos\frac{\pi}{7}\cos\frac{\pi}{5} - \sin\frac{\pi}{7}\sin\frac{\pi}{5}$
31. $\sin 60° \cos 15° + \cos 60° \sin 15°$
32. $\cos 130° \cos 40° - \sin 130° \sin 40°$
33. $\dfrac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$
34. $\dfrac{\tan 140° - \tan 60°}{1 + \tan 140° \tan 60°}$
35. $\dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$
36. $\cos 3x \cos 2y + \sin 3x \sin 2y$

In Exercises 37–42, find the exact value of the expressior

37. $\sin\frac{\pi}{12}\cos\frac{\pi}{4} + \cos\frac{\pi}{12}\sin\frac{\pi}{4}$
38. $\cos\frac{\pi}{16}\cos\frac{3\pi}{16} - \sin\frac{\pi}{16}\sin\frac{3\pi}{16}$
39. $\sin 120° \cos 60° - \cos 120° \sin 60°$
40. $\cos 120° \cos 30° + \sin 120° \sin 30°$
41. $\dfrac{\tan(5\pi/6) - \tan(\pi/6)}{1 + \tan(5\pi/6)\tan(\pi/6)}$
42. $\dfrac{\tan 25° + \tan 110°}{1 - \tan 25° \tan 110°}$

In Exercises 43–50, find the exact value of the trigono function given that $\sin u = \frac{5}{13}$ and $\cos v = -\frac{3}{5}$. (Both u are in Quadrant II.)

43. $\sin(u + v)$
44. $\cos(u - v)$
45. $\cos(u + v)$
46. $\sin(v - u)$
47. $\tan(u + v)$
48. $\csc(u - v)$
49. $\sec(v - u)$
50. $\cot(u + v)$

In Exercises 51–56, find the exact value of the trigon function given that $\sin u = -\frac{7}{25}$ and $\cos v = -\frac{4}{5}$. and v are in Quadrant III.)

51. $\cos(u + v)$
52. $\sin(u + v)$
53. $\tan(u - v)$
54. $\cot(v - u)$
55. $\csc(u - v)$
56. $\sec(v - u)$

In Exercises 57–60, write the trigonometric express algebraic expression.

57. $\sin(\arcsin x + \arccos x)$
58. $\sin(\arctan 2x - \arccos x)$
59. $\cos(\arccos x + \arcsin x)$
60. $\cos(\arccos x - \arctan x)$

In Exercises 61–70, prove the identity.

61. $\sin\left(\frac{\pi}{2} - x\right) = \cos x$ **62.** $\sin\left(\frac{\pi}{2} + x\right) = \cos x$

63. $\sin\left(\frac{\pi}{6} + x\right) = \frac{1}{2}(\cos x + \sqrt{3}\sin x)$

64. $\cos\left(\frac{5\pi}{4} - x\right) = -\frac{\sqrt{2}}{2}(\cos x + \sin x)$

65. $\cos(\pi - \theta) + \sin\left(\frac{\pi}{2} + \theta\right) = 0$

66. $\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan\theta}{1 + \tan\theta}$

67. $\cos(x + y)\cos(x - y) = \cos^2 x - \sin^2 y$

68. $\sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y$

69. $\sin(x + y) + \sin(x - y) = 2\sin x\cos y$

70. $\cos(x + y) + \cos(x - y) = 2\cos x\cos y$

In Exercises 71–74, simplify the expression algebraically and use a graphing utility to confirm your answer graphically.

71. $\cos\left(\frac{3\pi}{2} - x\right)$ **72.** $\cos(\pi + x)$

73. $\sin\left(\frac{3\pi}{2} + \theta\right)$ **74.** $\tan(\pi + \theta)$

In Exercises 75–84, find all solutions of the equation in the interval $[0, 2\pi)$.

75. $\sin(x + \pi) - \sin x + 1 = 0$

76. $\sin(x + \pi) - \sin x - 1 = 0$

77. $\cos(x + \pi) - \cos x - 1 = 0$

78. $\cos(x + \pi) - \cos x + 1 = 0$

79. $\sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$

80. $\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$

81. $\cos\left(x + \frac{\pi}{4}\right) - \cos\left(x - \frac{\pi}{4}\right) = 1$

82. $\tan(x + \pi) + 2\sin(x + \pi) = 0$

83. $\sin\left(x + \frac{\pi}{2}\right) - \cos^2 x = 0$

84. $\cos\left(x - \frac{\pi}{2}\right) + \sin^2 x = 0$

In Exercises 85–88, use a graphing utility to approximate the solutions in the interval $[0, 2\pi)$.

85. $\cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right) = 1$

86. $\tan(x + \pi) - \cos\left(x + \frac{\pi}{2}\right) = 0$

87. $\sin\left(x + \frac{\pi}{2}\right) + \cos^2 x = 0$

88. $\cos\left(x - \frac{\pi}{2}\right) - \sin^2 x = 0$

89. HARMONIC MOTION A weight is attached to a spring suspended vertically from a ceiling. When a driving force is applied to the system, the weight moves vertically from its equilibrium position, and this motion is modeled by

$$y = \frac{1}{3}\sin 2t + \frac{1}{4}\cos 2t$$

where y is the distance from equilibrium (in feet) and t is the time (in seconds).

(a) Use the identity

$$a\sin B\theta + b\cos B\theta = \sqrt{a^2 + b^2}\sin(B\theta + C)$$

where $C = \arctan(b/a)$, $a > 0$, to write the model in the form $y = \sqrt{a^2 + b^2}\sin(Bt + C)$.

(b) Find the amplitude of the oscillations of the weight.

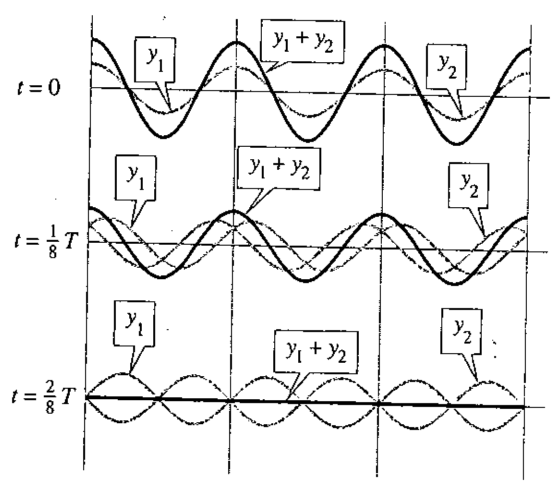
(c) Find the frequency of the oscillations of the weight.

90. STANDING WAVES The equation of a standing wave is obtained by adding the displacements of two waves traveling in opposite directions (see figure). Assume that each of the waves has amplitude A, period T, and wavelength λ. If the models for these waves are

$$y_1 = A\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \text{and} \quad y_2 = A\cos 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

show that

$$y_1 + y_2 = 2A\cos\frac{2\pi t}{T}\cos\frac{2\pi x}{\lambda}.$$



7.5 EXERCISES

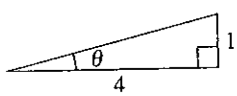
See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blank to complete the trigonometric formula.

1. $\sin 2u =$ ______
2. $\dfrac{1 + \cos 2u}{2} =$ ______
3. $\cos 2u =$ ______
4. $\dfrac{1 - \cos 2u}{1 + \cos 2u} =$ ______
5. $\sin \dfrac{u}{2} =$ ______
6. $\tan \dfrac{u}{2} =$ ______
7. $\cos u \cos v =$ ______
8. $\sin u \cos v =$ ______
9. $\sin u + \sin v =$ ______
10. $\cos u - \cos v =$ ______

SKILLS AND APPLICATIONS

In Exercises 11–18, use the figure to find the exact value of the trigonometric function.



11. $\cos 2\theta$
12. $\sin 2\theta$
13. $\tan 2\theta$
14. $\sec 2\theta$
15. $\csc 2\theta$
16. $\cot 2\theta$
17. $\sin 4\theta$
18. $\tan 4\theta$

In Exercises 19–28, find the exact solutions of the equation in the interval $[0, 2\pi)$.

19. $\sin 2x - \sin x = 0$
20. $\sin 2x + \cos x = 0$
21. $4 \sin x \cos x = 1$
22. $\sin 2x \sin x = \cos x$
23. $\cos 2x - \cos x = 0$
24. $\cos 2x + \sin x = 0$
25. $\sin 4x = -2 \sin 2x$
26. $(\sin 2x + \cos 2x)^2 = 1$
27. $\tan 2x - \cot x = 0$
28. $\tan 2x - 2 \cos x = 0$

In Exercises 29–36, use a double-angle formula to rewrite the expression.

29. $6 \sin x \cos x$
30. $\sin x \cos x$
31. $6 \cos^2 x - 3$
32. $\cos^2 x - \frac{1}{2}$
33. $4 - 8 \sin^2 x$
34. $10 \sin^2 x - 5$
35. $(\cos x + \sin x)(\cos x - \sin x)$
36. $(\sin x - \cos x)(\sin x + \cos x)$

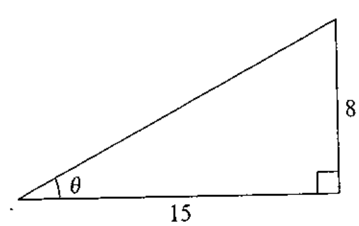
In Exercises 37–42, find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.

37. $\sin u = -\dfrac{3}{5}, \quad \dfrac{3\pi}{2} < u < 2\pi$
38. $\cos u = -\dfrac{4}{5}, \quad \dfrac{\pi}{2} < u < \pi$
39. $\tan u = \dfrac{3}{5}, \quad 0 < u < \dfrac{\pi}{2}$
40. $\cot u = \sqrt{2}, \quad \pi < u < \dfrac{3\pi}{2}$
41. $\sec u = -2, \quad \dfrac{\pi}{2} < u < \pi$
42. $\csc u = 3, \quad \dfrac{\pi}{2} < u < \pi$

In Exercises 43–52, use the power-reducing formulas to rewrite the expression in terms of the first power of the cosine.

43. $\cos^4 x$
44. $\sin^4 2x$
45. $\cos^4 2x$
46. $\sin^8 x$
47. $\tan^4 2x$
48. $\sin^2 x \cos^4 x$
49. $\sin^2 2x \cos^2 2x$
50. $\tan^2 2x \cos^4 2x$
51. $\sin^4 x \cos^2 x$
52. $\sin^4 x \cos^4 x$

In Exercises 53–58, use the figure to find the exact value of the trigonometric function.



53. $\cos \dfrac{\theta}{2}$
54. $\sin \dfrac{\theta}{2}$
55. $\tan \dfrac{\theta}{2}$
56. $\sec \dfrac{\theta}{2}$
57. $\csc \dfrac{\theta}{2}$
58. $\cot \dfrac{\theta}{2}$

In Exercises 59–66, use the half-angle formulas to determine the exact values of the sine, cosine, and tangent of the angle.

59. $75°$
60. $165°$
61. $112° 30'$
62. $67° 30'$
63. $\pi/8$
64. $\pi/12$
65. $3\pi/8$
66. $7\pi/12$

In Exercises 67–72, (a) determine the quadrant in which $u/2$ lies, and (b) find the exact values of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$ using the half-angle formulas.

67. $\cos u = \frac{7}{25}, \quad 0 < u < \frac{\pi}{2}$

68. $\sin u = \frac{5}{13}, \quad \frac{\pi}{2} < u < \pi$

69. $\tan u = -\frac{5}{12}, \quad \frac{3\pi}{2} < u < 2\pi$

70. $\cot u = 3, \quad \pi < u < \frac{3\pi}{2}$

71. $\csc u = -\frac{5}{3}, \quad \pi < u < \frac{3\pi}{2}$

72. $\sec u = \frac{7}{2}, \quad \frac{3\pi}{2} < u < 2\pi$

In Exercises 73–76, use the half-angle formulas to simplify the expression.

73. $\sqrt{\frac{1 - \cos 6x}{2}}$
74. $\sqrt{\frac{1 + \cos 4x}{2}}$
75. $-\sqrt{\frac{1 - \cos 8x}{1 + \cos 8x}}$
76. $-\sqrt{\frac{1 - \cos(x - 1)}{2}}$

In Exercises 77–80, find all solutions of the equation in the interval $[0, 2\pi)$. Use a graphing utility to graph the equation and verify the solutions.

77. $\sin \frac{x}{2} + \cos x = 0$
78. $\sin \frac{x}{2} + \cos x - 1 = 0$
79. $\cos \frac{x}{2} - \sin x = 0$
80. $\tan \frac{x}{2} - \sin x = 0$

In Exercises 81–90, use the product-to-sum formulas to write the product as a sum or difference.

81. $\sin \frac{\pi}{3} \cos \frac{\pi}{6}$
82. $4 \cos \frac{\pi}{3} \sin \frac{5\pi}{6}$
83. $10 \cos 75° \cos 15°$
84. $6 \sin 45° \cos 15°$
85. $\sin 5\theta \sin 3\theta$
86. $3 \sin(-4\alpha) \sin 6\alpha$
87. $7 \cos(-5\beta) \sin 3\beta$
88. $\cos 2\theta \cos 4\theta$
89. $\sin(x + y) \sin(x - y)$
90. $\sin(x + y) \cos(x - y)$

In Exercises 91–98, use the sum-to-product formulas to write the sum or difference as a product.

91. $\sin 3\theta + \sin \theta$
92. $\sin 5\theta - \sin 3\theta$
93. $\cos 6x + \cos 2x$
94. $\cos x + \cos 4x$
95. $\sin(\alpha + \beta) - \sin(\alpha - \beta)$
96. $\cos(\phi + 2\pi) + \cos \phi$
97. $\cos\left(\theta + \frac{\pi}{2}\right) - \cos\left(\theta - \frac{\pi}{2}\right)$
98. $\sin\left(x + \frac{\pi}{2}\right) + \sin\left(x - \frac{\pi}{2}\right)$

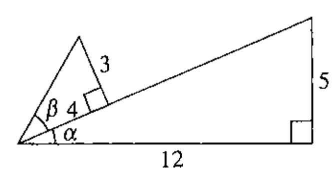
In Exercises 99–102, use the sum-to-product formulas to find the exact value of the expression.

99. $\sin 75° + \sin 15°$
100. $\cos 120° + \cos 60°$
101. $\cos \frac{3\pi}{4} - \cos \frac{\pi}{4}$
102. $\sin \frac{5\pi}{4} - \sin \frac{3\pi}{4}$

In Exercises 103–106, find all solutions of the equation in the interval $[0, 2\pi)$. Use a graphing utility to graph the equation and verify the solutions.

103. $\sin 6x + \sin 2x = 0$
104. $\cos 2x - \cos 6x = 0$
105. $\frac{\cos 2x}{\sin 3x - \sin x} - 1 = 0$
106. $\sin^2 3x - \sin^2 x = 0$

In Exercises 107–110, use the figure to find the exact value of the trigonometric function.



107. $\sin 2\alpha$
108. $\cos 2\beta$
109. $\cos(\beta/2)$
110. $\sin(\alpha + \beta)$

In Exercises 111–124, verify the identity.

111. $\csc 2\theta = \frac{\csc \theta}{2 \cos \theta}$
112. $\sec 2\theta = \frac{\sec^2 \theta}{2 - \sec^2 \theta}$
113. $\sin \frac{\alpha}{3} \cos \frac{\alpha}{3} = \frac{1}{2} \sin \frac{2\alpha}{3}$
114. $\frac{\cos 3\beta}{\cos \beta} = 1 - 4 \sin^2 \beta$
115. $1 + \cos 10y = 2 \cos^2 5y$
116. $\cos^4 x - \sin^4 x = \cos 2x$
117. $\cos 4\alpha = \cos^2 2\alpha - \sin^2 2\alpha$
118. $(\sin x + \cos x)^2 = 1 + \sin 2x$
119. $\tan \frac{u}{2} = \csc u - \cot u$
120. $\sec \frac{u}{2} = \pm\sqrt{\frac{2 \tan u}{\tan u + \sin u}}$

8.1 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Exercises with no solution: 26, 27, 30, 31
Exercises with two solutions: 18, 29, 34

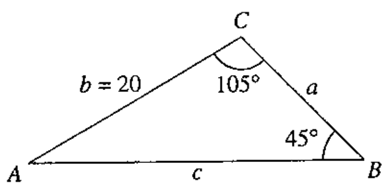
VOCABULARY: Fill in the blanks.

1. An ________ triangle is a triangle that has no right angle.
2. For triangle ABC, the Law of Sines is given by $\dfrac{a}{\sin A} =$ ________ $= \dfrac{c}{\sin C}$.
3. Two ________ and one ________ determine a unique triangle.
4. The area of an oblique triangle is given by $\frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C =$ ________ .

SKILLS AND APPLICATIONS

In Exercises 5–24, use the Law of Sines to solve the triangle. Round your answers to two decimal places.

5.



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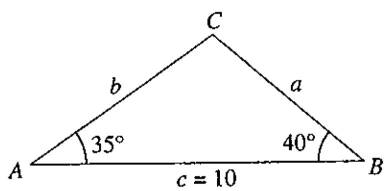

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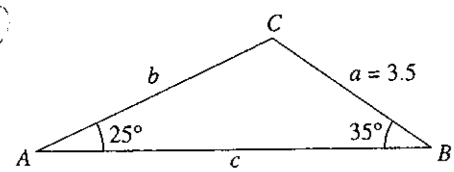

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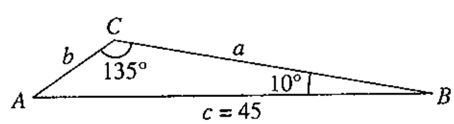


9. $A = 102.4°, \quad C = 16.7°, \quad a = 21.6$
10. $A = 24.3°, \quad C = 54.6°, \quad c = 2.68$
11. $A = 83° 20', \quad C = 54.6°, \quad c = 18.1$
12. $A = 5° 40', \quad B = 8° 15', \quad b = 4.8$
13. $A = 35°, \quad B = 65°, \quad c = 10$
14. $A = 120°, \quad B = 45°, \quad c = 16$
15. $A = 55°, \quad B = 42°, \quad c = \frac{3}{4}$
16. $B = 28°, \quad C = 104°, \quad a = 3\frac{5}{8}$
17. $A = 36°, \quad a = 8, \quad b = 5$
18. $A = 60°, \quad a = 9, \quad c = 10$
19. $B = 15° 30', \quad a = 4.5, \quad b = 6.8$
20. $B = 2° 45', \quad b = 6.2, \quad c = 5.8$
21. $A = 145°, \quad a = 14, \quad b = 4$
22. $A = 100°, \quad a = 125, \quad c = 10$
23. $A = 110° 15', \quad a = 48, \quad b = 16$
24. $C = 95.20°, \quad a = 35, \quad c = 50$

In Exercises 25–34, use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both. Round your answers to two decimal places.

25. $A = 110°, \quad a = 125, \quad b = 100$
26. $A = 110°, \quad a = 125, \quad b = 200$
27. $A = 76°, \quad a = 18, \quad b = 20$
28. $A = 76°, \quad a = 34, \quad b = 21$
29. $A = 58°, \quad a = 11.4, \quad b = 12.8$
30. $A = 58°, \quad a = 4.5, \quad b = 12.8$
31. $A = 120°, \quad a = b = 25$
32. $A = 120°, \quad a = 25, \quad b = 24$
33. $A = 45°, \quad a = b = 1$
34. $A = 25° 4', \quad a = 9.5, \quad b = 22$

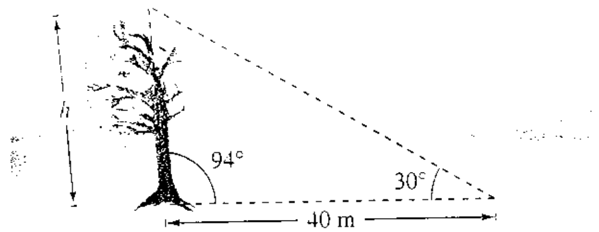
In Exercises 35–38, find values for b such that the triangle has (a) one solution, (b) two solutions, and (c) no solution.

35. $A = 36°, \quad a = 5$
36. $A = 60°, \quad a = 10$
37. $A = 10°, \quad a = 10.8$
38. $A = 88°, \quad a = 315.6$

In Exercises 39–44, find the area of the triangle having the indicated angle and sides.

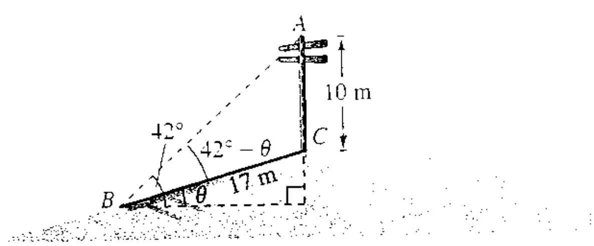
39. $C = 120°, \quad a = 4, \quad b = 6$
40. $B = 130°, \quad a = 62, \quad c = 20$
41. $A = 43° 45', \quad b = 57, \quad c = 85$
42. $A = 5° 15', \quad b = 4.5, \quad c = 22$
43. $B = 72° 30', \quad a = 105, \quad c = 64$
44. $C = 84° 30', \quad a = 16, \quad b = 20$

45. **HEIGHT** Because of prevailing winds, a tree grew so that it was leaning 4° from the vertical. At a point 40 meters from the tree, the angle of elevation to the top of the tree is 30° (see figure). Find the height h of the tree.

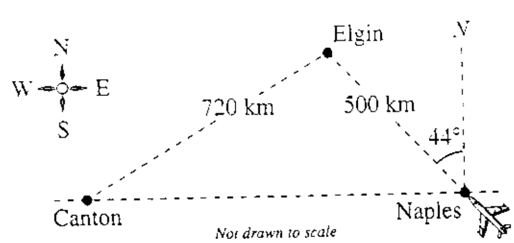


46. **HEIGHT** A flagpole at a right angle to the horizontal is located on a slope that makes an angle of 12° with the horizontal. The flagpole's shadow is 16 meters long and points directly up the slope. The angle of elevation from the tip of the shadow to the sun is 20°.
   (a) Draw a triangle to represent the situation. Show the known quantities on the triangle and use a variable to indicate the height of the flagpole.
   (b) Write an equation that can be used to find the height of the flagpole.
   (c) Find the height of the flagpole.

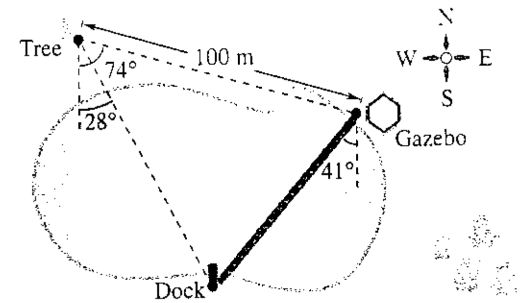
47. **ANGLE OF ELEVATION** A 10-meter utility pole casts a 17-meter shadow directly down a slope when the angle of elevation of the sun is 42° (see figure). Find θ, the angle of elevation of the ground.



48. **FLIGHT PATH** A plane flies 500 kilometers with a bearing of 316° from Naples to Elgin (see figure). The plane then flies 720 kilometers from Elgin to Canton (Canton is due west of Naples). Find the bearing of the flight from Elgin to Canton.



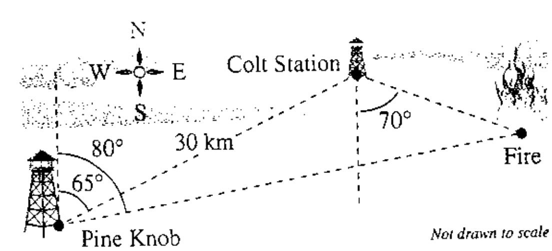
49. **BRIDGE DESIGN** A bridge is to be built across a small lake from a gazebo to a dock (see figure). The bearing from the gazebo to the dock is S 41° W. From a tree 100 meters from the gazebo, the bearings to the gazebo and the dock are S 74° E and S 28° E, respectively. Find the distance from the gazebo to the dock.



50. **RAILROAD TRACK DESIGN** The circular arc of a railroad curve has a chord of length 3000 feet corresponding to a central angle of 40°.
   (a) Draw a diagram that visually represents the situation. Show the known quantities on the diagram and use the variables r and s to represent the radius of the arc and the length of the arc, respectively.
   (b) Find the radius r of the circular arc.
   (c) Find the length s of the circular arc.

51. **GLIDE PATH** A pilot has just started on the glide path for landing at an airport with a runway of length 9000 feet. The angles of depression from the plane to the ends of the runway are 17.5° and 18.8°.
   (a) Draw a diagram that visually represents the situation.
   (b) Find the air distance the plane must travel until touching down on the near end of the runway.
   (c) Find the ground distance the plane must travel until touching down.
   (d) Find the altitude of the plane when the pilot begins the descent.

52. **LOCATING A FIRE** The bearing from the Pine Knob fire tower to the Colt Station fire tower is N 65° E, and the two towers are 30 kilometers apart. A fire spotted by rangers in each tower has a bearing of N 80° E from Pine Knob and S 70° E from Colt Station (see figure). Find the distance of the fire from each tower.



8.2 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. If you are given three sides of a triangle, you would use the Law of ________ to find the three angles of the triangle.
2. If you are given two angles and any side of a triangle, you would use the Law of ________ to solve the triangle.
3. The standard form of the Law of Cosines for $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$ is ________ .
4. The Law of Cosines can be used to establish a formula for finding the area of a triangle called ________ ________ Formula.

SKILLS AND APPLICATIONS

In Exercises 5–20, use the Law of Cosines to solve the triangle. Round your answers to two decimal places.

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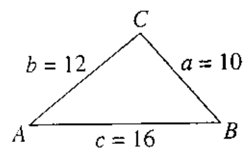

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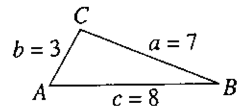

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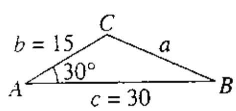

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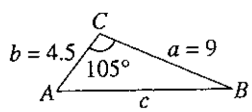
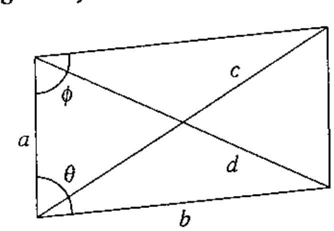


9. $a = 11, \quad b = 15, \quad c = 21$
10. $a = 55, \quad b = 25, \quad c = 72$
11. $a = 75.4, \quad b = 52, \quad c = 52$
12. $a = 1.42, \quad b = 0.75, \quad c = 1.25$
13. $A = 120°, \quad b = 6, \quad c = 7$
14. $A = 48°, \quad b = 3, \quad c = 14$
15. $B = 10° 35', \quad a = 40, \quad c = 30$
16. $B = 75° 20', \quad a = 6.2, \quad c = 9.5$
17. $B = 125° 40', \quad a = 37, \quad c = 37$
18. $C = 15° 15', \quad a = 7.45, \quad b = 2.15$
19. $C = 43°, \quad a = \frac{4}{9}, \quad b = \frac{7}{9}$
20. $C = 101°, \quad a = \frac{3}{8}, \quad b = \frac{3}{4}$

In Exercises 21–26, complete the table by solving the parallelogram shown in the figure. (The lengths of the diagonals are given by c and d.)



	a	b	c	d	θ	ϕ
21.	5	8			45°	
22.	25	35				120°
23.	10	14	20			
24.	40	60		80		
25.	15		25	20		
26.		25	50	35		

In Exercises 27–32, determine whether the Law of Sines or the Law of Cosines is needed to solve the triangle. Then solve the triangle.

27. $a = 8, \quad c = 5, \quad B = 40°$
28. $a = 10, \quad b = 12, \quad C = 70°$
29. $A = 24°, \quad a = 4, \quad b = 18$
30. $a = 11, \quad b = 13, \quad c = 7$
31. $A = 42°, \quad B = 35°, \quad c = 1.2$
32. $a = 160, \quad B = 12°, \quad C = 7°$

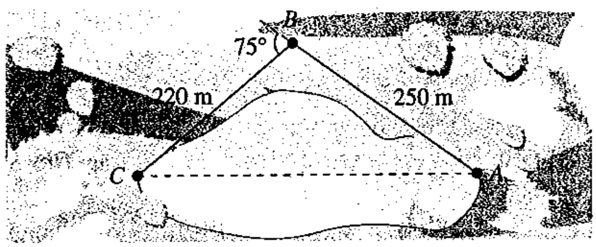
In Exercises 33–40, use Heron's Area Formula to find the area of the triangle.

33. $a = 8, \quad b = 12, \quad c = 17$
34. $a = 33, \quad b = 36, \quad c = 25$
35. $a = 2.5, \quad b = 10.2, \quad c = 9$
36. $a = 75.4, \quad b = 52, \quad c = 52$
37. $a = 12.32, \quad b = 8.46, \quad c = 15.05$
38. $a = 3.05, \quad b = 0.75, \quad c = 2.45$
39. $a = 1, \quad b = \frac{1}{2}, \quad c = \frac{3}{4}$
40. $a = \frac{3}{5}, \quad b = \frac{5}{8}, \quad c = \frac{3}{8}$

41. **NAVIGATION** A boat race runs along a triangular course marked by buoys A, B, and C. The race starts with the boats headed west for 3700 meters. The other two sides of the course lie to the north of the first side, and their lengths are 1700 meters and 3000 meters. Draw a figure that gives a visual representation of the situation, and find the bearings for the last two legs of the race.

42. **NAVIGATION** A plane flies 810 miles from Franklin to Centerville with a bearing of 75°. Then it flies 648 miles from Centerville to Rosemount with a bearing of 32°. Draw a figure that visually represents the situation, and find the straight-line distance and bearing from Franklin to Rosemount.

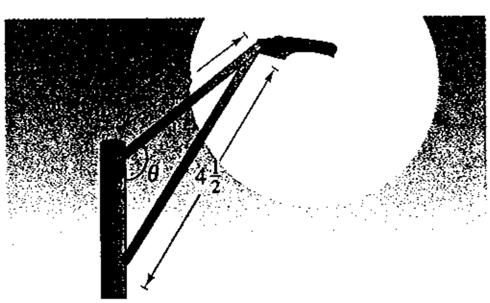
43. **SURVEYING** To approximate the length of a marsh, a surveyor walks 250 meters from point A to point B, then turns 75° and walks 220 meters to point C (see figure). Approximate the length AC of the marsh.



44. **SURVEYING** A triangular parcel of land has 115 meters of frontage, and the other boundaries have lengths of 76 meters and 92 meters. What angles does the frontage make with the two other boundaries?

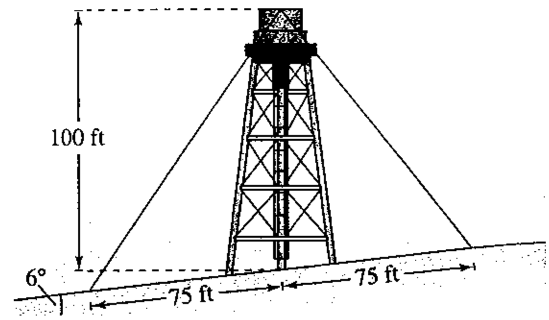
45. **SURVEYING** A triangular parcel of ground has sides of lengths 725 feet, 650 feet, and 575 feet. Find the measure of the largest angle.

46. **STREETLIGHT DESIGN** Determine the angle θ in the design of the streetlight shown in the figure.

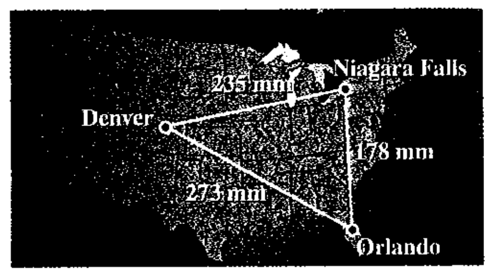


47. **DISTANCE** Two ships leave a port at 9 A.M. One travels at a bearing of N 53° W at 12 miles per hour, and the other travels at a bearing of S 67° W at 16 miles per hour. Approximate how far apart they are at noon that day.

48. **LENGTH** A 100-foot vertical tower is to be erected on the side of a hill that makes a 6° angle with the horizontal (see figure). Find the length of each of the two guy wires that will be anchored 75 feet uphill and downhill from the base of the tower.



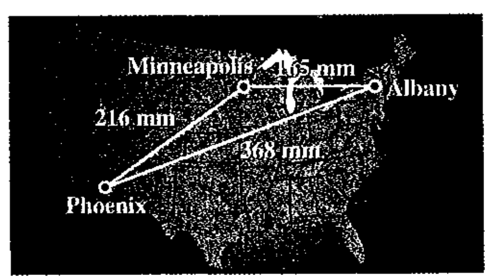
49. **NAVIGATION** On a map, Orlando is 178 millimeters due south of Niagara Falls, Denver is 273 millimeters from Orlando, and Denver is 235 millimeters from Niagara Falls (see figure).



(a) Find the bearing of Denver from Orlando.

(b) Find the bearing of Denver from Niagara Falls.

50. **NAVIGATION** On a map, Minneapolis is 165 millimeters due west of Albany, Phoenix is 216 millimeters from Minneapolis, and Phoenix is 368 millimeters from Albany (see figure).



(a) Find the bearing of Minneapolis from Phoenix.

(b) Find the bearing of Albany from Phoenix.

51. **BASEBALL** On a baseball diamond with 90-foot sides, the pitcher's mound is 60.5 feet from home plate. How far is it from the pitcher's mound to third base?

8.5 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

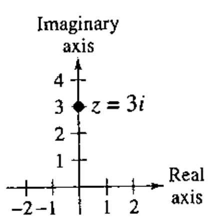
1. The _______ _______ of a complex number $a + bi$ is the distance between the origin $(0, 0)$ and the point (a, b).
2. The _______ _______ of a complex number $z = a + bi$ is given by $z = r(\cos\theta + i\sin\theta)$, where r is the _______ of z and θ is the _______ of z.
3. _______ Theorem states that if $z = r(\cos\theta + i\sin\theta)$ is a complex number and n is a positive integer, then $z^n = r^n(\cos n\theta + i\sin n\theta)$.
4. The complex number $u = a + bi$ is an _______ _______ of the complex number z if $z = u^n = (a + bi)^n$.

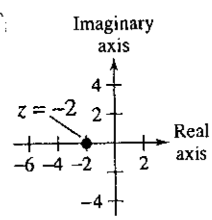
SKILLS AND APPLICATIONS

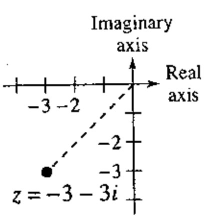
In Exercises 5–10, plot the complex number and find its absolute value.

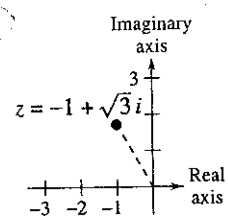
5. $-6 + 8i$
6. $5 - 12i$
7. $-7i$
8. -7
9. $4 - 6i$
10. $-8 + 3i$

In Exercises 11–14, write the complex number in trigonometric form.

11. 

12. 

13. 

14. 

In Exercises 15–32, represent the complex number graphically, and find the trigonometric form of the number.

15. $1 + i$
16. $5 - 5i$
17. $1 - \sqrt{3}i$
18. $4 - 4\sqrt{3}i$
19. $-2(1 + \sqrt{3}i)$
20. $\frac{5}{2}(\sqrt{3} - i)$
21. $-5i$
22. $12i$
23. $-7 + 4i$
24. $3 - i$
25. 2
26. 4
27. $2\sqrt{2} - i$
28. $-3 - i$
29. $5 + 2i$
30. $8 + 3i$
31. $-8 - 5\sqrt{3}i$
32. $-9 - 2\sqrt{10}i$

In Exercises 33–42, find the standard form of the complex number. Then represent the complex number graphically.

33. $2(\cos 60° + i\sin 60°)$
34. $5(\cos 135° + i\sin 135°)$
35. $\sqrt{48}[\cos(-30°) + i\sin(-30°)]$
36. $\sqrt{8}(\cos 225° + i\sin 225°)$
37. $\frac{9}{4}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$
38. $6\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$
39. $7(\cos 0 + i\sin 0)$
40. $8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$
41. $5[\cos(198° 45') + i\sin(198° 45')]$
42. $9.75[\cos(280° 30') + i\sin(280° 30')]$

In Exercises 43–46, use a graphing utility to represent the complex number in standard form.

43. $5\left(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9}\right)$
44. $10\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right)$
45. $2(\cos 155° + i\sin 155°)$
46. $9(\cos 58° + i\sin 58°)$

In Exercises 47–58, perform the operation and leave the result in trigonometric form.

47. $\left[2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]\left[6\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)\right]$
48. $\left[\frac{3}{4}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]\left[4\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right]$
49. $\left[\frac{5}{3}(\cos 120° + i\sin 120°)\right]\left[\frac{2}{3}(\cos 30° + i\sin 30°)\right]$
50. $\left[\frac{1}{2}(\cos 100° + i\sin 100°)\right]\left[\frac{4}{5}(\cos 300° + i\sin 300°)\right]$
51. $(\cos 80° + i\sin 80°)(\cos 330° + i\sin 330°)$
52. $(\cos 5° + i\sin 5°)(\cos 20° + i\sin 20°)$
53. $\dfrac{3(\cos 50° + i\sin 50°)}{9(\cos 20° + i\sin 20°)}$
54. $\dfrac{\cos 120° + i\sin 120°}{2(\cos 40° + i\sin 40°)}$
55. $\dfrac{\cos\pi + i\sin\pi}{\cos(\pi/3) + i\sin(\pi/3)}$
56. $\dfrac{5(\cos 4.3 + i\sin 4.3)}{4(\cos 2.1 + i\sin 2.1)}$
57. $\dfrac{12(\cos 92° + i\sin 92°)}{2(\cos 122° + i\sin 122°)}$
58. $\dfrac{6(\cos 40° + i\sin 40°)}{7(\cos 100° + i\sin 100°)}$

In Exercises 59–64, (a) write the trigonometric forms of the complex numbers, (b) perform the indicated operation using the trigonometric forms, and (c) perform the indicated operation using the standard forms, and check your result with that of part (b).

59. $(2 - 2i)(1 - i)$
60. $(\sqrt{3} + i)(1 + i)$
61. $-2i(1 + i)$
62. $3i(1 - \sqrt{2}i)$
63. $\dfrac{3 + 4i}{1 - \sqrt{3}i}$
64. $\dfrac{1 + \sqrt{3}i}{6 - 3i}$

In Exercises 65 and 66, represent the powers z, z^2, z^3, and z^4 graphically. Describe the pattern.

65. $z = \dfrac{\sqrt{2}}{2}(1 + i)$
66. $z = \dfrac{1}{2}(1 + \sqrt{3}i)$

In Exercises 67–82, use DeMoivre's Theorem to find the indicated power of the complex number. Write the result in standard form.

67. $(1 + i)^5$
68. $(2 + 2i)^6$
69. $(-1 + i)^6$
70. $(3 - 2i)^8$
71. $2(\sqrt{3} + i)^{10}$
72. $4(1 - \sqrt{3}i)^3$
73. $[5(\cos 20° + i \sin 20°)]^3$
74. $[3(\cos 60° + i \sin 60°)]^4$
75. $\left(\cos \dfrac{\pi}{4} + i \sin \dfrac{\pi}{4}\right)^{12}$
76. $\left[2\left(\cos \dfrac{\pi}{2} + i \sin \dfrac{\pi}{2}\right)\right]^8$
77. $[5(\cos 3.2 + i \sin 3.2)]^4$
78. $(\cos 0 + i \sin 0)^{20}$
79. $(3 - 2i)^5$
80. $(\sqrt{5} - 4i)^3$
81. $[3(\cos 15° + i \sin 15°)]^4$
82. $\left[2\left(\cos \dfrac{\pi}{8} + i \sin \dfrac{\pi}{8}\right)\right]^6$

In Exercises 83–98, (a) use the formula on page 634 to find the indicated roots of the complex number, (b) represent each of the roots graphically, and (c) write each of the roots in standard form.

83. Square roots of $5(\cos 120° + i \sin 120°)$
84. Square roots of $16(\cos 60° + i \sin 60°)$
85. Cube roots of $8\left(\cos \dfrac{2\pi}{3} + i \sin \dfrac{2\pi}{3}\right)$
86. Fifth roots of $32\left(\cos \dfrac{5\pi}{6} + i \sin \dfrac{5\pi}{6}\right)$
87. Cube roots of $-\frac{125}{2}(1 + \sqrt{3}i)$
88. Cube roots of $-4\sqrt{2}(-1 + i)$
89. Square roots of $-25i$
90. Fourth roots of $625i$
91. Fourth roots of 16
92. Fourth roots of i
93. Fifth roots of 1
94. Cube roots of 1000
95. Cube roots of -125
96. Fourth roots of -4
97. Fifth roots of $4(1 - i)$
98. Sixth roots of $64i$

In Exercises 99–106, use the formula on page 634 to find all the solutions of the equation and represent the solutions graphically.

99. $x^4 + i = 0$
100. $x^3 + 1 = 0$
101. $x^5 + 243 = 0$
102. $x^3 - 27 = 0$
103. $x^4 + 16i = 0$
104. $x^6 + 64i = 0$
105. $x^3 - (1 - i) = 0$
106. $x^4 + (1 + i) = 0$

EXPLORATION

TRUE OR FALSE? In Exercises 107 and 108, determine whether the statement is true or false. Justify your answer.

107. Geometrically, the nth roots of any complex number z are all equally spaced around the unit circle centered at the origin.

108. The product of two complex numbers is zero only when the modulus of one (or both) of the complex numbers is zero.

109. Given two complex numbers $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$, $z_2 \neq 0$, show that

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)].$$

110. Show that $\bar{z} = r[\cos(-\theta) + i \sin(-\theta)]$ is the complex conjugate of $z = r(\cos \theta + i \sin \theta)$.

111. Use the trigonometric forms of z and $\bar{z}$ in Exercise 110 to find (a) $z\bar{z}$ and (b) $z/\bar{z}$, $\bar{z} \neq 0$.

112. Show that the negative of $z = r(\cos \theta + i \sin \theta)$ is $-z = r[\cos(\theta + \pi) + i \sin(\theta + \pi)]$.

113. Show that $\frac{1}{2}(1 - \sqrt{3}i)$ is a ninth root of -1.

114. Show that $2^{-1/4}(1 - i)$ is a fourth root of -2.

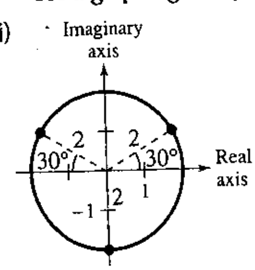
115. **THINK ABOUT IT** Explain how you can use DeMoivre's Theorem to solve the polynomial equation $x^4 + 16 = 0$. [*Hint:* Write -16 as $16(\cos \pi + i \sin \pi)$.]

116. **CAPSTONE** Use the graph of the roots of a complex number.

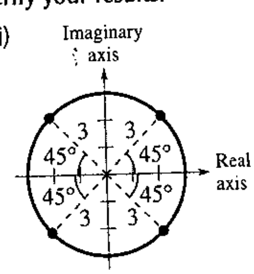
(a) Write each of the roots in trigonometric form.

(b) Identify the complex number whose roots are given. Use a graphing utility to verify your results.

(i)



(ii)



9.1 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.
Exercises containing systems with no solutions: 29, 30, 33, 45, 46, 57, 58

VOCABULARY: Fill in the blanks.

1. A set of two or more equations in two or more variables is called a ________ of ________.
2. A ________ of a system of equations is an ordered pair that satisfies each equation in the system.
3. Finding the set of all solutions to a system of equations is called ________ the system of equations.
4. The first step in solving a system of equations by the method of ________ is to solve one of the equations for one variable in terms of the other variable.
5. Graphically, the solution of a system of two equations is the ________ of ________ of the graphs of the two equations.
6. In business applications, the point at which the revenue equals costs is called the ________ point.

SKILLS AND APPLICATIONS

In Exercises 7–10, determine whether each ordered pair is a solution of the system of equations.

7. $\begin{cases} 2x - y = 4 \\ 8x + y = -9 \end{cases}$ (a) $(0, -4)$ (b) $(-2, 7)$ (c) $\left(\frac{3}{2}, -1\right)$ (d) $\left(-\frac{1}{2}, -5\right)$

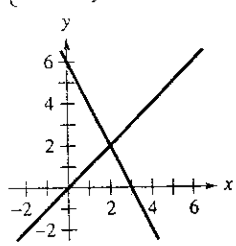
8. $\begin{cases} 4x^2 + y = 3 \\ -x - y = 11 \end{cases}$ (a) $(2, -13)$ (b) $(2, -9)$ (c) $\left(-\frac{3}{2}, -\frac{31}{3}\right)$ (d) $\left(-\frac{7}{4}, -\frac{37}{4}\right)$

9. $\begin{cases} y = -4e^x \\ 7x - y = 4 \end{cases}$ (a) $(-4, 0)$ (b) $(0, -4)$ (c) $(0, -2)$ (d) $(-1, -3)$

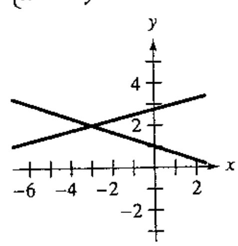
10. $\begin{cases} -\log x + 3 = y \\ \frac{1}{9}x + y = \frac{28}{9} \end{cases}$ (a) $\left(9, \frac{37}{9}\right)$ (b) $(10, 2)$ (c) $(1, 3)$ (d) $(2, 4)$

In Exercises 11–20, solve the system by the method of substitution. Check your solution(s) graphically.

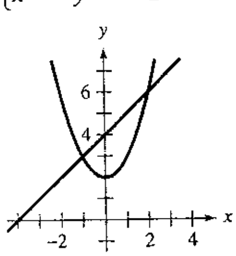
11. $\begin{cases} 2x + y = 6 \\ -x + y = 0 \end{cases}$



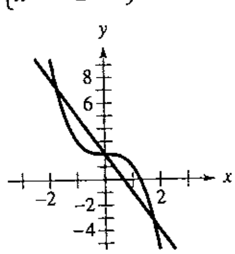
12. $\begin{cases} x - 4y = -11 \\ x + 3y = 3 \end{cases}$



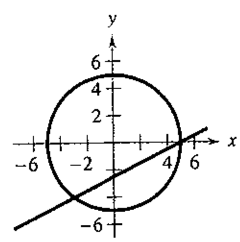
13. $\begin{cases} x - y = -4 \\ x^2 - y = -2 \end{cases}$



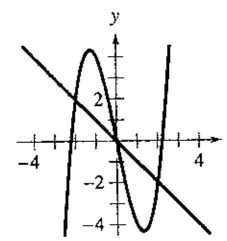
14. $\begin{cases} 3x + y = 2 \\ x^3 - 2 + y = 0 \end{cases}$



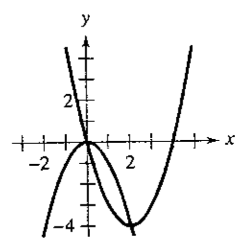
15. $\begin{cases} -\frac{1}{2}x + y = -\frac{5}{2} \\ x^2 + y^2 = 25 \end{cases}$



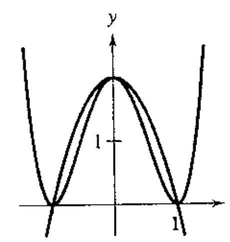
16. $\begin{cases} x + y = 0 \\ x^3 - 5x - y = 0 \end{cases}$



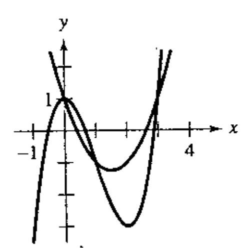
17. $\begin{cases} x^2 + y = 0 \\ x^2 - 4x - y = 0 \end{cases}$



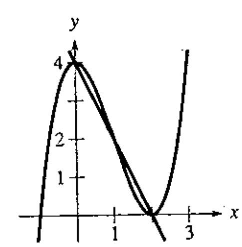
18. $\begin{cases} y = -2x^2 + 2 \\ y = 2(x^4 - 2x^2 + 1) \end{cases}$



19. $\begin{cases} y = x^3 - 3x^2 + 1 \\ y = x^2 - 3x + 1 \end{cases}$



20. $\begin{cases} y = x^3 - 3x^2 + 4 \\ y = -2x + 4 \end{cases}$



In Exercises 21–34, solve the system by the method of substitution.

21. $\begin{cases} x - y = 2 \\ 6x - 5y = 16 \end{cases}$

22. $\begin{cases} x + 4y = 3 \\ 2x - 7y = -24 \end{cases}$

23. $\begin{cases} 2x - y + 2 = 0 \\ 4x + y - 5 = 0 \end{cases}$

24. $\begin{cases} 6x - 3y - 4 = 0 \\ x + 2y - 4 = 0 \end{cases}$

25. $\begin{cases} 1.5x + 0.8y = 2.3 \\ 0.3x - 0.2y = 0.1 \end{cases}$

26. $\begin{cases} 0.5x + 3.2y = 9.0 \\ 0.2x - 1.6y = -3.6 \end{cases}$

27. $\begin{cases} \frac{1}{5}x + \frac{1}{2}y = 8 \\ x + y = 20 \end{cases}$

28. $\begin{cases} \frac{1}{2}x + \frac{3}{4}y = 10 \\ \frac{3}{4}x - y = 4 \end{cases}$

29. $\begin{cases} 6x + 5y = -3 \\ -x - \frac{5}{6}y = -7 \end{cases}$

30. $\begin{cases} -\frac{2}{3}x + y = 2 \\ 2x - 3y = 6 \end{cases}$

31. $\begin{cases} x^2 - y = 0 \\ 2x + y = 0 \end{cases}$

32. $\begin{cases} x - 2y = 0 \\ 3x - y^2 = 0 \end{cases}$

33. $\begin{cases} x - y = -1 \\ x^2 - y = -4 \end{cases}$

34. $\begin{cases} y = -x \\ y = x^3 + 3x^2 + 2x \end{cases}$

In Exercises 35–48, solve the system graphically.

35. $\begin{cases} -x + 2y = -2 \\ 3x + y = 20 \end{cases}$

36. $\begin{cases} x + y = 0 \\ 3x - 2y = 5 \end{cases}$

37. $\begin{cases} x - 3y = -3 \\ 5x + 3y = -6 \end{cases}$

38. $\begin{cases} -x + 2y = -7 \\ x - y = 2 \end{cases}$

39. $\begin{cases} x + y = 4 \\ x^2 + y^2 - 4x = 0 \end{cases}$

40. $\begin{cases} -x + y = 3 \\ x^2 - 6x - 27 + y^2 = 0 \end{cases}$

41. $\begin{cases} x - y + 3 = 0 \\ x^2 - 4x + 7 = y \end{cases}$

42. $\begin{cases} y^2 - 4x + 11 = 0 \\ -\frac{1}{2}x + y = -\frac{1}{2} \end{cases}$

43. $\begin{cases} 7x + 8y = 24 \\ x - 8y = 8 \end{cases}$

44. $\begin{cases} x - y = 0 \\ 5x - 2y = 6 \end{cases}$

45. $\begin{cases} 3x - 2y = 0 \\ x^2 - y^2 = 4 \end{cases}$

46. $\begin{cases} 2x - y + 3 = 0 \\ x^2 + y^2 - 4x = 0 \end{cases}$

47. $\begin{cases} x^2 + y^2 = 25 \\ 3x^2 - 16y = 0 \end{cases}$

48. $\begin{cases} x^2 + y^2 = 25 \\ (x - 8)^2 + y^2 = 41 \end{cases}$

In Exercises 49–54, use a graphing utility to solve the system of equations. Find the solution(s) accurate to two decimal places.

49. $\begin{cases} y = e^x \\ x - y + 1 = 0 \end{cases}$

50. $\begin{cases} y = -4e^{-x} \\ y + 3x + 8 = 0 \end{cases}$

51. $\begin{cases} x + 2y = 8 \\ y = \log_2 x \end{cases}$

52. $\begin{cases} y + 2 = \ln(x - 1) \\ 3y + 2x = 9 \end{cases}$

53. $\begin{cases} x^2 + y^2 = 169 \\ x^2 - 8y = 104 \end{cases}$

54. $\begin{cases} x^2 + y^2 = 4 \\ 2x^2 - y = 2 \end{cases}$

In Exercises 55–64, solve the system graphically or algebraically. Explain your choice of method.

55. $\begin{cases} y = 2x \\ y = x^2 + 1 \end{cases}$

56. $\begin{cases} x^2 + y^2 = 25 \\ 2x + y = 10 \end{cases}$

57. $\begin{cases} x - 2y = 4 \\ x^2 - y = 0 \end{cases}$

58. $\begin{cases} y = (x + 1)^3 \\ y = \sqrt{x - 1} \end{cases}$

59. $\begin{cases} y - e^{-x} = 1 \\ y - \ln x = 3 \end{cases}$

60. $\begin{cases} x^2 + y = 4 \\ e^x - y = 0 \end{cases}$

61. $\begin{cases} y = x^4 - 2x^2 + 1 \\ y = 1 - x^2 \end{cases}$

62. $\begin{cases} y = x^3 - 2x^2 + x - 1 \\ y = -x^2 + 3x - 1 \end{cases}$

63. $\begin{cases} xy - 1 = 0 \\ 2x - 4y + 7 = 0 \end{cases}$

64. $\begin{cases} x - 2y = 1 \\ y = \sqrt{x - 1} \end{cases}$

BREAK-EVEN ANALYSIS In Exercises 65 and 66, find the sales necessary to break even ($R = C$) for the cost C of producing x units and the revenue R obtained by selling x units. (Round to the nearest whole unit.)

65. $C = 8650x + 250{,}000$, $R = 9950x$

66. $C = 5.5\sqrt{x} + 10{,}000$, $R = 3.29x$

67. **BREAK-EVEN ANALYSIS** A small software company invests \$25,000 to produce a software package that will sell for \$69.95. Each unit can be produced for \$45.25.

(a) How many units must be sold to break even?

(b) How many units must be sold to make a profit of \$100,000?

68. **BREAK-EVEN ANALYSIS** A small fast-food restaurant invests \$10,000 to produce a new food item that will sell for \$3.99. Each item can be produced for \$1.90.

(a) How many items must be sold to break even?

(b) How many items must be sold to make a profit of \$12,000?

69. **DVD RENTALS** The weekly rentals for a newly released DVD of an animated film at a local video store decreased each week. At the same time, the weekly rentals for a newly released DVD of a horror film increased each week. Models that approximate the weekly rentals R for each DVD are

$$\begin{cases} R = 360 - 24x & \text{Animated film} \\ R = 24 + 18x & \text{Horror film} \end{cases}$$

where x represents the number of weeks each DVD was in the store, with $x = 1$ corresponding to the first week.

(a) After how many weeks will the rentals for the two movies be equal?

(b) Use a table to solve the system of equations numerically. Compare your result with that of part (a).

70. **SALES** The total weekly sales for a newly released portable media player (PMP) increased each week. At the same time, the total weekly sales for another newly released PMP decreased each week. Models that approximate the total weekly sales S (in thousands of units) for each PMP are

$$\begin{cases} S = 15x + 50 & \text{PMP 1} \\ S = -20x + 190 & \text{PMP 2} \end{cases}$$

where x represents the number of weeks each PMP was in stores, with $x = 0$ corresponding to the PMP sales on the day each PMP was first released in stores.

9.2 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.
Exercises containing systems with no solutions: 9, 10, 21, 34
Exercises containing systems with infinitely many solutions: 11, 12, 22, 23, 24, 33

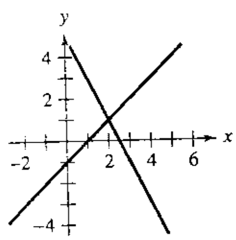
VOCABULARY: Fill in the blanks.

1. The first step in solving a system of equations by the method of ________ is to obtain coefficients for x (or y) that differ only in sign.
2. Two systems of equations that have the same solution set are called ________ systems.
3. A system of linear equations that has at least one solution is called ________, whereas a system of linear equations that has no solution is called ________.
4. In business applications, the ________ ________ is defined as the price p and the number of units x that satisfy both the demand and supply equations.

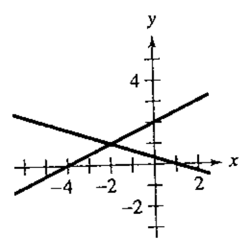
SKILLS AND APPLICATIONS

In Exercises 5–12, solve the system by the method of elimination. Label each line with its equation. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

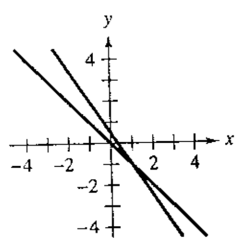
5. $\begin{cases} 2x + y = 5 \\ x - y = 1 \end{cases}$



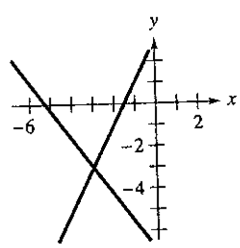
6. $\begin{cases} x + 3y = 1 \\ -x + 2y = 4 \end{cases}$



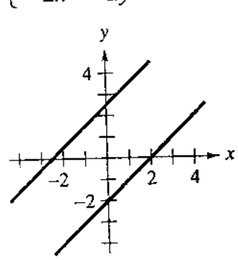
7. $\begin{cases} x + y = 0 \\ 3x + 2y = 1 \end{cases}$



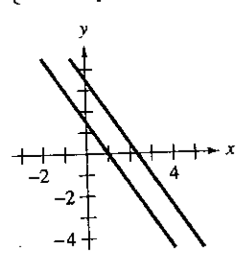
8. $\begin{cases} 2x - y = -3 \\ 4x + 3y = -21 \end{cases}$



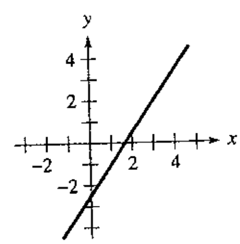
9. $\begin{cases} x - y = 2 \\ -2x + 2y = 5 \end{cases}$



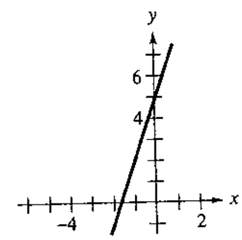
10. $\begin{cases} 3x + 2y = 3 \\ 6x + 4y = 14 \end{cases}$



11. $\begin{cases} 3x - 2y = 5 \\ -6x + 4y = -10 \end{cases}$



12. $\begin{cases} 9x - 3y = -15 \\ -3x + y = 5 \end{cases}$



In Exercises 13–30, solve the system by the method of elimination and check any solutions algebraically.

13. $\begin{cases} x + 2y = 6 \\ x - 2y = 2 \end{cases}$

14. $\begin{cases} 3x - 5y = 8 \\ 2x + 5y = 22 \end{cases}$

15. $\begin{cases} 5x + 3y = 6 \\ 3x - y = 5 \end{cases}$

16. $\begin{cases} x + 5y = 10 \\ 3x - 10y = -5 \end{cases}$

17. $\begin{cases} 3x + 2y = 10 \\ 2x + 5y = 3 \end{cases}$

18. $\begin{cases} 2r + 4s = 5 \\ 16r + 50s = 55 \end{cases}$

19. $\begin{cases} 5u + 6v = 24 \\ 3u + 5v = 18 \end{cases}$

20. $\begin{cases} 3x + 11y = 4 \\ -2x - 5y = 9 \end{cases}$

21. $\begin{cases} \frac{9}{5}x + \frac{6}{5}y = 4 \\ 9x + 6y = 3 \end{cases}$

22. $\begin{cases} \frac{3}{4}x + y = \frac{1}{8} \\ \frac{9}{4}x + 3y = \frac{3}{8} \end{cases}$

23. $\begin{cases} -5x + 6y = -3 \\ 20x - 24y = 12 \end{cases}$

24. $\begin{cases} 7x + 8y = 6 \\ -14x - 16y = -12 \end{cases}$

25. $\begin{cases} 0.2x - 0.5y = -27.8 \\ 0.3x + 0.4y = 68.7 \end{cases}$

26. $\begin{cases} 0.05x - 0.03y = 0.21 \\ 0.07x + 0.02y = 0.16 \end{cases}$

27. $\begin{cases} 4b + 3m = 3 \\ 3b + 11m = 13 \end{cases}$

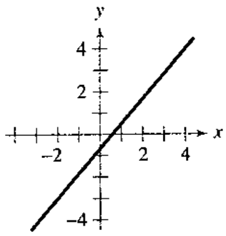
28. $\begin{cases} 2x + 5y = 8 \\ 5x + 8y = 10 \end{cases}$

29. $\begin{cases} \dfrac{x+3}{4} + \dfrac{y-1}{3} = 1 \\ 2x - y = 12 \end{cases}$

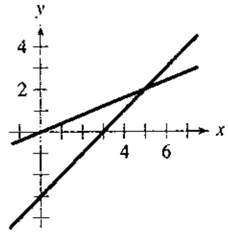
30. $\begin{cases} \dfrac{x-1}{2} + \dfrac{y+2}{3} = 4 \\ x - 2y = 5 \end{cases}$

In Exercises 31–34, match the system of linear equations with its graph. Describe the number of solutions and state whether the system is consistent or inconsistent. [The graphs are labeled (a), (b), (c) and (d).]

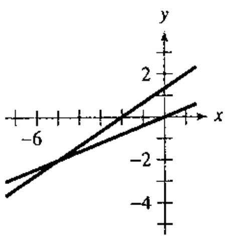
(a)



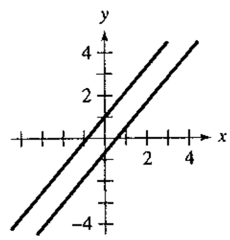
(b)



(c)



(d)



31. $\begin{cases} 2x - 5y = 0 \\ x - y = 3 \end{cases}$

32. $\begin{cases} 2x - 5y = 0 \\ 2x - 3y = -4 \end{cases}$

33. $\begin{cases} -7x + 6y = -4 \\ 14x - 12y = 8 \end{cases}$

34. $\begin{cases} 7x - 6y = -6 \\ -7x + 6y = -4 \end{cases}$

In Exercises 35–42, use any method to solve the system.

35. $\begin{cases} 3x - 5y = 7 \\ 2x + y = 9 \end{cases}$

36. $\begin{cases} -x + 3y = 17 \\ 4x + 3y = 7 \end{cases}$

37. $\begin{cases} y = 2x - 5 \\ y = 5x - 11 \end{cases}$

38. $\begin{cases} 7x + 3y = 16 \\ y = x + 2 \end{cases}$

39. $\begin{cases} x - 5y = 21 \\ 6x + 5y = 21 \end{cases}$

40. $\begin{cases} y = -2x - 17 \\ y = 2 - 3x \end{cases}$

41. $\begin{cases} -5x + 9y = 13 \\ y = x - 4 \end{cases}$

42. $\begin{cases} 4x - 3y = 6 \\ -5x + 7y = -1 \end{cases}$

43. AIRPLANE SPEED An airplane flying into a headwind travels the 1800-mile flying distance between Pittsburgh, Pennsylvania and Phoenix, Arizona in 3 hours and 36 minutes. On the return flight, the distance is traveled in 3 hours. Find the airspeed of the plane and the speed of the wind, assuming that both remain constant.

44. AIRPLANE SPEED Two planes start from Los Angeles International Airport and fly in opposite directions. The second plane starts $\frac{1}{2}$ hour after the first plane, but its speed is 80 kilometers per hour faster. Find the airspeed of each plane if 2 hours after the first plane departs the planes are 3200 kilometers apart.

SUPPLY AND DEMAND In Exercises 45–48, find the equilibrium point of the demand and supply equations. The equilibrium point is the price p and number of units x that satisfy both the demand and supply equations.

	Demand	*Supply*
45.	$p = 500 - 0.4x$	$p = 380 + 0.1x$
46.	$p = 100 - 0.05x$	$p = 25 + 0.1x$
47.	$p = 140 - 0.00002x$	$p = 80 + 0.00001x$
48.	$p = 400 - 0.0002x$	$p = 225 + 0.0005x$

49. NUTRITION Two cheeseburgers and one small order of French fries from a fast-food restaurant contain a total of 830 calories. Three cheeseburgers and two small orders of French fries contain a total of 1360 calories. Find the caloric content of each item.

50. NUTRITION One eight-ounce glass of apple juice and one eight-ounce glass of orange juice contain a total of 177.4 milligrams of vitamin C. Two eight-ounce glasses of apple juice and three eight-ounce glasses of orange juice contain a total of 436.7 milligrams of vitamin C. How much vitamin C is in an eight-ounce glass of each type of juice?

51. ACID MIXTURE Thirty liters of a 40% acid solution is obtained by mixing a 25% solution with a 50% solution.

(a) Write a system of equations in which one equation represents the amount of final mixture required and the other represents the percent of acid in the final mixture. Let x and y represent the amounts of the 25% and 50% solutions, respectively.

(b) Use a graphing utility to graph the two equations in part (a) in the same viewing window. As the amount of the 25% solution increases, how does the amount of the 50% solution change?

(c) How much of each solution is required to obtain the specified concentration of the final mixture?

52. FUEL MIXTURE Five hundred gallons of 89-octane gasoline is obtained by mixing 87-octane gasoline with 92-octane gasoline.

(a) Write a system of equations in which one equation represents the amount of final mixture required and the other represents the amounts of 87- and 92-octane gasolines in the final mixture. Let x and y represent the numbers of gallons of 87-octane and 92-octane gasolines, respectively.

(b) Use a graphing utility to graph the two equations in part (a) in the same viewing window. As the amount of 87-octane gasoline increases, how does the amount of 92-octane gasoline change?

(c) How much of each type of gasoline is required to obtain the 500 gallons of 89-octane gasoline?

53. **INVESTMENT PORTFOLIO** A total of \$24,000 is invested in two corporate bonds that pay 3.5% and 5% simple interest. The investor wants an annual interest income of \$930 from the investments. What amount should be invested in the 3.5% bond?

54. **INVESTMENT PORTFOLIO** A total of \$32,000 is invested in two municipal bonds that pay 5.75% and 6.25% simple interest. The investor wants an annual interest income of \$1900 from the investments. What amount should be invested in the 5.75% bond?

55. **PRESCRIPTIONS** The numbers of prescriptions P (in thousands) filled at two pharmacies from 2006 through 2010 are shown in the table.

Year	Pharmacy A	Pharmacy B
2006	19.2	20.4
2007	19.6	20.8
2008	20.0	21.1
2009	20.6	21.5
2010	21.3	22.0

(a) Use a graphing utility to create a scatter plot of the data for pharmacy A and use the *regression* feature to find a linear model. Let t represent the year, with $t = 6$ corresponding to 2006. Repeat the procedure for pharmacy B.

(b) Assuming the numbers for the given five years are representative of future years, will the number of prescriptions filled at pharmacy A ever exceed the number of prescriptions filled at pharmacy B? If so, when?

56. **DATA ANALYSIS** A store manager wants to know the demand for a product as a function of the price. The daily sales for different prices of the product are shown in the table.

Price, x	Demand, y
\$1.00	45
\$1.20	37
\$1.50	23

(a) Find the least squares regression line $y = ax + b$ for the data by solving the system for a and b.

$$\begin{cases} 3.00b + 3.70a = 105.00 \\ 3.70b + 4.69a = 123.90 \end{cases}$$

(b) Use the regression feature of a graphing utility to confirm the result in part (a).

(c) Use the graphing utility to plot the data and graph the linear model from part (a) in the same viewing window.

(d) Use the linear model from part (a) to predict the demand when the price is \$1.75.

FITTING A LINE TO DATA In Exercises 57–60, find the least squares regression line $y = ax + b$ for the points

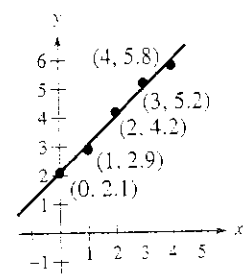
$(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$

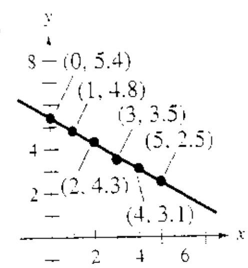
by solving the system for a and b.

$$nb + \left(\sum_{i=1}^{n} x_i\right)a = \left(\sum_{i=1}^{n} y_i\right)$$

$$\left(\sum_{i=1}^{n} x_i\right)b + \left(\sum_{i=1}^{n} x_i^2\right)a = \left(\sum_{i=1}^{n} x_i y_i\right)$$

Then use a graphing utility to confirm the result. (If you are unfamiliar with summation notation, look at the discussion in Section 11.1 or in Appendix B at the website for this text at *academic.cengage.com*.)

57.


58.


59. (0, 8), (1, 6), (2, 4), (3, 2)

60. (1, 0.0), (2, 1.1), (3, 2.3), (4, 3.8), (5, 4.0), (6, 5.5), (7, 6.7), (8, 6.9)

61. **DATA ANALYSIS** An agricultural scientist used four test plots to determine the relationship between wheat yield y (in bushels per acre) and the amount of fertilizer x (in hundreds of pounds per acre). The results are shown in the table.

Fertilizer, x	Yield, y
1.0	32
1.5	41
2.0	48
2.5	53

(a) Use the technique demonstrated in Exercises 57–60 to set up a system of equations for the data and to find the least squares regression line $y = ax + b$.

(b) Use the linear model to predict the yield for a fertilizer application of 160 pounds per acre.

10.1 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.
Exercises containing systems with no solutions: 81, 82; Exercises containing systems with infinitely many solutions: 70, 78, 79, 80, 85, 86, 89, 90

VOCABULARY: Fill in the blanks.

1. A rectangular array of real numbers that can be used to solve a system of linear equations is called a ________.
2. A matrix is ________ if the number of rows equals the number of columns.
3. For a square matrix, the entries $a_{11}, a_{22}, a_{33}, \ldots, a_{nn}$ are the ________ ________ entries.
4. A matrix with only one row is called a ________ matrix, and a matrix with only one column is called a ________ matrix.
5. The matrix derived from a system of linear equations is called the ________ matrix of the system.
6. The matrix derived from the coefficients of a system of linear equations is called the ________ matrix of the system.
7. Two matrices are called ________ if one of the matrices can be obtained from the other by a sequence of elementary row operations.
8. A matrix in row-echelon form is in ________ ________ ________ if every column that has a leading 1 has zeros in every position above and below its leading 1.

SKILLS AND APPLICATIONS

In Exercises 9–14, determine the order of the matrix.

9. $\begin{bmatrix} 7 & 0 \end{bmatrix}$

10. $\begin{bmatrix} 5 & -3 & 8 & 7 \end{bmatrix}$

11. $\begin{bmatrix} 2 \\ 36 \\ 3 \end{bmatrix}$

12. $\begin{bmatrix} -3 & 7 & 15 & 0 \\ 0 & 0 & 3 & 3 \\ 1 & 1 & 6 & 7 \end{bmatrix}$

13. $\begin{bmatrix} 33 & 45 \\ -9 & 20 \end{bmatrix}$

14. $\begin{bmatrix} -7 & 6 & 4 \\ 0 & -5 & 1 \end{bmatrix}$

In Exercises 15–20, write the augmented matrix for the system of linear equations.

15. $\begin{cases} 4x - 3y = -5 \\ -x + 3y = 12 \end{cases}$

16. $\begin{cases} 7x + 4y = 22 \\ 5x - 9y = 15 \end{cases}$

17. $\begin{cases} x + 10y - 2z = 2 \\ 5x - 3y + 4z = 0 \\ 2x + y = 6 \end{cases}$

18. $\begin{cases} -x - 8y + 5z = 8 \\ -7x - 15z = -38 \\ 3x - y + 8z = 20 \end{cases}$

19. $\begin{cases} 7x - 5y + z = 13 \\ 19x - 8z = 10 \end{cases}$

20. $\begin{cases} 9x + 2y - 3z = 20 \\ -25y + 11z = -5 \end{cases}$

In Exercises 21–26, write the system of linear equations represented by the augmented matrix. (Use variables x, y, z, and w, if applicable.)

21. $\left[\begin{array}{cc:c} 1 & 2 & 7 \\ 2 & -3 & 4 \end{array}\right]$

22. $\left[\begin{array}{cc:c} 7 & -5 & 0 \\ 8 & 3 & -2 \end{array}\right]$

23. $\left[\begin{array}{ccc:c} 2 & 0 & 5 & -12 \\ 0 & 1 & -2 & 7 \\ 6 & 3 & 0 & 2 \end{array}\right]$

24. $\left[\begin{array}{ccc:c} 4 & -5 & -1 & 18 \\ -11 & 0 & 6 & 25 \\ 3 & 8 & 0 & -29 \end{array}\right]$

25. $\left[\begin{array}{cccc:c} 9 & 12 & 3 & 0 & 0 \\ -2 & 18 & 5 & 2 & 10 \\ 1 & 7 & -8 & 0 & -4 \\ 3 & 0 & 2 & 0 & -10 \end{array}\right]$

26. $\left[\begin{array}{cccc:c} 6 & 2 & -1 & -5 & -25 \\ -1 & 0 & 7 & 3 & 7 \\ 4 & -1 & -10 & 6 & 23 \\ 0 & 8 & 1 & -11 & -21 \end{array}\right]$

In Exercises 27–34, fill in the blank(s) using elementary row operations to form a row-equivalent matrix.

27. $\begin{bmatrix} 1 & 4 & 3 \\ 2 & 10 & 5 \end{bmatrix}$
$\begin{bmatrix} 1 & 4 & 3 \\ 0 & \quad & -1 \end{bmatrix}$

28. $\begin{bmatrix} 3 & 6 & 8 \\ 4 & -3 & 6 \end{bmatrix}$
$\begin{bmatrix} 1 & \quad & \frac{8}{3} \\ 4 & -3 & 6 \end{bmatrix}$

29. $\begin{bmatrix} 1 & 1 & 1 \\ 5 & -2 & 4 \end{bmatrix}$
$\begin{bmatrix} 1 & 1 & 1 \\ 0 & \quad & -1 \end{bmatrix}$

30. $\begin{bmatrix} -3 & 3 & 12 \\ 18 & -8 & 4 \end{bmatrix}$
$\begin{bmatrix} 1 & -1 & \quad \\ 18 & -8 & 4 \end{bmatrix}$

31. $\begin{bmatrix} 1 & 5 & 4 & -1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -7 \end{bmatrix}$
$\begin{bmatrix} 1 & 0 & \quad & \quad \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -7 \end{bmatrix}$

32. $\begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & -1 & 0 & 7 \\ 0 & 0 & -1 & 3 \end{bmatrix}$
$\begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & 1 & 0 & \quad \\ 0 & 0 & 1 & \quad \end{bmatrix}$

33. $\begin{bmatrix} 1 & 1 & 4 & -1 \\ 3 & 8 & 10 & 3 \\ -2 & 1 & 12 & 6 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 4 & -1 \\ 0 & 5 & & \\ 0 & 3 & & \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 4 & -1 \\ 0 & 1 & -\frac{2}{5} & \frac{6}{5} \\ 0 & 3 & & \end{bmatrix}$

34. $\begin{bmatrix} 2 & 4 & 8 & 3 \\ 1 & -1 & -3 & 2 \\ 2 & 6 & 4 & 9 \end{bmatrix}$

$\begin{bmatrix} 1 & & & \\ 1 & -1 & -3 & 2 \\ 2 & 6 & 4 & 9 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 4 & \frac{3}{2} \\ 0 & & -7 & \frac{1}{2} \\ 0 & 2 & & \end{bmatrix}$

In Exercises 35–38, identify the elementary row operation(s) being performed to obtain the new row-equivalent matrix.

35. *Original Matrix* $\begin{bmatrix} -2 & 5 & 1 \\ 3 & -1 & -8 \end{bmatrix}$ *New Row-Equivalent Matrix* $\begin{bmatrix} 13 & 0 & -39 \\ 3 & -1 & -8 \end{bmatrix}$

36. *Original Matrix* $\begin{bmatrix} 3 & -1 & -4 \\ -4 & 3 & 7 \end{bmatrix}$ *New Row-Equivalent Matrix* $\begin{bmatrix} 3 & -1 & -4 \\ 5 & 0 & -5 \end{bmatrix}$

37. *Original Matrix* $\begin{bmatrix} 0 & -1 & -5 & 5 \\ -1 & 3 & -7 & 6 \\ 4 & -5 & 1 & 3 \end{bmatrix}$ *New Row-Equivalent Matrix* $\begin{bmatrix} -1 & 3 & -7 & 6 \\ 0 & -1 & -5 & 5 \\ 0 & 7 & -27 & 27 \end{bmatrix}$

38. *Original Matrix* $\begin{bmatrix} -1 & -2 & 3 & -2 \\ 2 & -5 & 1 & -7 \\ 5 & 4 & -7 & 6 \end{bmatrix}$ *New Row-Equivalent Matrix* $\begin{bmatrix} -1 & -2 & 3 & -2 \\ 0 & -9 & 7 & -11 \\ 0 & -6 & 8 & -4 \end{bmatrix}$

39. Perform the sequence of row operations on the matrix. What did the operations accomplish?

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & -4 \\ 3 & 1 & -1 \end{bmatrix}$$

(a) Add -2 times R_1 to R_2.
(b) Add -3 times R_1 to R_3.
(c) Add -1 times R_2 to R_3.
(d) Multiply R_2 by $-\frac{1}{5}$.
(e) Add -2 times R_2 to R_1.

40. Perform the sequence of row operations on the matrix. What did the operations accomplish?

$$\begin{bmatrix} 7 & 1 \\ 0 & 2 \\ -3 & 4 \\ 4 & 1 \end{bmatrix}$$

(a) Add R_3 to R_4.
(b) Interchange R_1 and R_4.
(c) Add 3 times R_1 to R_3.
(d) Add -7 times R_1 to R_4.
(e) Multiply R_2 by $\frac{1}{2}$.
(f) Add the appropriate multiples of R_2 to R_1, R_3, and R

In Exercises 41–44, determine whether the matrix is i row-echelon form. If it is, determine if it is also in reduce row-echelon form.

41. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **42.** $\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

43. $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ **44.** $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

In Exercises 45–48, write the matrix in row-echelon forn (Remember that the row-echelon form of a matrix is nc unique.)

45. $\begin{bmatrix} 1 & 1 & 0 & 5 \\ -2 & -1 & 2 & -10 \\ 3 & 6 & 7 & 14 \end{bmatrix}$ **46.** $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 7 & -5 & 14 \\ -2 & -1 & -3 & 8 \end{bmatrix}$

47. $\begin{bmatrix} 1 & -1 & -1 & 1 \\ 5 & -4 & 1 & 8 \\ -6 & 8 & 18 & 0 \end{bmatrix}$ **48.** $\begin{bmatrix} 1 & -3 & 0 & -7 \\ -3 & 10 & 1 & 23 \\ 4 & -10 & 2 & -24 \end{bmatrix}$

In Exercises 49–54, use the matrix capabilities of a graphir utility to write the matrix in *reduced* row-echelon form.

49. $\begin{bmatrix} 3 & 3 & 3 \\ -1 & 0 & -4 \\ 2 & 4 & -2 \end{bmatrix}$ **50.** $\begin{bmatrix} 1 & 3 & 2 \\ 5 & 15 & 9 \\ 2 & 6 & 10 \end{bmatrix}$

51. $\begin{bmatrix} 1 & 2 & 3 & -5 \\ 1 & 2 & 4 & -9 \\ -2 & -4 & -4 & 3 \\ 4 & 8 & 11 & -14 \end{bmatrix}$

52. $\begin{bmatrix} -2 & 3 & -1 & -2 \\ 4 & -2 & 5 & 8 \\ 1 & 5 & -2 & 0 \\ 3 & 8 & -10 & -30 \end{bmatrix}$

53. $\begin{bmatrix} -3 & 5 & 1 & 12 \\ 1 & -1 & 1 & 4 \end{bmatrix}$ **54.** $\begin{bmatrix} 5 & 1 & 2 & 4 \\ -1 & 5 & 10 & -32 \end{bmatrix}$

In Exercises 55–58, write the system of linear equatio represented by the augmented matrix. Then use bac substitution to solve. (Use variables x, y, and z, if applicabl

55. $\left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -3 \end{array}\right]$ **56.** $\left[\begin{array}{cc|c} 1 & 5 & 0 \\ 0 & 1 & -1 \end{array}\right]$

57. $\begin{bmatrix} 1 & -1 & 2 & \vdots & 4 \\ 0 & 1 & -1 & \vdots & 2 \\ 0 & 0 & 1 & \vdots & -2 \end{bmatrix}$ **58.** $\begin{bmatrix} 1 & 2 & -2 & \vdots & -1 \\ 0 & 1 & 1 & \vdots & 9 \\ 0 & 0 & 1 & \vdots & -3 \end{bmatrix}$

In Exercises 59–62, an augmented matrix that represents a system of linear equations (in variables x, y, and z, if applicable) has been reduced using Gauss-Jordan elimination. Write the solution represented by the augmented matrix.

59. $\begin{bmatrix} 1 & 0 & \vdots & 3 \\ 0 & 1 & \vdots & -4 \end{bmatrix}$ **60.** $\begin{bmatrix} 1 & 0 & \vdots & -6 \\ 0 & 1 & \vdots & 10 \end{bmatrix}$

61. $\begin{bmatrix} 1 & 0 & 0 & \vdots & -4 \\ 0 & 1 & 0 & \vdots & -10 \\ 0 & 0 & 1 & \vdots & 4 \end{bmatrix}$ **62.** $\begin{bmatrix} 1 & 0 & 0 & \vdots & 5 \\ 0 & 1 & 0 & \vdots & -3 \\ 0 & 0 & 1 & \vdots & 0 \end{bmatrix}$

In Exercises 63–84, use matrices to solve the system of equations (if possible). Use Gaussian elimination with back-substitution or Gauss-Jordan elimination.

63. $\begin{cases} x + 2y = 7 \\ 2x + y = 8 \end{cases}$ **64.** $\begin{cases} 2x + 6y = 16 \\ 2x + 3y = 7 \end{cases}$

65. $\begin{cases} 3x - 2y = -27 \\ x + 3y = 13 \end{cases}$ **66.** $\begin{cases} -x + y = 4 \\ 2x - 4y = -34 \end{cases}$

67. $\begin{cases} -2x + 6y = -22 \\ x + 2y = -9 \end{cases}$ **68.** $\begin{cases} 5x - 5y = -5 \\ -2x - 3y = 7 \end{cases}$

69. $\begin{cases} 8x - 4y = 7 \\ 5x + 2y = 1 \end{cases}$ **70.** $\begin{cases} x - 3y = 5 \\ -2x + 6y = -10 \end{cases}$

71. $\begin{cases} x - 3z = -2 \\ 3x + y - 2z = 5 \\ 2x + 2y + z = 4 \end{cases}$ **72.** $\begin{cases} 2x - y + 3z = 24 \\ 2y - z = 14 \\ 7x - 5y = 6 \end{cases}$

73. $\begin{cases} -x + y - z = -14 \\ 2x - y + z = 21 \\ 3x + 2y + z = 19 \end{cases}$ **74.** $\begin{cases} 2x + 2y - z = 2 \\ x - 3y + z = -28 \\ -x + y = 14 \end{cases}$

75. $\begin{cases} x + 2y - 3z = -28 \\ 4y + 2z = 0 \\ -x + y - z = -5 \end{cases}$ **76.** $\begin{cases} 3x - 2y + z = 15 \\ -x + y + 2z = -10 \\ x - y - 4z = 14 \end{cases}$

77. $\begin{cases} x + 2y = 0 \\ -x - y = 0 \end{cases}$ **78.** $\begin{cases} x + 2y = 0 \\ 2x + 4y = 0 \end{cases}$

79. $\begin{cases} x + 2y + z = 8 \\ 3x + 7y + 6z = 26 \end{cases}$ **80.** $\begin{cases} x + y + 4z = 5 \\ 2x + y - z = 9 \end{cases}$

81. $\begin{cases} -x + y = -22 \\ 3x + 4y = 4 \\ 4x - 8y = 32 \end{cases}$ **82.** $\begin{cases} x + 2y = 0 \\ x + y = 6 \\ 3x - 2y = 8 \end{cases}$

83. $\begin{cases} 3x + 2y - z + w = 0 \\ x - y + 4z + 2w = 25 \\ -2x + y + 2z - w = 2 \\ x + y + z + w = 6 \end{cases}$

84. $\begin{cases} x - 4y + 3z - 2w = 9 \\ 3x - 2y + z - 4w = -13 \\ -4x + 3y - 2z + w = -4 \\ -2x + y - 4z + 3w = -10 \end{cases}$

In Exercises 85–90, use the matrix capabilities of a graphing utility to reduce the augmented matrix corresponding to the system of equations, and solve the system.

85. $\begin{cases} 3x + 3y + 12z = 6 \\ x + y + 4z = 2 \\ 2x + 5y + 20z = 10 \\ -x + 2y + 8z = 4 \end{cases}$ **86.** $\begin{cases} 2x + 10y + 2z = 6 \\ x + 5y + 2z = 6 \\ x + 5y + z = 3 \\ -3x - 15y - 3z = -9 \end{cases}$

87. $\begin{cases} 2x + y - z + 2w = -6 \\ 3x + 4y + w = 1 \\ x + 5y + 2z + 6w = -3 \\ 5x + 2y - z - w = 3 \end{cases}$

88. $\begin{cases} x + 2y + 2z + 4w = 11 \\ 3x + 6y + 5z + 12w = 30 \\ x + 3y - 3z + 2w = -5 \\ 6x - y - z + w = -9 \end{cases}$

89. $\begin{cases} x + y + z + w = 0 \\ 2x + 3y + z - 2w = 0 \\ 3x + 5y + z = 0 \end{cases}$

90. $\begin{cases} x + 2y + z + 3w = 0 \\ x - y + w = 0 \\ y - z + 2w = 0 \end{cases}$

In Exercises 91–94, determine whether the two systems of linear equations yield the same solution. If so, find the solution using matrices.

91. (a) $\begin{cases} x - 2y + z = -6 \\ y - 5z = 16 \\ z = -3 \end{cases}$ (b) $\begin{cases} x + y - 2z = 6 \\ y + 3z = -8 \\ z = -3 \end{cases}$

92. (a) $\begin{cases} x - 3y + 4z = -11 \\ y - z = -4 \\ z = 2 \end{cases}$ (b) $\begin{cases} x + 4y = -11 \\ y + 3z = 4 \\ z = 2 \end{cases}$

93. (a) $\begin{cases} x - 4y + 5z = 27 \\ y - 7z = -54 \\ z = 8 \end{cases}$ (b) $\begin{cases} x - 6y + z = 15 \\ y + 5z = 42 \\ z = 8 \end{cases}$

94. (a) $\begin{cases} x + 3y - z = 19 \\ y + 6z = -18 \\ z = -4 \end{cases}$ (b) $\begin{cases} x - y + 3z = -15 \\ y - 2z = 14 \\ z = -4 \end{cases}$

In Exercises 95–98, use a system of equations to find the quadratic function $f(x) = ax^2 + bx + c$ that satisfies the equations. Solve the system using matrices.

95. $f(1) = 1, f(2) = -1, f(3) = -5$

96. $f(1) = 2, f(2) = 9, f(3) = 20$

10.2 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.
Exercises in which the operation is not possible: 17(a), 17(b), 17(d), 18(a), 18(b), 18(d), 33, 40, 44, 45, 51(c), 52(c), 79, 80, 81, 84

VOCABULARY

In Exercises 1–4, fill in the blanks.

1. Two matrices are ________ if all of their corresponding entries are equal.
2. When performing matrix operations, real numbers are often referred to as ________.
3. A matrix consisting entirely of zeros is called a ________ matrix and is denoted by ________.
4. The $n \times n$ matrix consisting of 1's on its main diagonal and 0's elsewhere is called the ________ matrix of order $n \times n$.

In Exercises 5 and 6, match the matrix property with the correct form. A, B, and C are matrices of order $m \times n$, and c and d are scalars.

5. (a) $1A = A$
   (b) $A + (B + C) = (A + B) + C$
   (c) $(c + d)A = cA + dA$
   (d) $(cd)A = c(dA)$
   (e) $A + B = B + A$

   (i) Distributive Property
   (ii) Commutative Property of Matrix Addition
   (iii) Scalar Identity Property
   (iv) Associative Property of Matrix Addition
   (v) Associative Property of Scalar Multiplication

6. (a) $A + O = A$
   (b) $c(AB) = A(cB)$
   (c) $A(B + C) = AB + AC$
   (d) $A(BC) = (AB)C$

   (i) Distributive Property
   (ii) Additive Identity of Matrix Addition
   (iii) Associative Property of Matrix Multiplication
   (iv) Associative Property of Scalar Multiplication

SKILLS AND APPLICATIONS

In Exercises 7–10, find x and y.

7. $\begin{bmatrix} x & -2 \\ 7 & y \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ 7 & 22 \end{bmatrix}$

8. $\begin{bmatrix} -5 & x \\ y & 8 \end{bmatrix} = \begin{bmatrix} -5 & 13 \\ 12 & 8 \end{bmatrix}$

9. $\begin{bmatrix} 16 & 4 & 5 & 4 \\ -3 & 13 & 15 & 6 \\ 0 & 2 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 16 & 4 & 2x+1 & 4 \\ -3 & 13 & 15 & 3x \\ 0 & 2 & 3y-5 & 0 \end{bmatrix}$

10. $\begin{bmatrix} x+2 & 8 & -3 \\ 1 & 2y & 2x \\ 7 & -2 & y+2 \end{bmatrix} = \begin{bmatrix} 2x+6 & 8 & -3 \\ 1 & 18 & -8 \\ 7 & -2 & 11 \end{bmatrix}$

In Exercises 11–18, if possible, find (a) $A + B$, (b) $A - B$, (c) $3A$, and (d) $3A - 2B$.

11. $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix}$

12. $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix}$

13. $A = \begin{bmatrix} 8 & -1 \\ 2 & 3 \\ -4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 6 \\ -1 & -5 \\ 1 & 10 \end{bmatrix}$

14. $A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 6 & 9 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0 & -5 \\ -3 & 4 & -7 \end{bmatrix}$

15. $A = \begin{bmatrix} 4 & 5 & -1 & 3 & 4 \\ 1 & 2 & -2 & -1 & 0 \end{bmatrix}$,
$B = \begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ -6 & 8 & 2 & -3 & -7 \end{bmatrix}$

16. $A = \begin{bmatrix} -1 & 4 & 0 \\ 3 & -2 & 2 \\ 5 & 4 & -1 \\ 0 & 8 & -6 \\ -4 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 5 & 1 \\ 2 & -4 & -7 \\ 10 & -9 & -1 \\ 3 & 2 & -4 \\ 0 & 1 & -2 \end{bmatrix}$

17. $A = \begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 8 & -1 \\ 4 & -3 \end{bmatrix}$

18. $A = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$, $B = \begin{bmatrix} -4 & 6 & 2 \end{bmatrix}$

In Exercises 19–24, evaluate the expression.

19. $\begin{bmatrix} -5 & 0 \\ 3 & -6 \end{bmatrix} + \begin{bmatrix} 7 & 1 \\ -2 & -1 \end{bmatrix} + \begin{bmatrix} -10 & -8 \\ 14 & 6 \end{bmatrix}$

20. $\begin{bmatrix} 6 & 8 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ -3 & -1 \end{bmatrix} + \begin{bmatrix} -11 & -7 \\ 2 & -1 \end{bmatrix}$

21. $4\left(\begin{bmatrix} -4 & 0 & 1 \\ 0 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -2 \\ 3 & -6 & 0 \end{bmatrix}\right)$

22. $\frac{1}{2}([5 \quad -2 \quad 4 \quad 0] + [14 \quad 6 \quad -18 \quad 9])$

23. $-3\left(\begin{bmatrix} 0 & -3 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} -6 & 3 \\ 8 & 1 \end{bmatrix}\right) - 2\begin{bmatrix} 4 & -4 \\ 7 & -9 \end{bmatrix}$

24. $-\begin{bmatrix} 4 & 11 \\ -2 & -1 \\ 9 & 3 \end{bmatrix} + \frac{1}{6}\left(\begin{bmatrix} -5 & -1 \\ 3 & 4 \\ 0 & 13 \end{bmatrix} + \begin{bmatrix} 7 & 5 \\ -9 & -1 \\ 6 & -1 \end{bmatrix}\right)$

In Exercises 25–28, use the matrix capabilities of a graphing utility to evaluate the expression. Round your results to three decimal places, if necessary.

25. $\frac{3}{7}\begin{bmatrix} 2 & 5 \\ -1 & -4 \end{bmatrix} + 6\begin{bmatrix} -3 & 0 \\ 2 & 2 \end{bmatrix}$

26. $55\left(\begin{bmatrix} 14 & -11 \\ -22 & 19 \end{bmatrix} + \begin{bmatrix} -22 & 20 \\ 13 & 6 \end{bmatrix}\right)$

27. $-\begin{bmatrix} 3.211 & 6.829 \\ -1.004 & 4.914 \\ 0.055 & -3.889 \end{bmatrix} - \begin{bmatrix} -1.630 & -3.090 \\ 5.256 & 8.335 \\ -9.768 & 4.251 \end{bmatrix}$

28. $-\begin{bmatrix} 10 & 15 \\ -20 & 10 \\ 12 & 4 \end{bmatrix} + \frac{1}{8}\left(\begin{bmatrix} -13 & 11 \\ 7 & 0 \\ 6 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 13 \\ -3 & 8 \\ -14 & 15 \end{bmatrix}\right)$

In Exercises 29–32, solve for X in the equation, given

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix}.$$

29. $X = 3A - 2B$

30. $2X = 2A - B$

31. $2X + 3A = B$

32. $2A + 4B = -2X$

In Exercises 33–40, if possible, find AB and state the order of the result.

33. $A = \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 & 0 \\ 4 & 0 & 2 \\ 8 & -1 & 7 \end{bmatrix}$

34. $A = \begin{bmatrix} 0 & -1 & 2 \\ 6 & 0 & 3 \\ 7 & -1 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 4 & -5 \\ 1 & 6 \end{bmatrix}$

35. $A = \begin{bmatrix} -1 & 6 \\ -4 & 5 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 0 & 9 \end{bmatrix}$

36. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

37. $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 7 \end{bmatrix}$, $B = \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & -\frac{1}{8} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$

38. $A = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 0 & -3 \\ 0 & 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 & -11 & 4 \\ 8 & 16 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

39. $A = \begin{bmatrix} 10 \\ 12 \end{bmatrix}$, $B = [6 \quad -2 \quad 1 \quad 6]$

40. $A = \begin{bmatrix} 1 & 0 & 3 & -2 \\ 6 & 13 & 8 & -17 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 6 \\ 4 & 2 \end{bmatrix}$

In Exercises 41–46, use the matrix capabilities of a graphing utility to find AB, if possible.

41. $A = \begin{bmatrix} 7 & 5 & -4 \\ -2 & 5 & 1 \\ 10 & -4 & -7 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 & 3 \\ 8 & 1 & 4 \\ -4 & 2 & -8 \end{bmatrix}$

42. $A = \begin{bmatrix} 11 & -12 & 4 \\ 14 & 10 & 12 \\ 6 & -2 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 12 & 10 \\ -5 & 12 \\ 15 & 16 \end{bmatrix}$

43. $A = \begin{bmatrix} -3 & 8 & -6 & 8 \\ -12 & 15 & 9 & 6 \\ 5 & -1 & 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 & 6 \\ 24 & 15 & 14 \\ 16 & 10 & 21 \\ 8 & -4 & 10 \end{bmatrix}$

44. $A = \begin{bmatrix} -2 & 4 & 8 \\ 21 & 5 & 6 \\ 13 & 2 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ -7 & 15 \\ 32 & 14 \\ 0.5 & 1.6 \end{bmatrix}$

45. $A = \begin{bmatrix} 9 & 10 & -38 & 18 \\ 100 & -50 & 250 & 75 \end{bmatrix}$,

$B = \begin{bmatrix} 52 & -85 & 27 & 45 \\ 40 & -35 & 60 & 82 \end{bmatrix}$

46. $A = \begin{bmatrix} 16 & -18 \\ -4 & 13 \\ -9 & 21 \end{bmatrix}$, $B = \begin{bmatrix} -7 & 20 & -1 \\ 7 & 15 & 26 \end{bmatrix}$

In Exercises 47–52, if possible, find (a) AB, (b) BA, and (c) A^2. (*Note:* $A^2 = AA$.)

47. $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix}$

48. $A = \begin{bmatrix} 6 & 3 \\ -2 & -4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$

49. $A = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$

50. $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}$

51. $A = \begin{bmatrix} 7 \\ 8 \\ -1 \end{bmatrix}$, $B = [1 \quad 1 \quad 2]$

52. $A = [3 \quad 2 \quad 1]$, $B = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$

In Exercises 53–56, evaluate the expression. Use the matrix capabilities of a graphing utility to verify your answer.

53. $\begin{bmatrix} 3 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$

54. $-3\left(\begin{bmatrix} 6 & 5 & -1 \\ 1 & -2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ -1 & -3 \\ 4 & 1 \end{bmatrix}\right)$

55. $\begin{bmatrix} 0 & 2 & -2 \\ 4 & 1 & 2 \end{bmatrix}\left(\begin{bmatrix} 4 & 0 \\ 0 & -1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ -3 & 5 \\ 0 & -3 \end{bmatrix}\right)$

56. $\begin{bmatrix} 3 \\ -1 \\ 5 \\ 7 \end{bmatrix}([5 \quad -6] + [7 \quad -1] + [-8 \quad 9])$

In Exercises 57–64, (a) write the system of linear equations as a matrix equation, $AX = B$, and (b) use Gauss-Jordan elimination on the augmented matrix $[A \ \vdots \ B]$ to solve for the matrix X.

57. $\begin{cases} -x_1 + x_2 = 4 \\ -2x_1 + x_2 = 0 \end{cases}$

58. $\begin{cases} 2x_1 + 3x_2 = 5 \\ x_1 + 4x_2 = 10 \end{cases}$

59. $\begin{cases} -2x_1 - 3x_2 = -4 \\ 6x_1 + x_2 = -36 \end{cases}$

60. $\begin{cases} -4x_1 + 9x_2 = -13 \\ x_1 - 3x_2 = 12 \end{cases}$

61. $\begin{cases} x_1 - 2x_2 + 3x_3 = 9 \\ -x_1 + 3x_2 - x_3 = -6 \\ 2x_1 - 5x_2 + 5x_3 = 17 \end{cases}$

62. $\begin{cases} x_1 + x_2 - 3x_3 = -1 \\ -x_1 + 2x_2 = 1 \\ x_1 - x_2 + x_3 = 2 \end{cases}$

63. $\begin{cases} x_1 - 5x_2 + 2x_3 = -20 \\ -3x_1 + x_2 - x_3 = 8 \\ -2x_2 + 5x_3 = -16 \end{cases}$

64. $\begin{cases} x_1 - x_2 + 4x_3 = 17 \\ x_1 + 3x_2 = -11 \\ -6x_2 + 5x_3 = 40 \end{cases}$

65. MANUFACTURING A corporation has three factories, each of which manufactures acoustic guitars and electric guitars. The number of units of guitars produced at factory j in one day is represented by a_{ij} in the matrix

$$A = \begin{bmatrix} 70 & 50 & 25 \\ 35 & 100 & 70 \end{bmatrix}.$$

Find the production levels if production is increased by 20%.

66. MANUFACTURING A corporation has four factories, each of which manufactures sport utility vehicles and pickup trucks. The number of units of vehicle i produced at factory j in one day is represented by a_{ij} in the matrix

$$A = \begin{bmatrix} 100 & 90 & 70 & 30 \\ 40 & 20 & 60 & 60 \end{bmatrix}.$$

Find the production levels if production is increased by 10%.

67. AGRICULTURE A fruit grower raises two crops, apples and peaches. Each of these crops is sent to three different outlets for sale. These outlets are The Farmer's Market, The Fruit Stand, and The Fruit Farm. The numbers of bushels of apples sent to the three outlets are 125, 100, and 75, respectively. The numbers of bushels of peaches sent to the three outlets are 100, 175, and 125, respectively. The profit per bushel for apples is \$3.50 and the profit per bushel for peaches is \$6.00.

(a) Write a matrix A that represents the number of bushels of each crop i that are shipped to each outlet j. State what each entry a_{ij} of the matrix represents.

(b) Write a matrix B that represents the profit per bushel of each fruit. State what each entry b_{ij} of the matrix represents.

(c) Find the product BA and state what each entry of the matrix represents.

68. REVENUE An electronics manufacturer produces three models of LCD televisions, which are shipped to two warehouses. The numbers of units of model i that are shipped to warehouse j are represented by a_{ij} in the matrix

$$A = \begin{bmatrix} 5{,}000 & 4{,}000 \\ 6{,}000 & 10{,}000 \\ 8{,}000 & 5{,}000 \end{bmatrix}.$$

The prices per unit are represented by the matrix

$$B = [\$699.95 \quad \$899.95 \quad \$1099.95].$$

Compute BA and interpret the result.

69. INVENTORY A company sells five models of computers through three retail outlets. The inventories are represented by S.

$$S = \begin{array}{c} \text{Model} \\ \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{array} \\ \begin{bmatrix} 3 & 2 & 2 & 3 & 0 \\ 0 & 2 & 3 & 4 & 3 \\ 4 & 2 & 1 & 3 & 2 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ 1 \\ 2 \\ 3 \end{array} \ \text{Outlet}$$

The wholesale and retail prices are represented by T.

$$T = \begin{array}{c} \text{Price} \\ \begin{array}{cc} \text{Wholesale} & \text{Retail} \end{array} \\ \begin{bmatrix} \$840 & \$1100 \\ \$1200 & \$1350 \\ \$1450 & \$1650 \\ \$2650 & \$3000 \\ \$3050 & \$3200 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \end{array} \ \text{Model}$$

Compute ST and interpret the result.

10.3 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.
Exercises in which the inverse matrix does not exist: 21, 22, 30, 37; Exercise containing system with no solution: 53; Exercises containing systems with infinitely many solutions: 59, 60

VOCABULARY: Fill in the blanks.

1. In a ________ matrix, the number of rows equals the number of columns.
2. If there exists an $n \times n$ matrix A^{-1} such that $AA^{-1} = I_n = A^{-1}A$, then A^{-1} is called the ________ of A.
3. If a matrix A has an inverse, it is called invertible or ________; if it does not have an inverse, it is called ________.
4. If A is an invertible matrix, the system of linear equations represented by $AX = B$ has a unique solution given by $X =$ ________.

SKILLS AND APPLICATIONS

In Exercises 5–12, show that B is the inverse of A.

5. $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

6. $A = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

7. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

8. $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, B = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$

9. $A = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}$

10. $A = \begin{bmatrix} -4 & 1 & 5 \\ -1 & 2 & 4 \\ 0 & -1 & -1 \end{bmatrix}, B = \begin{bmatrix} -\frac{1}{2} & 1 & \frac{3}{2} \\ \frac{1}{4} & -1 & -\frac{11}{4} \\ -\frac{1}{4} & 1 & \frac{7}{4} \end{bmatrix}$

11. $A = \begin{bmatrix} -2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{bmatrix}, B = \frac{1}{3}\begin{bmatrix} -4 & -5 & 3 \\ -4 & -8 & 3 \\ 1 & 2 & 0 \end{bmatrix}$

12. $A = \begin{bmatrix} -1 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & 2 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix},$

$B = \frac{1}{3}\begin{bmatrix} -3 & 1 & 1 & -3 \\ -3 & -1 & 2 & -3 \\ 0 & 1 & 1 & 0 \\ -3 & -2 & 1 & 0 \end{bmatrix}$

In Exercises 13–24, find the inverse of the matrix (if it exists).

13. $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

14. $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

15. $\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$

16. $\begin{bmatrix} -7 & 33 \\ 4 & -19 \end{bmatrix}$

17. $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

18. $\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$

19. $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix}$

20. $\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$

21. $\begin{bmatrix} -5 & 0 & 0 \\ 2 & 0 & 0 \\ -1 & 5 & 7 \end{bmatrix}$

22. $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & 0 \\ 2 & 5 & 5 \end{bmatrix}$

23. $\begin{bmatrix} -8 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & -5 \end{bmatrix}$

24. $\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

In Exercises 25–34, use the matrix capabilities of a graphing utility to find the inverse of the matrix (if it exists).

25. $\begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ -5 & -7 & -15 \end{bmatrix}$

26. $\begin{bmatrix} 10 & 5 & -7 \\ -5 & 1 & 4 \\ 3 & 2 & -2 \end{bmatrix}$

27. $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ -2 & 0 & 3 \end{bmatrix}$

28. $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 2 \\ -4 & 4 & 3 \end{bmatrix}$

29. $\begin{bmatrix} -\frac{1}{2} & \frac{3}{4} & \frac{1}{4} \\ 1 & 0 & -\frac{3}{2} \\ 0 & -1 & \frac{1}{2} \end{bmatrix}$

30. $\begin{bmatrix} -\frac{5}{6} & \frac{1}{3} & \frac{11}{6} \\ 0 & \frac{2}{3} & 2 \\ 1 & -\frac{1}{2} & -\frac{5}{2} \end{bmatrix}$

31. $\begin{bmatrix} 0.1 & 0.2 & 0.3 \\ -0.3 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.4 \end{bmatrix}$

32. $\begin{bmatrix} 0.6 & 0 & -0.3 \\ 0.7 & -1 & 0.2 \\ 1 & 0 & -0.9 \end{bmatrix}$

33. $\begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 \\ 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$

34. $\begin{bmatrix} 1 & -2 & -1 & -2 \\ 3 & -5 & -2 & -3 \\ 2 & -5 & -2 & -5 \\ -1 & 4 & 4 & 11 \end{bmatrix}$

In Exercises 35–40, use the formula on page 763 to find the inverse of the 2×2 matrix (if it exists).

35. $\begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$

36. $\begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix}$

37. $\begin{bmatrix} -4 & -6 \\ 2 & 3 \end{bmatrix}$

38. $\begin{bmatrix} -12 & 3 \\ 5 & -2 \end{bmatrix}$

39. $\begin{bmatrix} \frac{7}{2} & -\frac{3}{4} \\ \frac{1}{5} & \frac{4}{5} \end{bmatrix}$ **40.** $\begin{bmatrix} -\frac{1}{4} & \frac{9}{4} \\ \frac{5}{3} & \frac{8}{9} \end{bmatrix}$

In Exercises 41–44, use the inverse matrix found in Exercise 15 to solve the system of linear equations.

41. $\begin{cases} x - 2y = 5 \\ 2x - 3y = 10 \end{cases}$ **42.** $\begin{cases} x - 2y = 0 \\ 2x - 3y = 3 \end{cases}$

43. $\begin{cases} x - 2y = 4 \\ 2x - 3y = 2 \end{cases}$ **44.** $\begin{cases} x - 2y = 1 \\ 2x - 3y = -2 \end{cases}$

In Exercises 45 and 46, use the inverse matrix found in Exercise 19 to solve the system of linear equations.

45. $\begin{cases} x + y + z = 0 \\ 3x + 5y + 4z = 5 \\ 3x + 6y + 5z = 2 \end{cases}$ **46.** $\begin{cases} x + y + z = -1 \\ 3x + 5y + 4z = 2 \\ 3x + 6y + 5z = 0 \end{cases}$

In Exercises 47 and 48, use the inverse matrix found in Exercise 34 to solve the system of linear equations.

47. $\begin{cases} x_1 - 2x_2 - x_3 - 2x_4 = 0 \\ 3x_1 - 5x_2 - 2x_3 - 3x_4 = 1 \\ 2x_1 - 5x_2 - 2x_3 - 5x_4 = -1 \\ -x_1 + 4x_2 + 4x_3 + 11x_4 = 2 \end{cases}$

48. $\begin{cases} x_1 - 2x_2 - x_3 - 2x_4 = 1 \\ 3x_1 - 5x_2 - 2x_3 - 3x_4 = -2 \\ 2x_1 - 5x_2 - 2x_3 - 5x_4 = 0 \\ -x_1 + 4x_2 + 4x_3 + 11x_4 = -3 \end{cases}$

In Exercises 49 and 50, use a graphing utility to solve the system of linear equations using an inverse matrix.

49. $\begin{aligned} x_1 + 2x_2 - x_3 + 3x_4 - x_5 &= -3 \\ x_1 - 3x_2 + x_3 + 2x_4 - x_5 &= -3 \\ 2x_1 + x_2 + x_3 - 3x_4 + x_5 &= 6 \\ x_1 - x_2 + 2x_3 + x_4 - x_5 &= 2 \\ 2x_1 + x_2 - x_3 + 2x_4 + x_5 &= -3 \end{aligned}$

50. $\begin{aligned} x_1 + x_2 - x_3 + 3x_4 - x_5 &= 3 \\ 2x_1 + x_2 + x_3 + x_4 + x_5 &= 4 \\ x_1 + x_2 - x_3 + 2x_4 - x_5 &= 3 \\ 2x_1 + x_2 + 4x_3 + x_4 - x_5 &= -1 \\ 3x_1 + x_2 + x_3 - 2x_4 + x_5 &= 5 \end{aligned}$

In Exercises 51–58, use an inverse matrix to solve (if possible) the system of linear equations.

51. $\begin{cases} 3x + 4y = -2 \\ 5x + 3y = 4 \end{cases}$ **52.** $\begin{cases} 18x + 12y = 13 \\ 30x + 24y = 23 \end{cases}$

53. $\begin{cases} -0.4x + 0.8y = 1.6 \\ 2x - 4y = 5 \end{cases}$ **54.** $\begin{cases} 0.2x - 0.6y = 2.4 \\ -x + 1.4y = -8.8 \end{cases}$

55. $\begin{cases} -\frac{1}{4}x + \frac{3}{8}y = -2 \\ \frac{3}{2}x + \frac{3}{4}y = -12 \end{cases}$ **56.** $\begin{cases} \frac{5}{6}x - y = -20 \\ \frac{4}{3}x - \frac{7}{2}y = -51 \end{cases}$

57. $\begin{cases} 4x - y + z = -5 \\ 2x + 2y + 3z = 10 \\ 5x - 2y + 6z = 1 \end{cases}$ **58.** $\begin{cases} 4x - 2y + 3z = -2 \\ 2x + 2y + 5z = 16 \\ 8x - 5y - 2z = 4 \end{cases}$

In Exercises 59–62, use the matrix capabilities of a graphing utility to solve (if possible) the system of linear equations.

59. $\begin{cases} 5x - 3y + 2z = 2 \\ 2x + 2y - 3z = 3 \\ x - 7y + 8z = -4 \end{cases}$ **60.** $\begin{cases} 2x + 3y + 5z = 4 \\ 3x + 5y + 9z = 7 \\ 5x + 9y + 17z = 13 \end{cases}$

61. $\begin{cases} 3x - 2y + z = -29 \\ -4x + y - 3z = 37 \\ x - 5y + z = -24 \end{cases}$

62. $\begin{cases} -8x + 7y - 10z = -151 \\ 12x + 3y - 5z = 86 \\ 15x - 9y + 2z = 187 \end{cases}$

In Exercises 63 and 64, show that the matrix is invertible and find its inverse.

63. $A = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$ **64.** $A = \begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{bmatrix}$

INVESTMENT PORTFOLIO In Exercises 65–68, consider a person who invests in AAA-rated bonds, A-rated bonds, and B-rated bonds. The average yields are 6.5% on AAA bonds, 7% on A bonds, and 9% on B bonds. The person invests twice as much in B bonds as in A bonds. Let x, y, and z represent the amounts invested in AAA, A, and B bonds, respectively.

$$\begin{cases} x + y + z = \textbf{(total investment)} \\ 0.065x + 0.07y + 0.09z = \textbf{(annual return)} \\ 2y - z = 0 \end{cases}$$

Use the inverse of the coefficient matrix of this system to find the amount invested in each type of bond.

	Total Investment	*Annual Return*
65.	\$10,000	\$705
66.	\$10,000	\$760
67.	\$12,000	\$835
68.	\$500,000	\$38,000

PRODUCTION In Exercises 69–72, a small home business creates muffins, bones, and cookies for dogs. In addition to other ingredients, each muffin requires 2 units of beef, 3 units of chicken, and 2 units of liver. Each bone requires 1 unit of beef, 1 unit of chicken, and 1 unit of liver. Each cookie requires 2 units of beef, 1 unit of chicken, and 1.5 units of liver. Find the numbers of muffins, bones, and cookies that the company can create with the given amounts of ingredients.

10.4 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. Both det(A) and $|A|$ represent the ________ of the matrix A.
2. The ________ M_{ij} of the entry a_{ij} is the determinant of the matrix obtained by deleting the ith row and jth column of the square matrix A.
3. The ________ C_{ij} of the entry a_{ij} of the square matrix A is given by $(-1)^{i+j}M_{ij}$.
4. The method of finding the determinant of a matrix of order 2×2 or greater is called ________ by ________.

SKILLS AND APPLICATIONS

In Exercises 5–20, find the determinant of the matrix.

5. $[4]$
6. $[-10]$
7. $\begin{bmatrix} 8 & 4 \\ 2 & 3 \end{bmatrix}$
8. $\begin{bmatrix} -9 & 0 \\ 6 & 2 \end{bmatrix}$
9. $\begin{bmatrix} 6 & 2 \\ -5 & 3 \end{bmatrix}$
10. $\begin{bmatrix} 3 & -3 \\ 4 & -8 \end{bmatrix}$
11. $\begin{bmatrix} -7 & 0 \\ 3 & 0 \end{bmatrix}$
12. $\begin{bmatrix} 4 & -3 \\ 0 & 0 \end{bmatrix}$
13. $\begin{bmatrix} 2 & 6 \\ 0 & 3 \end{bmatrix}$
14. $\begin{bmatrix} 2 & -3 \\ -6 & 9 \end{bmatrix}$
15. $\begin{bmatrix} -3 & -2 \\ -6 & -1 \end{bmatrix}$
16. $\begin{bmatrix} 4 & 7 \\ -2 & 5 \end{bmatrix}$
17. $\begin{bmatrix} -7 & 6 \\ \frac{1}{2} & 3 \end{bmatrix}$
18. $\begin{bmatrix} 0 & 6 \\ -3 & 2 \end{bmatrix}$
19. $\begin{bmatrix} -\frac{1}{2} & \frac{1}{3} \\ -6 & \frac{1}{3} \end{bmatrix}$
20. $\begin{bmatrix} \frac{2}{3} & \frac{4}{3} \\ -1 & -\frac{1}{3} \end{bmatrix}$

In Exercises 21–24, use the matrix capabilities of a graphing utility to find the determinant of the matrix.

21. $\begin{bmatrix} 0.3 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 \\ -0.4 & 0.4 & 0.3 \end{bmatrix}$
22. $\begin{bmatrix} 0.1 & 0.2 & 0.3 \\ -0.3 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.4 \end{bmatrix}$
23. $\begin{bmatrix} 0.9 & 0.7 & 0 \\ -0.1 & 0.3 & 1.3 \\ -2.2 & 4.2 & 6.1 \end{bmatrix}$
24. $\begin{bmatrix} 0.1 & 0.1 & -4.3 \\ 7.5 & 6.2 & 0.7 \\ 0.3 & 0.6 & -1.2 \end{bmatrix}$

In Exercises 25–32, find all (a) minors and (b) cofactors of the matrix.

25. $\begin{bmatrix} 4 & 5 \\ 3 & -6 \end{bmatrix}$
26. $\begin{bmatrix} 0 & 10 \\ 3 & -4 \end{bmatrix}$
27. $\begin{bmatrix} 3 & 1 \\ -2 & -4 \end{bmatrix}$
28. $\begin{bmatrix} -6 & 5 \\ 7 & -2 \end{bmatrix}$
29. $\begin{bmatrix} 4 & 0 & 2 \\ -3 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$
30. $\begin{bmatrix} 1 & -1 & 0 \\ 3 & 2 & 5 \\ 4 & -6 & 4 \end{bmatrix}$
31. $\begin{bmatrix} -4 & 6 & 3 \\ 7 & -2 & 8 \\ 1 & 0 & -5 \end{bmatrix}$
32. $\begin{bmatrix} -2 & 9 & 4 \\ 7 & -6 & 0 \\ 6 & 7 & -6 \end{bmatrix}$

In Exercises 33–38, find the determinant of the matrix by the method of expansion by cofactors. Expand using the indicated row or column.

33. $\begin{bmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 2 & -3 & 1 \end{bmatrix}$
(a) Row 1
(b) Column 2

34. $\begin{bmatrix} -3 & 4 & 2 \\ 6 & 3 & 1 \\ 4 & -7 & -8 \end{bmatrix}$
(a) Row 2
(b) Column 3

35. $\begin{bmatrix} 5 & 0 & -3 \\ 0 & 12 & 4 \\ 1 & 6 & 3 \end{bmatrix}$
(a) Row 2
(b) Column 2

36. $\begin{bmatrix} 10 & -5 & 5 \\ 30 & 0 & 10 \\ 0 & 10 & 1 \end{bmatrix}$
(a) Row 3
(b) Column 1

37. $\begin{bmatrix} 6 & 0 & -3 & 5 \\ 4 & 13 & 6 & -8 \\ -1 & 0 & 7 & 4 \\ 8 & 6 & 0 & 2 \end{bmatrix}$
(a) Row 2
(b) Column 2

38. $\begin{bmatrix} 10 & 8 & 3 & -7 \\ 4 & 0 & 5 & -6 \\ 0 & 3 & 2 & 7 \\ 1 & 0 & -3 & 2 \end{bmatrix}$
(a) Row 3
(b) Column 1

In Exercises 39–54, find the determinant of the matrix. Expand by cofactors on the row or column that appears to make the computations easiest.

39. $\begin{bmatrix} 2 & -1 & 0 \\ 4 & 2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$
40. $\begin{bmatrix} -2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{bmatrix}$
41. $\begin{bmatrix} 6 & 3 & -7 \\ 0 & 0 & 0 \\ 4 & -6 & 3 \end{bmatrix}$
42. $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ -2 & 0 & 3 \end{bmatrix}$
43. $\begin{bmatrix} -1 & 8 & -3 \\ 0 & 3 & -6 \\ 0 & 0 & 3 \end{bmatrix}$
44. $\begin{bmatrix} 1 & 0 & 0 \\ -1 & -1 & 0 \\ 4 & 11 & 5 \end{bmatrix}$

45. $\begin{bmatrix} 1 & 4 & -2 \\ 3 & 2 & 0 \\ -1 & 4 & 3 \end{bmatrix}$

46. $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & 4 \\ 1 & 0 & 2 \end{bmatrix}$

47. $\begin{bmatrix} 2 & 4 & 6 \\ 0 & 3 & 1 \\ 0 & 0 & -5 \end{bmatrix}$

48. $\begin{bmatrix} -3 & 0 & 0 \\ 7 & 11 & 0 \\ 1 & 2 & 2 \end{bmatrix}$

49. $\begin{bmatrix} 2 & 6 & 6 & 2 \\ 2 & 7 & 3 & 6 \\ 1 & 5 & 0 & 1 \\ 3 & 7 & 0 & 7 \end{bmatrix}$

50. $\begin{bmatrix} 3 & 6 & -5 & 4 \\ -2 & 0 & 6 & 0 \\ 1 & 1 & 2 & 2 \\ 0 & 3 & -1 & -1 \end{bmatrix}$

51. $\begin{bmatrix} 5 & 3 & 0 & 6 \\ 4 & 6 & 4 & 12 \\ 0 & 2 & -3 & 4 \\ 0 & 1 & -2 & 2 \end{bmatrix}$

52. $\begin{bmatrix} 1 & 4 & 3 & 2 \\ -5 & 6 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 3 & -2 & 1 & 5 \end{bmatrix}$

53. $\begin{bmatrix} 3 & 2 & 4 & -1 & 5 \\ -2 & 0 & 1 & 3 & 2 \\ 1 & 0 & 0 & 4 & 0 \\ 6 & 0 & 2 & -1 & 0 \\ 3 & 0 & 5 & 1 & 0 \end{bmatrix}$

54. $\begin{bmatrix} 5 & 2 & 0 & 0 & -2 \\ 0 & 1 & 4 & 3 & 2 \\ 0 & 0 & 2 & 6 & 3 \\ 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$

In Exercises 55–62, use the matrix capabilities of a graphing utility to evaluate the determinant.

55. $\begin{vmatrix} 3 & 8 & -7 \\ 0 & -5 & 4 \\ 8 & 1 & 6 \end{vmatrix}$

56. $\begin{vmatrix} 5 & -8 & 0 \\ 9 & 7 & 4 \\ -8 & 7 & 1 \end{vmatrix}$

57. $\begin{vmatrix} 7 & 0 & -14 \\ -2 & 5 & 4 \\ -6 & 2 & 12 \end{vmatrix}$

58. $\begin{vmatrix} 3 & 0 & 0 \\ -2 & 5 & 0 \\ 12 & 5 & 7 \end{vmatrix}$

59. $\begin{vmatrix} 1 & -1 & 8 & 4 \\ 2 & 6 & 0 & -4 \\ 2 & 0 & 2 & 6 \\ 0 & 2 & 8 & 0 \end{vmatrix}$

60. $\begin{vmatrix} 0 & -3 & 8 & 2 \\ 8 & 1 & -1 & 6 \\ -4 & 6 & 0 & 9 \\ -7 & 0 & 0 & 14 \end{vmatrix}$

61. $\begin{vmatrix} 3 & -2 & 4 & 3 & 1 \\ -1 & 0 & 2 & 1 & 0 \\ 5 & -1 & 0 & 3 & 2 \\ 4 & 7 & -8 & 0 & 0 \\ 1 & 2 & 3 & 0 & 2 \end{vmatrix}$

62. $\begin{vmatrix} -2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & -4 \end{vmatrix}$

In Exercises 63–70, find (a) $|A|$, (b) $|B|$, (c) AB, and (d) $|AB|$.

63. $A = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

64. $A = \begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

65. $A = \begin{bmatrix} 4 & 0 \\ 3 & -2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix}$

66. $A = \begin{bmatrix} 5 & 4 \\ 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 6 \\ 1 & -2 \end{bmatrix}$

67. $A = \begin{bmatrix} 0 & 1 & 2 \\ -3 & -2 & 1 \\ 0 & 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 & 0 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}$

68. $A = \begin{bmatrix} 3 & 2 & 0 \\ -1 & -3 & 4 \\ -2 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 0 & 1 \\ 0 & 2 & -1 \\ -2 & -1 & 1 \end{bmatrix}$

69. $A = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

70. $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & 4 \\ 0 & 1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$

In Exercises 71–76, evaluate the determinant(s) to verify the equation.

71. $\begin{vmatrix} w & x \\ y & z \end{vmatrix} = -\begin{vmatrix} y & z \\ w & x \end{vmatrix}$

72. $\begin{vmatrix} w & cx \\ y & cz \end{vmatrix} = c\begin{vmatrix} w & x \\ y & z \end{vmatrix}$

73. $\begin{vmatrix} w & x \\ y & z \end{vmatrix} = \begin{vmatrix} w & x + cw \\ y & z + cy \end{vmatrix}$

74. $\begin{vmatrix} w & x \\ cw & cx \end{vmatrix} = 0$

75. $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (y - x)(z - x)(z - y)$

76. $\begin{vmatrix} a + b & a & a \\ a & a + b & a \\ a & a & a + b \end{vmatrix} = b^2(3a + b)$

In Exercises 77–84, solve for x.

77. $\begin{vmatrix} x & 2 \\ 1 & x \end{vmatrix} = 2$

78. $\begin{vmatrix} x & 4 \\ -1 & x \end{vmatrix} = 20$

79. $\begin{vmatrix} x & 1 \\ 2 & x - 2 \end{vmatrix} = -1$

80. $\begin{vmatrix} x + 1 & 2 \\ -1 & x \end{vmatrix} = 4$

81. $\begin{vmatrix} x - 1 & 2 \\ 3 & x - 2 \end{vmatrix} = 0$

82. $\begin{vmatrix} x - 2 & -1 \\ -3 & x \end{vmatrix} = 0$

83. $\begin{vmatrix} x + 3 & 2 \\ 1 & x + 2 \end{vmatrix} = 0$

84. $\begin{vmatrix} x + 4 & -2 \\ 7 & x - 5 \end{vmatrix} = 0$

10.5 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.
Exercise in which Cramer's Rule does not apply: 9
Exercises in which the points are not collinear: 40, 41, 44

OCABULARY: Fill in the blanks.

1. The method of using determinants to solve a system of linear equations is called ________ ________.
2. Three points are ________ if the points lie on the same line.
3. The area A of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by ________.
4. A message written according to a secret code is called a ________.
5. To encode a message, choose an invertible matrix A and multiply the ________ row matrices by A (on the right) to obtain ________ row matrices.
6. If a message is encoded using an invertible matrix A, then the message can be decoded by multiplying the coded row matrices by ________ (on the right).

KILLS AND APPLICATIONS

In Exercises 7–16, use Cramer's Rule to solve (if possible) the system of equations.

7. $\begin{cases} -7x + 11y = -1 \\ 3x - 9y = 9 \end{cases}$ 8. $\begin{cases} 4x - 3y = -10 \\ 6x + 9y = 12 \end{cases}$

9. $\begin{cases} 3x + 2y = -2 \\ 6x + 4y = 4 \end{cases}$ 10. $\begin{cases} 6x - 5y = 17 \\ -13x + 3y = -76 \end{cases}$

11. $\begin{cases} -0.4x + 0.8y = 1.6 \\ 0.2x + 0.3y = 2.2 \end{cases}$ 12. $\begin{cases} 2.4x - 1.3y = 14.63 \\ -4.6x + 0.5y = -11.51 \end{cases}$

13. $\begin{cases} 4x - y + z = -5 \\ 2x + 2y + 3z = 10 \\ 5x - 2y + 6z = 1 \end{cases}$ 14. $\begin{cases} 4x - 2y + 3z = -2 \\ 2x + 2y + 5z = 16 \\ 8x - 5y - 2z = 4 \end{cases}$

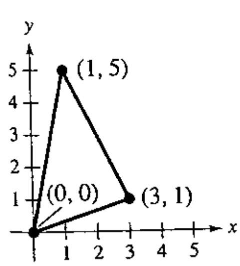
15. $\begin{cases} x + 2y + 3z = -3 \\ -2x + y - z = 6 \\ 3x - 3y + 2z = -11 \end{cases}$ 16. $\begin{cases} 5x - 4y + z = -14 \\ -x + 2y - 2z = 10 \\ 3x + y + z = 1 \end{cases}$

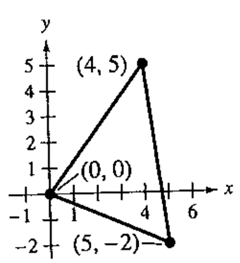
In Exercises 17–20, use a graphing utility and Cramer's Rule to solve (if possible) the system of equations.

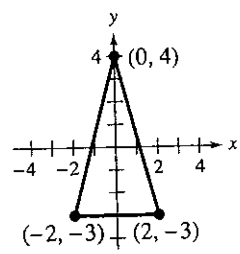
17. $\begin{cases} 3x + 3y + 5z = 1 \\ 3x + 5y + 9z = 2 \\ 5x + 9y + 17z = 4 \end{cases}$ 18. $\begin{cases} x + 2y - z = -7 \\ 2x - 2y - 2z = -8 \\ -x + 3y + 4z = 8 \end{cases}$

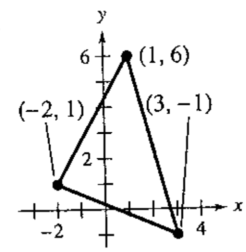
19. $\begin{cases} 2x - y + z = 5 \\ x - 2y - z = 1 \\ 3x + y + z = 4 \end{cases}$ 20. $\begin{cases} 3x - y - 3z = 1 \\ 2x + y + 2z = -4 \\ x + y - z = 5 \end{cases}$

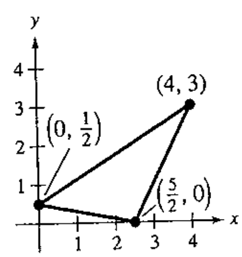
In Exercises 21–32, use a determinant and the given vertices of a triangle to find the area of the triangle.

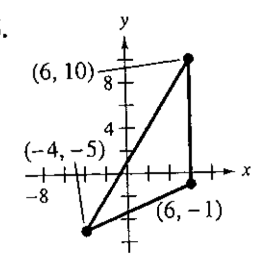
21.


22.


23.


24.


25.


26.


27. $(-2, 4), (2, 3), (-1, 5)$ 28. $(0, -2), (-1, 4), (3, 5)$

29. $(-3, 5), (2, 6), (3, -5)$ 30. $(-2, 4), (1, 5), (3, -2)$

31. $(-4, 2), \left(0, \frac{7}{2}\right), \left(3, -\frac{1}{2}\right)$ 32. $\left(\frac{9}{2}, 0\right), (2, 6), \left(0, -\frac{3}{2}\right)$

In Exercises 33 and 34, find a value of y such that the triangle with the given vertices has an area of 4 square units.

33. $(-5, 1), (0, 2), (-2, y)$ 34. $(-4, 2), (-3, 5), (-1, y)$

In Exercises 35 and 36, find a value of y such that the triangle with the given vertices has an area of 6 square units.

35. $(-2, -3), (1, -1), (-8, y)$

36. $(1, 0), (5, -3), (-3, y)$

37. **AREA OF A REGION** A large region of forest has been infested with gypsy moths. The region is roughly triangular, as shown in the figure on the next page. From the northernmost vertex A of the region, the distances to the other vertices are 25 miles south and 10 miles east (for vertex B), and 20 miles south and 28 miles east (for vertex C). Use a graphing utility to approximate the number of square miles in this region.