P.3 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1 and 2, use the graphs of f and g to answer the following.

(a) Identify the domains and ranges of f and g.

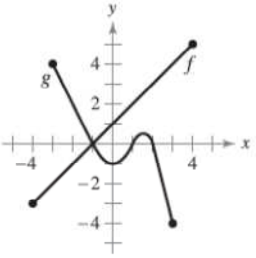
(b) Identify $f(-2)$ and $g(3)$.

(c) For what value(s) of x is $f(x) = g(x)$?

(d) Estimate the solution(s) of $f(x) = 2$.

(e) Estimate the solutions of $g(x) = 0$.

1.



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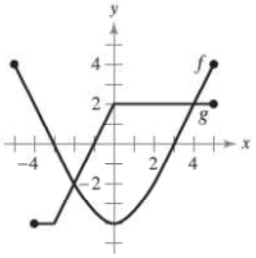


In Exercises 3–12, evaluate (if possible) the function at the given value(s) of the independent variable. Simplify the results.

3. $f(x) = 7x - 4$
(a) $f(0)$
(b) $f(-3)$
(c) $f(b)$
(d) $f(x-1)$

4. $f(x) = \sqrt{x+5}$
(a) $f(-4)$
(b) $f(11)$
(c) $f(-8)$
(d) $f(x + \Delta x)$

5. $g(x) = 5 - x^2$
(a) $g(0)$
(b) $g(\sqrt{5})$
(c) $g(-2)$
(d) $g(t-1)$

6. $g(x) = x^2(x-4)$
(a) $g(4)$
(b) $g(\frac{3}{2})$
(c) $g(c)$
(d) $g(t+4)$

7. $f(x) = \cos 2x$
(a) $f(0)$
(b) $f(-\pi/4)$
(c) $f(\pi/3)$

8. $f(x) = \sin x$
(a) $f(\pi)$
(b) $f(5\pi/4)$
(c) $f(2\pi/3)$

9. $f(x) = x^3$
$$\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

10. $f(x) = 3x - 1$
$$\frac{f(x) - f(1)}{x - 1}$$

11. $f(x) = \dfrac{1}{\sqrt{x-1}}$
$$\frac{f(x) - f(2)}{x - 2}$$

12. $f(x) = x^3 - x$
$$\frac{f(x) - f(1)}{x - 1}$$

In Exercises 13–20, find the domain and range of the function.

13. $f(x) = 4x^2$

14. $g(x) = x^2 - 5$

15. $g(x) = \sqrt{6x}$

16. $h(x) = -\sqrt{x+3}$

17. $f(t) = \sec \dfrac{\pi t}{4}$

18. $h(t) = \cot t$

19. $f(x) = \dfrac{3}{x}$

20. $g(x) = \dfrac{2}{x-1}$

In Exercises 21–26, find the domain of the function.

21. $f(x) = \sqrt{x} + \sqrt{1-x}$

22. $f(x) = \sqrt{x^2 - 3x + 2}$

23. $g(x) = \dfrac{2}{1 - \cos x}$

24. $h(x) = \dfrac{1}{\sin x - \frac{1}{2}}$

25. $f(x) = \dfrac{1}{|x+3|}$

26. $g(x) = \dfrac{1}{|x^2 - 4|}$

In Exercises 27–30, evaluate the function as indicated. Determine its domain and range.

27. $f(x) = \begin{cases} 2x+1, & x < 0 \\ 2x+2, & x \ge 0 \end{cases}$
(a) $f(-1)$ (b) $f(0)$ (c) $f(2)$ (d) $f(t^2 + 1)$

28. $f(x) = \begin{cases} x^2+2, & x \le 1 \\ 2x^2+2, & x > 1 \end{cases}$
(a) $f(-2)$ (b) $f(0)$ (c) $f(1)$ (d) $f(s^2 + 2)$

29. $f(x) = \begin{cases} |x|+1, & x < 1 \\ -x+1, & x \ge 1 \end{cases}$
(a) $f(-3)$ (b) $f(1)$ (c) $f(3)$ (d) $f(b^2 + 1)$

30. $f(x) = \begin{cases} \sqrt{x+4}, & x \le 5 \\ (x-5)^2, & x > 5 \end{cases}$
(a) $f(-3)$ (b) $f(0)$ (c) $f(5)$ (d) $f(10)$

In Exercises 31–38, sketch a graph of the function and find its domain and range. Use a graphing utility to verify your graph.

31. $f(x) = 4 - x$

32. $g(x) = \dfrac{4}{x}$

33. $h(x) = \sqrt{x-6}$

34. $f(x) = \frac{1}{4}x^3 + 3$

35. $f(x) = \sqrt{9 - x^2}$

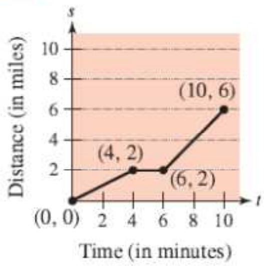
36. $f(x) = x + \sqrt{4 - x^2}$

37. $g(t) = 3 \sin \pi t$

38. $h(\theta) = -5 \cos \dfrac{\theta}{2}$

WRITING ABOUT CONCEPTS

39. The graph of the distance that a student drives in a 10-minute trip to school is shown in the figure. Give a verbal description of characteristics of the student's drive to school.

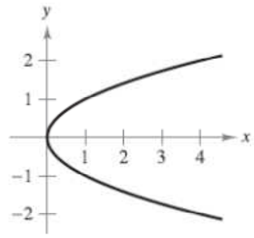


WRITING ABOUT CONCEPTS (continued)

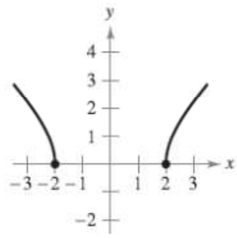
40. A student who commutes 27 miles to attend college remembers, after driving a few minutes, that a term paper that is due has been forgotten. Driving faster than usual, the student returns home, picks up the paper, and once again starts toward school. Sketch a possible graph of the student's distance from home as a function of time.

In Exercises 41–44, use the Vertical Line Test to determine whether y is a function of x. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

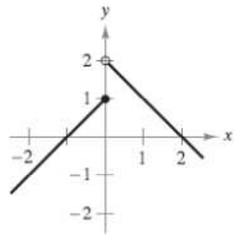
41. $x - y^2 = 0$



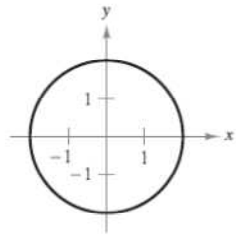
42. $\sqrt{x^2 - 4} - y = 0$



43. $y = \begin{cases} x + 1, & x \le 0 \\ -x + 2, & x > 0 \end{cases}$



44. $x^2 + y^2 = 4$



In Exercises 45–48, determine whether y is a function of x.

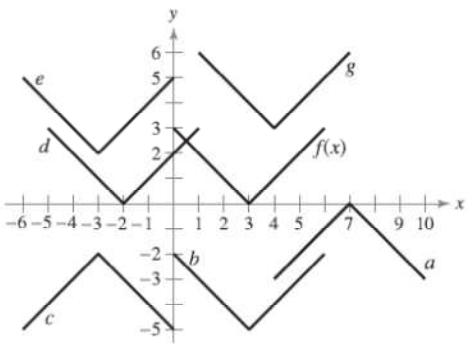
45. $x^2 + y^2 = 16$

46. $x^2 + y = 16$

47. $y^2 = x^2 - 1$

48. $x^2y - x^2 + 4y = 0$

In Exercises 49–54, use the graph of $y = f(x)$ to match the function with its graph.



49. $y = f(x + 5)$

50. $y = f(x) - 5$

51. $y = -f(-x) - 2$

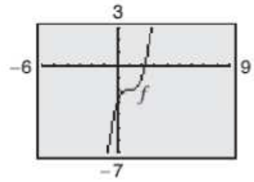
52. $y = -f(x - 4)$

53. $y = f(x + 6) + 2$

54. $y = f(x - 1) + 3$

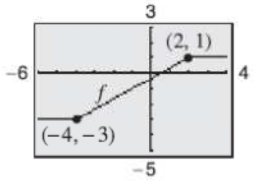
55. Use the graph of f shown in the figure to sketch the graph of each function. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

(a) $f(x + 3)$ (b) $f(x - 1)$
(c) $f(x) + 2$ (d) $f(x) - 4$
(e) $3f(x)$ (f) $\frac{1}{4}f(x)$



56. Use the graph of f shown in the figure to sketch the graph of each function. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

(a) $f(x - 4)$ (b) $f(x + 2)$
(c) $f(x) + 4$ (d) $f(x) - 1$
(e) $2f(x)$ (f) $\frac{1}{2}f(x)$



57. Use the graph of $f(x) = \sqrt{x}$ to sketch the graph of each function. In each case, describe the transformation.

(a) $y = \sqrt{x} + 2$ (b) $y = -\sqrt{x}$ (c) $y = \sqrt{x - 2}$

58. Specify a sequence of transformations that will yield each graph of h from the graph of the function $f(x) = \sin x$.

(a) $h(x) = \sin\left(x + \dfrac{\pi}{2}\right) + 1$ (b) $h(x) = -\sin(x - 1)$

59. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$, evaluate each expression.

(a) $f(g(1))$ (b) $g(f(1))$ (c) $g(f(0))$
(d) $f(g(-4))$ (e) $f(g(x))$ (f) $g(f(x))$

60. Given $f(x) = \sin x$ and $g(x) = \pi x$, evaluate each expression.

(a) $f(g(2))$ (b) $f\left(g\left(\dfrac{1}{2}\right)\right)$ (c) $g(f(0))$
(d) $g\left(f\left(\dfrac{\pi}{4}\right)\right)$ (e) $f(g(x))$ (f) $g(f(x))$

In Exercises 61–64, find the composite functions $(f \circ g)$ and $(g \circ f)$. What is the domain of each composite function? Are the two composite functions equal?

61. $f(x) = x^2$
$g(x) = \sqrt{x}$

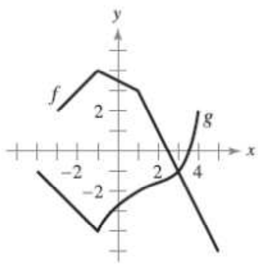
62. $f(x) = x^2 - 1$
$g(x) = \cos x$

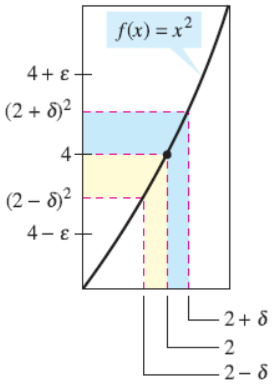
63. $f(x) = \dfrac{3}{x}$
$g(x) = x^2 - 1$

64. $f(x) = \dfrac{1}{x}$
$g(x) = \sqrt{x + 2}$

65. Use the graphs of f and g to evaluate each expression. If the result is undefined, explain why.

(a) $(f \circ g)(3)$ (b) $g(f(2))$
(c) $g(f(5))$ (d) $(f \circ g)(-3)$
(e) $(g \circ f)(-1)$ (f) $f(g(-1))$





The limit of $f(x)$ as x approaches 2 is 4.
Figure 1.15

EXAMPLE 8 Using the ε-δ Definition of Limit

Use the ε-δ definition of limit to prove that

$$\lim_{x \to 2} x^2 = 4.$$

Solution You must show that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that

$$|x^2 - 4| < \varepsilon \text{ whenever } 0 < |x - 2| < \delta.$$

To find an appropriate δ, begin by writing $|x^2 - 4| = |x - 2||x + 2|$. For all x in the interval $(1, 3)$, $x + 2 < 5$ and thus $|x + 2| < 5$. So, letting δ be the minimum of $\varepsilon/5$ and 1, it follows that, whenever $0 < |x - 2| < \delta$, you have

$$|x^2 - 4| = |x - 2||x + 2| < \left(\frac{\varepsilon}{5}\right)(5) = \varepsilon$$

as shown in Figure 1.15. ■

Throughout this chapter you will use the ε-δ definition of limit primarily to prove theorems about limits and to establish the existence or nonexistence of particular types of limits. For *finding* limits, you will learn techniques that are easier to use than the ε-δ definition of limit.

1.2 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–8, complete the table and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result.

1. $\lim_{x \to 4} \dfrac{x - 4}{x^2 - 3x - 4}$

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$						

2. $\lim_{x \to 2} \dfrac{x - 2}{x^2 - 4}$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$						

3. $\lim_{x \to 0} \dfrac{\sqrt{x + 6} - \sqrt{6}}{x}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

4. $\lim_{x \to -5} \dfrac{\sqrt{4 - x} - 3}{x + 5}$

x	-5.1	-5.01	-5.001	-4.999	-4.99	-4.9
$f(x)$						

5. $\lim_{x \to 3} \dfrac{[1/(x + 1)] - (1/4)}{x - 3}$

x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$						

6. $\lim_{x \to 4} \dfrac{[x/(x + 1)] - (4/5)}{x - 4}$

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$						

7. $\lim_{x \to 0} \dfrac{\sin x}{x}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

8. $\lim_{x \to 0} \dfrac{\cos x - 1}{x}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

In Exercises 9–14, create a table of values for the function and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result.

9. $\lim_{x\to 1} \frac{x-2}{x^2+x-6}$

10. $\lim_{x\to -3} \frac{x+3}{x^2+7x+12}$

11. $\lim_{x\to 1} \frac{x^4-1}{x^6-1}$

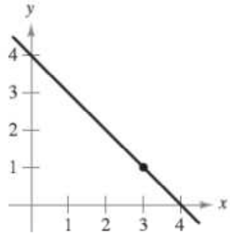
12. $\lim_{x\to -2} \frac{x^3+8}{x+2}$

13. $\lim_{x\to 0} \frac{\sin 2x}{x}$

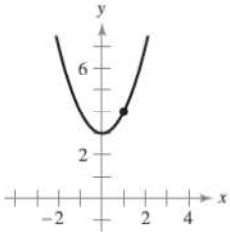
14. $\lim_{x\to 0} \frac{\tan x}{\tan 2x}$

In Exercises 15–24, use the graph to find the limit (if it exists). If the limit does not exist, explain why.

15. $\lim_{x\to 3} (4-x)$

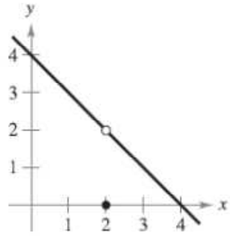


16. $\lim_{x\to 1} (x^2+3)$



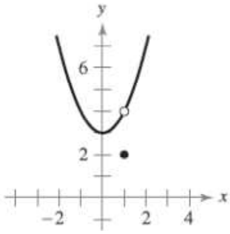
17. $\lim_{x\to 2} f(x)$

$$f(x) = \begin{cases} 4-x, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

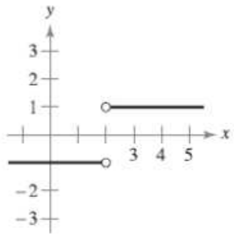


18. $\lim_{x\to 1} f(x)$

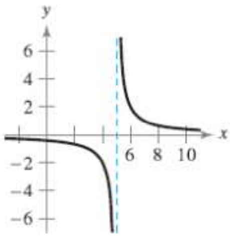
$$f(x) = \begin{cases} x^2+3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$



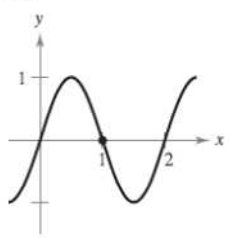
19. $\lim_{x\to 2} \frac{|x-2|}{x-2}$



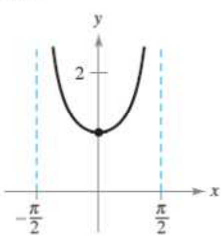
20. $\lim_{x\to 5} \frac{2}{x-5}$



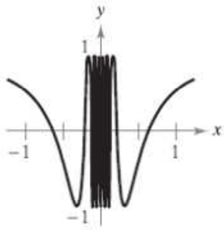
21. $\lim_{x\to 1} \sin \pi x$



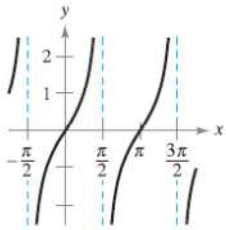
22. $\lim_{x\to 0} \sec x$



23. $\lim_{x\to 0} \cos \frac{1}{x}$



24. $\lim_{x\to \pi/2} \tan x$



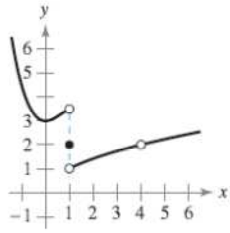
In Exercises 25 and 26, use the graph of the function f to decide whether the value of the given quantity exists. If it does, find it. If not, explain why.

25. (a) $f(1)$

(b) $\lim_{x\to 1} f(x)$

(c) $f(4)$

(d) $\lim_{x\to 4} f(x)$



26. (a) $f(-2)$

(b) $\lim_{x\to -2} f(x)$

(c) $f(0)$

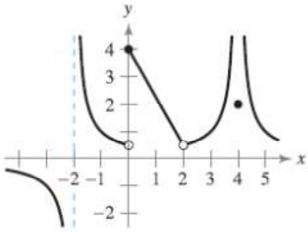
(d) $\lim_{x\to 0} f(x)$

(e) $f(2)$

(f) $\lim_{x\to 2} f(x)$

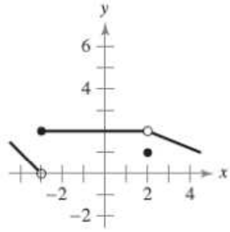
(g) $f(4)$

(h) $\lim_{x\to 4} f(x)$



In Exercises 27 and 28, use the graph of f to identify the values of c for which $\lim_{x\to c} f(x)$ exists.

27.



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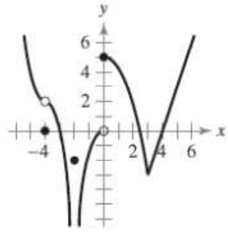


In Exercises 29 and 30, sketch the graph of f. Then identify the values of c for which $\lim_{x\to c} f(x)$ exists.

29. $f(x) = \begin{cases} x^2, & x \leq 2 \\ 8-2x, & 2 < x < 4 \\ 4, & x \geq 4 \end{cases}$

30. $f(x) = \begin{cases} \sin x, & x < 0 \\ 1-\cos x, & 0 \leq x \leq \pi \\ \cos x, & x > \pi \end{cases}$

In Exercises 31 and 32, sketch a graph of a function f that satisfies the given values. (There are many correct answers.)

31. $f(0)$ is undefined.

$\lim_{x \to 0} f(x) = 4$

$f(2) = 6$

$\lim_{x \to 2} f(x) = 3$

32. $f(-2) = 0$

$f(2) = 0$

$\lim_{x \to -2} f(x) = 0$

$\lim_{x \to 2} f(x)$ does not exist.

33. ***Modeling Data*** For a long distance phone call, a hotel charges \$9.99 for the first minute and \$0.79 for each additional minute or fraction thereof. A formula for the cost is given by

$$C(t) = 9.99 - 0.79[\![-(t-1)]\!]$$

where t is the time in minutes.

(*Note:* $[\![x]\!]$ = greatest integer n such that $n \le x$. For example, $[\![3.2]\!] = 3$ and $[\![-1.6]\!] = -2$.)

(a) Use a graphing utility to graph the cost function for $0 < t \le 6$.

(b) Use the graph to complete the table and observe the behavior of the function as t approaches 3.5. Use the graph and the table to find

$$\lim_{t \to 3.5} C(t).$$

t	3	3.3	3.4	3.5	3.6	3.7	4
C				?			

(c) Use the graph to complete the table and observe the behavior of the function as t approaches 3.

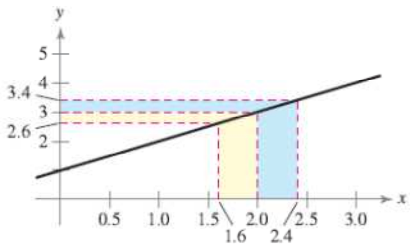
t	2	2.5	2.9	3	3.1	3.5	4
C				?			

Does the limit of $C(t)$ as t approaches 3 exist? Explain.

34. Repeat Exercise 33 for

$$C(t) = 5.79 - 0.99[\![-(t-1)]\!].$$

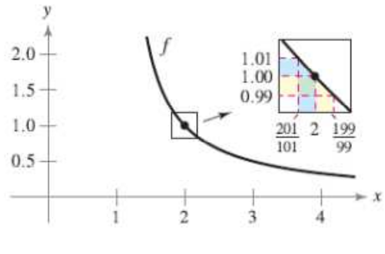
35. The graph of $f(x) = x + 1$ is shown in the figure. Find δ such that if $0 < |x - 2| < \delta$ then $|f(x) - 3| < 0.4$.



36. The graph of

$$f(x) = \frac{1}{x-1}$$

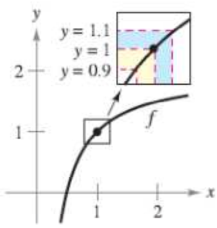
is shown in the figure. Find δ such that if $0 < |x - 2| < \delta$ then $|f(x) - 1| < 0.01$.



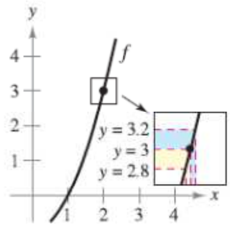
37. The graph of

$$f(x) = 2 - \frac{1}{x}$$

is shown in the figure. Find δ such that if $0 < |x - 1| < \delta$ then $|f(x) - 1| < 0.1$.



38. The graph of $f(x) = x^2 - 1$ is shown in the figure. Find δ such that if $0 < |x - 2| < \delta$ then $|f(x) - 3| < 0.2$.



In Exercises 39–42, find the limit L. Then find $\delta > 0$ such that $|f(x) - L| < 0.01$ whenever $0 < |x - c| < \delta$.

39. $\lim_{x \to 2} (3x + 2)$

40. $\lim_{x \to 4} \left(4 - \frac{x}{2}\right)$

41. $\lim_{x \to 2} (x^2 - 3)$

42. $\lim_{x \to 5} (x^2 + 4)$

The symbol indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system. The solutions of other exercises may also be facilitated by use of appropriate technology.

In Exercises 43–54, find the limit L. Then use the ε-δ definition to prove that the limit is L.

43. $\lim_{x\to 4} (x + 2)$

44. $\lim_{x\to -3} (2x + 5)$

45. $\lim_{x\to -4} \left(\frac{1}{2}x - 1\right)$

46. $\lim_{x\to 1} \left(\frac{2}{5}x + 7\right)$

47. $\lim_{x\to 6} 3$

48. $\lim_{x\to 2} (-1)$

49. $\lim_{x\to 0} \sqrt[3]{x}$

50. $\lim_{x\to 4} \sqrt{x}$

51. $\lim_{x\to -5} |x - 5|$

52. $\lim_{x\to 6} |x - 6|$

53. $\lim_{x\to 1} (x^2 + 1)$

54. $\lim_{x\to -3} (x^2 + 3x)$

55. What is the limit of $f(x) = 4$ as x approaches π?

56. What is the limit of $g(x) = x$ as x approaches π?

Writing **In Exercises 57–60, use a graphing utility to graph the function and estimate the limit (if it exists). What is the domain of the function? Can you detect a possible error in determining the domain of a function solely by analyzing the graph generated by a graphing utility? Write a short paragraph about the importance of examining a function analytically as well as graphically.**

57. $f(x) = \dfrac{\sqrt{x+5} - 3}{x - 4}$

$\lim_{x\to 4} f(x)$

58. $f(x) = \dfrac{x - 3}{x^2 - 4x + 3}$

$\lim_{x\to 3} f(x)$

59. $f(x) = \dfrac{x - 9}{\sqrt{x} - 3}$

$\lim_{x\to 9} f(x)$

60. $f(x) = \dfrac{x - 3}{x^2 - 9}$

$\lim_{x\to 3} f(x)$

WRITING ABOUT CONCEPTS

61. Write a brief description of the meaning of the notation

$\lim_{x\to 8} f(x) = 25.$

62. The definition of limit on page 52 requires that f is a function defined on an open interval containing c, except possibly at c. Why is this requirement necessary?

63. Identify three types of behavior associated with the nonexistence of a limit. Illustrate each type with a graph of a function.

CAPSTONE

64. (a) If $f(2) = 4$, can you conclude anything about the limit of $f(x)$ as x approaches 2? Explain your reasoning.

(b) If the limit of $f(x)$ as x approaches 2 is 4, can you conclude anything about $f(2)$? Explain your reasoning.

65. ***Jewelry*** A jeweler resizes a ring so that its inner circumference is 6 centimeters.

(a) What is the radius of the ring?

(b) If the ring's inner circumference can vary between 5.5 centimeters and 6.5 centimeters, how can the radius vary?

(c) Use the ε-δ definition of limit to describe this situation. Identify ε and δ.

66. ***Sports*** A sporting goods manufacturer designs a golf ball having a volume of 2.48 cubic inches.

(a) What is the radius of the golf ball?

(b) If the ball's volume can vary between 2.45 cubic inches and 2.51 cubic inches, how can the radius vary?

(c) Use the ε-δ definition of limit to describe this situation. Identify ε and δ.

67. Consider the function $f(x) = (1 + x)^{1/x}$. Estimate the limit

$$\lim_{x\to 0} (1 + x)^{1/x}$$

by evaluating f at x-values near 0. Sketch the graph of f.

68. Consider the function

$$f(x) = \frac{|x + 1| - |x - 1|}{x}.$$

Estimate

$$\lim_{x\to 0} \frac{|x + 1| - |x - 1|}{x}$$

by evaluating f at x-values near 0. Sketch the graph of f.

69. ***Graphical Analysis*** The statement

$$\lim_{x\to 2} \frac{x^2 - 4}{x - 2} = 4$$

means that for each $\varepsilon > 0$ there corresponds a $\delta > 0$ such that if $0 < |x - 2| < \delta$, then

$$\left|\frac{x^2 - 4}{x - 2} - 4\right| < \varepsilon.$$

If $\varepsilon = 0.001$, then

$$\left|\frac{x^2 - 4}{x - 2} - 4\right| < 0.001.$$

Use a graphing utility to graph each side of this inequality. Use the *zoom* feature to find an interval $(2 - \delta, 2 + \delta)$ such that the graph of the left side is below the graph of the right side of the inequality.

1.3 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–4, use a graphing utility to graph the function and visually estimate the limits.

1. $h(x) = -x^2 + 4x$
   (a) $\lim_{x\to 4} h(x)$
   (b) $\lim_{x\to -1} h(x)$

2. $g(x) = \dfrac{12(\sqrt{x} - 3)}{x - 9}$
   (a) $\lim_{x\to 4} g(x)$
   (b) $\lim_{x\to 0} g(x)$

3. $f(x) = x \cos x$
   (a) $\lim_{x\to 0} f(x)$
   (b) $\lim_{x\to \pi/3} f(x)$

4. $f(t) = t|t - 4|$
   (a) $\lim_{t\to 4} f(t)$
   (b) $\lim_{t\to -1} f(t)$

In Exercises 5–22, find the limit.

5. $\lim_{x\to 2} x^3$
6. $\lim_{x\to -2} x^4$
7. $\lim_{x\to 0} (2x - 1)$
8. $\lim_{x\to -3} (3x + 2)$
9. $\lim_{x\to -3} (x^2 + 3x)$
10. $\lim_{x\to 1} (-x^2 + 1)$
11. $\lim_{x\to -3} (2x^2 + 4x + 1)$
12. $\lim_{x\to 1} (3x^3 - 2x^2 + 4)$
13. $\lim_{x\to 3} \sqrt{x + 1}$
14. $\lim_{x\to 4} \sqrt[3]{x + 4}$
15. $\lim_{x\to -4} (x + 3)^2$
16. $\lim_{x\to 0} (2x - 1)^3$
17. $\lim_{x\to 2} \dfrac{1}{x}$
18. $\lim_{x\to -3} \dfrac{2}{x + 2}$
19. $\lim_{x\to 1} \dfrac{x}{x^2 + 4}$
20. $\lim_{x\to 1} \dfrac{2x - 3}{x + 5}$
21. $\lim_{x\to 7} \dfrac{3x}{\sqrt{x + 2}}$
22. $\lim_{x\to 2} \dfrac{\sqrt{x + 2}}{x - 4}$

In Exercises 23–26, find the limits.

23. $f(x) = 5 - x,\ g(x) = x^3$
    (a) $\lim_{x\to 1} f(x)$ (b) $\lim_{x\to 4} g(x)$ (c) $\lim_{x\to 1} g(f(x))$

24. $f(x) = x + 7,\ g(x) = x^2$
    (a) $\lim_{x\to -3} f(x)$ (b) $\lim_{x\to 4} g(x)$ (c) $\lim_{x\to -3} g(f(x))$

25. $f(x) = 4 - x^2,\ g(x) = \sqrt{x + 1}$
    (a) $\lim_{x\to 1} f(x)$ (b) $\lim_{x\to 3} g(x)$ (c) $\lim_{x\to 1} g(f(x))$

26. $f(x) = 2x^2 - 3x + 1,\ g(x) = \sqrt[3]{x + 6}$
    (a) $\lim_{x\to 4} f(x)$ (b) $\lim_{x\to 21} g(x)$ (c) $\lim_{x\to 4} g(f(x))$

In Exercises 27–36, find the limit of the trigonometric function.

27. $\lim_{x\to \pi/2} \sin x$
28. $\lim_{x\to \pi} \tan x$
29. $\lim_{x\to 1} \cos \dfrac{\pi x}{3}$
30. $\lim_{x\to 2} \sin \dfrac{\pi x}{2}$
31. $\lim_{x\to 0} \sec 2x$
32. $\lim_{x\to \pi} \cos 3x$
33. $\lim_{x\to 5\pi/6} \sin x$
34. $\lim_{x\to 5\pi/3} \cos x$
35. $\lim_{x\to 3} \tan\left(\dfrac{\pi x}{4}\right)$
36. $\lim_{x\to 7} \sec\left(\dfrac{\pi x}{6}\right)$

In Exercises 37–40, use the information to evaluate the limits.

37. $\lim_{x\to c} f(x) = 3$
    $\lim_{x\to c} g(x) = 2$
    (a) $\lim_{x\to c} [5g(x)]$
    (b) $\lim_{x\to c} [f(x) + g(x)]$
    (c) $\lim_{x\to c} [f(x)g(x)]$
    (d) $\lim_{x\to c} \dfrac{f(x)}{g(x)}$

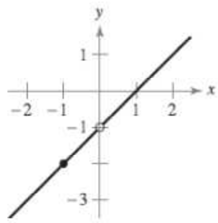
38. $\lim_{x\to c} f(x) = \frac{3}{2}$
    $\lim_{x\to c} g(x) = \frac{1}{2}$
    (a) $\lim_{x\to c} [4f(x)]$
    (b) $\lim_{x\to c} [f(x) + g(x)]$
    (c) $\lim_{x\to c} [f(x)g(x)]$
    (d) $\lim_{x\to c} \dfrac{f(x)}{g(x)}$

39. $\lim_{x\to c} f(x) = 4$
    (a) $\lim_{x\to c} [f(x)]^3$
    (b) $\lim_{x\to c} \sqrt{f(x)}$
    (c) $\lim_{x\to c} [3f(x)]$
    (d) $\lim_{x\to c} [f(x)]^{3/2}$

40. $\lim_{x\to c} f(x) = 27$
    (a) $\lim_{x\to c} \sqrt[3]{f(x)}$
    (b) $\lim_{x\to c} \dfrac{f(x)}{18}$
    (c) $\lim_{x\to c} [f(x)]^2$
    (d) $\lim_{x\to c} [f(x)]^{2/3}$

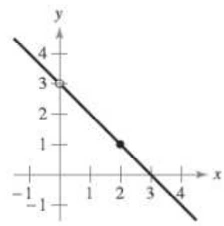
In Exercises 41–44, use the graph to determine the limit visually (if it exists). Write a simpler function that agrees with the given function at all but one point.

41. $g(x) = \dfrac{x^2 - x}{x}$



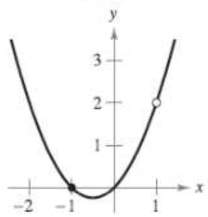
(a) $\lim_{x\to 0} g(x)$
(b) $\lim_{x\to -1} g(x)$

42. $h(x) = \dfrac{-x^2 + 3x}{x}$



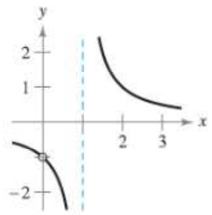
(a) $\lim_{x\to 2} h(x)$
(b) $\lim_{x\to 0} h(x)$

43. $g(x) = \dfrac{x^3 - x}{x - 1}$



(a) $\lim_{x\to 1} g(x)$
(b) $\lim_{x\to -1} g(x)$

44. $f(x) = \dfrac{x}{x^2 - x}$



(a) $\lim_{x\to 1} f(x)$
(b) $\lim_{x\to 0} f(x)$

In Exercises 45–48, find the limit of the function (if it exists). Write a simpler function that agrees with the given function at all but one point. Use a graphing utility to confirm your result.

45. $\lim_{x \to -1} \frac{x^2 - 1}{x + 1}$

46. $\lim_{x \to -1} \frac{2x^2 - x - 3}{x + 1}$

47. $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

48. $\lim_{x \to -1} \frac{x^3 + 1}{x + 1}$

In Exercises 49–64, find the limit (if it exists).

49. $\lim_{x \to 0} \frac{x}{x^2 - x}$

50. $\lim_{x \to 0} \frac{3x}{x^2 + 2x}$

51. $\lim_{x \to 4} \frac{x - 4}{x^2 - 16}$

52. $\lim_{x \to 3} \frac{3 - x}{x^2 - 9}$

53. $\lim_{x \to -3} \frac{x^2 + x - 6}{x^2 - 9}$

54. $\lim_{x \to 4} \frac{x^2 - 5x + 4}{x^2 - 2x - 8}$

55. $\lim_{x \to 4} \frac{\sqrt{x + 5} - 3}{x - 4}$

56. $\lim_{x \to 3} \frac{\sqrt{x + 1} - 2}{x - 3}$

57. $\lim_{x \to 0} \frac{\sqrt{x + 5} - \sqrt{5}}{x}$

58. $\lim_{x \to 0} \frac{\sqrt{2 + x} - \sqrt{2}}{x}$

59. $\lim_{x \to 0} \frac{[1/(3 + x)] - (1/3)}{x}$

60. $\lim_{x \to 0} \frac{[1/(x + 4)] - (1/4)}{x}$

61. $\lim_{\Delta x \to 0} \frac{2(x + \Delta x) - 2x}{\Delta x}$

62. $\lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$

63. $\lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - 2(x + \Delta x) + 1 - (x^2 - 2x + 1)}{\Delta x}$

64. $\lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x}$

In Exercises 65–76, determine the limit of the trigonometric function (if it exists).

65. $\lim_{x \to 0} \frac{\sin x}{5x}$

66. $\lim_{x \to 0} \frac{3(1 - \cos x)}{x}$

67. $\lim_{x \to 0} \frac{\sin x(1 - \cos x)}{x^2}$

68. $\lim_{\theta \to 0} \frac{\cos \theta \tan \theta}{\theta}$

69. $\lim_{x \to 0} \frac{\sin^2 x}{x}$

70. $\lim_{x \to 0} \frac{\tan^2 x}{x}$

71. $\lim_{h \to 0} \frac{(1 - \cos h)^2}{h}$

72. $\lim_{\phi \to \pi} \phi \sec \phi$

73. $\lim_{x \to \pi/2} \frac{\cos x}{\cot x}$

74. $\lim_{x \to \pi/4} \frac{1 - \tan x}{\sin x - \cos x}$

75. $\lim_{t \to 0} \frac{\sin 3t}{2t}$

76. $\lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$ $\left[\textit{Hint}: \text{Find } \lim_{x \to 0} \left(\frac{2 \sin 2x}{2x}\right)\left(\frac{3x}{3 \sin 3x}\right).\right]$

Graphical, Numerical, and Analytic Analysis **In Exercises 77–84, use a graphing utility to graph the function and estimate the limit. Use a table to reinforce your conclusion. Then find the limit by analytic methods.**

77. $\lim_{x \to 0} \frac{\sqrt{x + 2} - \sqrt{2}}{x}$

78. $\lim_{x \to 16} \frac{4 - \sqrt{x}}{x - 16}$

79. $\lim_{x \to 0} \frac{[1/(2 + x)] - (1/2)}{x}$

80. $\lim_{x \to 2} \frac{x^5 - 32}{x - 2}$

81. $\lim_{t \to 0} \frac{\sin 3t}{t}$

82. $\lim_{x \to 0} \frac{\cos x - 1}{2x^2}$

83. $\lim_{x \to 0} \frac{\sin x^2}{x}$

84. $\lim_{x \to 0} \frac{\sin x}{\sqrt[3]{x}}$

In Exercises 85–88, find $\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$.

85. $f(x) = 3x - 2$

86. $f(x) = \sqrt{x}$

87. $f(x) = \frac{1}{x + 3}$

88. $f(x) = x^2 - 4x$

In Exercises 89 and 90, use the Squeeze Theorem to find $\lim_{x \to c} f(x)$.

89. $c = 0$

$4 - x^2 \le f(x) \le 4 + x^2$

90. $c = a$

$b - |x - a| \le f(x) \le b + |x - a|$

In Exercises 91–96, use a graphing utility to graph the given function and the equations $y = |x|$ and $y = -|x|$ in the same viewing window. Using the graphs to observe the Squeeze Theorem visually, find $\lim_{x \to 0} f(x)$.

91. $f(x) = x \cos x$

92. $f(x) = |x \sin x|$

93. $f(x) = |x| \sin x$

94. $f(x) = |x| \cos x$

95. $f(x) = x \sin \frac{1}{x}$

96. $h(x) = x \cos \frac{1}{x}$

WRITING ABOUT CONCEPTS

97. In the context of finding limits, discuss what is meant by two functions that agree at all but one point.

98. Give an example of two functions that agree at all but one point.

99. What is meant by an indeterminate form?

100. In your own words, explain the Squeeze Theorem.

101. ***Writing*** Use a graphing utility to graph

$$f(x) = x, \quad g(x) = \sin x, \quad \text{and} \quad h(x) = \frac{\sin x}{x}$$

in the same viewing window. Compare the magnitudes of $f(x)$ and $g(x)$ when x is close to 0. Use the comparison to write a short paragraph explaining why

$$\lim_{x \to 0} h(x) = 1.$$

102. ***Writing*** Use a graphing utility to graph

$$f(x) = x, \ g(x) = \sin^2 x, \ \text{and } h(x) = \frac{\sin^2 x}{x}$$

in the same viewing window. Compare the magnitudes of $f(x)$ and $g(x)$ when x is close to 0. Use the comparison to write a short paragraph explaining why $\lim_{x\to 0} h(x) = 0$.

Free-Falling Object **In Exercises 103 and 104, use the position function $s(t) = -16t^2 + 500$, which gives the height (in feet) of an object that has fallen for t seconds from a height of 500 feet. The velocity at time $t = a$ seconds is given by**

$$\lim_{t\to a} \frac{s(a) - s(t)}{a - t}.$$

103. If a construction worker drops a wrench from a height of 500 feet, how fast will the wrench be falling after 2 seconds?

104. If a construction worker drops a wrench from a height of 500 feet, when will the wrench hit the ground? At what velocity will the wrench impact the ground?

Free-Falling Object **In Exercises 105 and 106, use the position function $s(t) = -4.9t^2 + 200$, which gives the height (in meters) of an object that has fallen from a height of 200 meters. The velocity at time $t = a$ seconds is given by**

$$\lim_{t\to a} \frac{s(a) - s(t)}{a - t}.$$

105. Find the velocity of the object when $t = 3$.

106. At what velocity will the object impact the ground?

107. Find two functions f and g such that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0} [f(x) + g(x)]$ does exist.

108. Prove that if $\lim_{x\to c} f(x)$ exists and $\lim_{x\to c} [f(x) + g(x)]$ does not exist, then $\lim_{x\to c} g(x)$ does not exist.

109. Prove Property 1 of Theorem 1.1.

110. Prove Property 3 of Theorem 1.1. (You may use Property 3 of Theorem 1.2.)

111. Prove Property 1 of Theorem 1.2.

112. Prove that if $\lim_{x\to c} f(x) = 0$, then $\lim_{x\to c} |f(x)| = 0$.

113. Prove that if $\lim_{x\to c} f(x) = 0$ and $|g(x)| \le M$ for a fixed number M and all $x \ne c$, then $\lim_{x\to c} f(x)g(x) = 0$.

114. (a) Prove that if $\lim_{x\to c} |f(x)| = 0$, then $\lim_{x\to c} f(x) = 0$.

(*Note:* This is the converse of Exercise 112.)

(b) Prove that if $\lim_{x\to c} f(x) = L$, then $\lim_{x\to c} |f(x)| = |L|$.

[*Hint:* Use the inequality $\big||f(x)| - |L|\big| \le |f(x) - L|$.]

115. ***Think About It*** Find a function f to show that the converse of Exercise 114(b) is not true. [*Hint:* Find a function f such that $\lim_{x\to c} |f(x)| = |L|$ but $\lim_{x\to c} f(x)$ does not exist.]

CAPSTONE

116. Let $f(x) = \begin{cases} 3, & x \ne 2 \\ 5, & x = 2 \end{cases}$. Find $\lim_{x\to 2} f(x)$.

True or False? **In Exercises 117–122, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

117. $\lim_{x\to 0} \frac{|x|}{x} = 1$

118. $\lim_{x\to \pi} \frac{\sin x}{x} = 1$

119. If $f(x) = g(x)$ for all real numbers other than $x = 0$, and $\lim_{x\to 0} f(x) = L$, then $\lim_{x\to 0} g(x) = L$.

120. If $\lim_{x\to c} f(x) = L$, then $f(c) = L$.

121. $\lim_{x\to 2} f(x) = 3$, where $f(x) = \begin{cases} 3, & x \le 2 \\ 0, & x > 2 \end{cases}$

122. If $f(x) < g(x)$ for all $x \ne a$, then

$\lim_{x\to a} f(x) < \lim_{x\to a} g(x)$.

123. Prove the second part of Theorem 1.9.

$$\lim_{x\to 0} \frac{1 - \cos x}{x} = 0$$

124. Let $f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$

and

$$g(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ x, & \text{if } x \text{ is irrational.} \end{cases}$$

Find (if possible) $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$.

125. ***Graphical Reasoning*** Consider $f(x) = \dfrac{\sec x - 1}{x^2}$.

(a) Find the domain of f.

(b) Use a graphing utility to graph f. Is the domain of f obvious from the graph? If not, explain.

(c) Use the graph of f to approximate $\lim_{x\to 0} f(x)$.

(d) Confirm your answer to part (c) analytically.

126. ***Approximation***

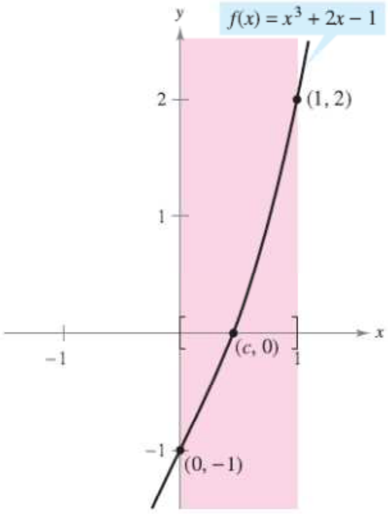
(a) Find $\lim_{x\to 0} \dfrac{1 - \cos x}{x^2}$.

(b) Use your answer to part (a) to derive the approximation $\cos x \approx 1 - \frac{1}{2}x^2$ for x near 0.

(c) Use your answer to part (b) to approximate $\cos(0.1)$.

(d) Use a calculator to approximate $\cos(0.1)$ to four decimal places. Compare the result with part (c).

127. ***Think About It*** When using a graphing utility to generate a table to approximate $\lim_{x\to 0} [(\sin x)/x]$, a student concluded that the limit was 0.01745 rather than 1. Determine the probable cause of the error.



f is continuous on $[0, 1]$ with $f(0) < 0$ and $f(1) > 0$.
Figure 1.37

EXAMPLE 8 An Application of the Intermediate Value Theorem

Use the Intermediate Value Theorem to show that the polynomial function $f(x) = x^3 + 2x - 1$ has a zero in the interval $[0, 1]$.

Solution Note that f is continuous on the closed interval $[0, 1]$. Because

$$f(0) = 0^3 + 2(0) - 1 = -1 \quad \text{and} \quad f(1) = 1^3 + 2(1) - 1 = 2$$

it follows that $f(0) < 0$ and $f(1) > 0$. You can therefore apply the Intermediate Value Theorem to conclude that there must be some c in $[0, 1]$ such that

$$f(c) = 0 \qquad f \text{ has a zero in the closed interval } [0, 1].$$

as shown in Figure 1.37. ■

The **bisection method** for approximating the real zeros of a continuous function is similar to the method used in Example 8. If you know that a zero exists in the closed interval $[a, b]$, the zero must lie in the interval $[a, (a + b)/2]$ or $[(a + b)/2, b]$. From the sign of $f([a + b]/2)$, you can determine which interval contains the zero. By repeatedly bisecting the interval, you can "close in" on the zero of the function.

TECHNOLOGY You can also use the *zoom* feature of a graphing utility to approximate the real zeros of a continuous function. By repeatedly zooming in on the point where the graph crosses the x-axis, and adjusting the x-axis scale, you can approximate the zero of the function to any desired accuracy. The zero of $x^3 + 2x - 1$ is approximately 0.453, as shown in Figure 1.38.

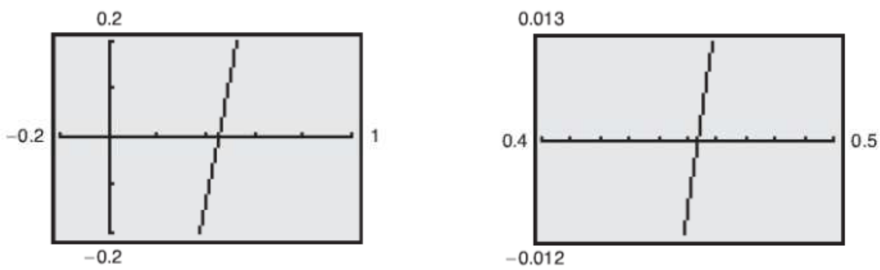


Figure 1.38 Zooming in on the zero of $f(x) = x^3 + 2x - 1$

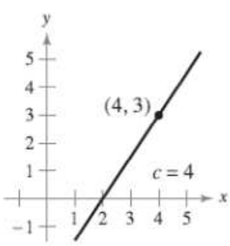
1.4 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–6, use the graph to determine the limit, and discuss the continuity of the function.

(a) $\lim\limits_{x \to c^+} f(x)$ (b) $\lim\limits_{x \to c^-} f(x)$ (c) $\lim\limits_{x \to c} f(x)$

1.



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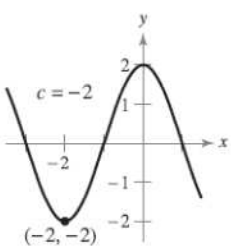

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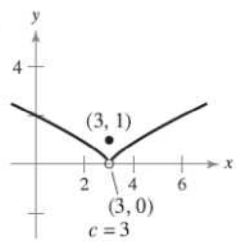

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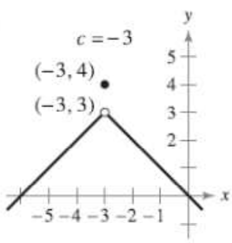
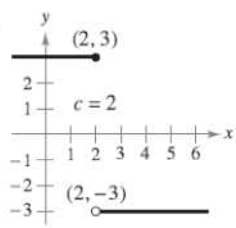
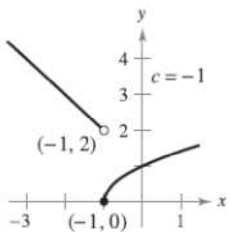


5.



6.



In Exercises 7–26, find the limit (if it exists). If it does not exist, explain why.

7. $\lim_{x\to 8^+} \dfrac{1}{x+8}$

8. $\lim_{x\to 5^-} \dfrac{-3}{x+5}$

9. $\lim_{x\to 5^+} \dfrac{x-5}{x^2-25}$

10. $\lim_{x\to 2^+} \dfrac{2-x}{x^2-4}$

11. $\lim_{x\to -3^-} \dfrac{x}{\sqrt{x^2-9}}$

12. $\lim_{x\to 9^-} \dfrac{\sqrt{x}-3}{x-9}$

13. $\lim_{x\to 0^-} \dfrac{|x|}{x}$

14. $\lim_{x\to 10^+} \dfrac{|x-10|}{x-10}$

15. $\lim_{\Delta x\to 0^-} \dfrac{\dfrac{1}{x+\Delta x} - \dfrac{1}{x}}{\Delta x}$

16. $\lim_{\Delta x\to 0^+} \dfrac{(x+\Delta x)^2 + x + \Delta x - (x^2+x)}{\Delta x}$

17. $\lim_{x\to 3^-} f(x)$, where $f(x) = \begin{cases} \dfrac{x+2}{2}, & x \le 3 \\ \dfrac{12-2x}{3}, & x > 3 \end{cases}$

18. $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \ge 2 \end{cases}$

19. $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} x^3 + 1, & x < 1 \\ x + 1, & x \ge 1 \end{cases}$

20. $\lim_{x\to 1^+} f(x)$, where $f(x) = \begin{cases} x, & x \le 1 \\ 1 - x, & x > 1 \end{cases}$

21. $\lim_{x\to \pi} \cot x$

22. $\lim_{x\to \pi/2} \sec x$

23. $\lim_{x\to 4^-} (5[\![x]\!] - 7)$

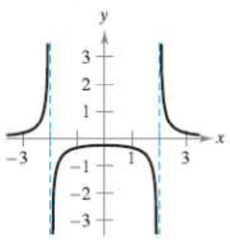
24. $\lim_{x\to 2^+} (2x - [\![x]\!])$

25. $\lim_{x\to 3} (2 - [\![-x]\!])$

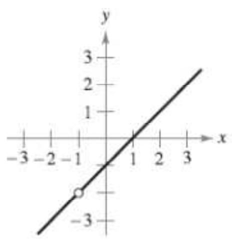
26. $\lim_{x\to 1} \left(1 - \left[\!\!\left[-\dfrac{x}{2}\right]\!\!\right]\right)$

In Exercises 27–30, discuss the continuity of each function.

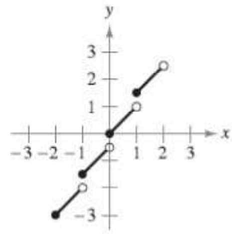
27. $f(x) = \dfrac{1}{x^2-4}$



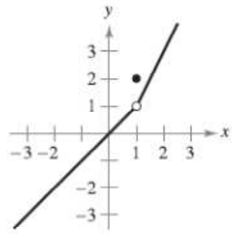
28. $f(x) = \dfrac{x^2-1}{x+1}$



29. $f(x) = \frac{1}{2}[\![x]\!] + x$



30. $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$



In Exercises 31–34, discuss the continuity of the function on the closed interval.

Function	*Interval*
31. $g(x) = \sqrt{49 - x^2}$	$[-7, 7]$
32. $f(t) = 3 - \sqrt{9 - t^2}$	$[-3, 3]$
33. $f(x) = \begin{cases} 3 - x, & x \le 0 \\ 3 + \frac{1}{2}x, & x > 0 \end{cases}$	$[-1, 4]$
34. $g(x) = \dfrac{1}{x^2 - 4}$	$[-1, 2]$

In Exercises 35–60, find the x-values (if any) at which f is not continuous. Which of the discontinuities are removable?

35. $f(x) = \dfrac{6}{x}$

36. $f(x) = \dfrac{3}{x-2}$

37. $f(x) = x^2 - 9$

38. $f(x) = x^2 - 2x + 1$

39. $f(x) = \dfrac{1}{4 - x^2}$

40. $f(x) = \dfrac{1}{x^2 + 1}$

41. $f(x) = 3x - \cos x$

42. $f(x) = \cos \dfrac{\pi x}{2}$

43. $f(x) = \dfrac{x}{x^2 - x}$

44. $f(x) = \dfrac{x}{x^2 - 1}$

45. $f(x) = \dfrac{x}{x^2 + 1}$

46. $f(x) = \dfrac{x - 6}{x^2 - 36}$

47. $f(x) = \dfrac{x + 2}{x^2 - 3x - 10}$

48. $f(x) = \dfrac{x - 1}{x^2 + x - 2}$

49. $f(x) = \dfrac{|x + 7|}{x + 7}$

50. $f(x) = \dfrac{|x - 8|}{x - 8}$

51. $f(x) = \begin{cases} x, & x \le 1 \\ x^2, & x > 1 \end{cases}$

52. $f(x) = \begin{cases} -2x + 3, & x < 1 \\ x^2, & x \ge 1 \end{cases}$

53. $f(x) = \begin{cases} \frac{1}{2}x + 1, & x \le 2 \\ 3 - x, & x > 2 \end{cases}$

54. $f(x) = \begin{cases} -2x, & x \le 2 \\ x^2 - 4x + 1, & x > 2 \end{cases}$

55. $f(x) = \begin{cases} \tan \dfrac{\pi x}{4}, & |x| < 1 \\ x, & |x| \ge 1 \end{cases}$

56. $f(x) = \begin{cases} \csc \dfrac{\pi x}{6}, & |x - 3| \le 2 \\ 2, & |x - 3| > 2 \end{cases}$

57. $f(x) = \csc 2x$ **58.** $f(x) = \tan \dfrac{\pi x}{2}$

59. $f(x) = [\![x - 8]\!]$ **60.** $f(x) = 5 - [\![x]\!]$

In Exercises 61 and 62, use a graphing utility to graph the function. From the graph, estimate

$$\lim_{x\to 0^+} f(x) \quad \text{and} \quad \lim_{x\to 0^-} f(x).$$

Is the function continuous on the entire real line? Explain.

61. $f(x) = \dfrac{|x^2 - 4|x}{x + 2}$ **62.** $f(x) = \dfrac{|x^2 + 4x|(x + 2)}{x + 4}$

In Exercises 63–68, find the constant *a*, or the constants *a* and *b*, such that the function is continuous on the entire real line.

63. $f(x) = \begin{cases} 3x^2, & x \ge 1 \\ ax - 4, & x < 1 \end{cases}$

64. $f(x) = \begin{cases} 3x^3, & x \le 1 \\ ax + 5, & x > 1 \end{cases}$

65. $f(x) = \begin{cases} x^3, & x \le 2 \\ ax^2, & x > 2 \end{cases}$

66. $g(x) = \begin{cases} \dfrac{4 \sin x}{x}, & x < 0 \\ a - 2x, & x \ge 0 \end{cases}$

67. $f(x) = \begin{cases} 2, & x \le -1 \\ ax + b, & -1 < x < 3 \\ -2, & x \ge 3 \end{cases}$

68. $g(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & x \ne a \\ 8, & x = a \end{cases}$

In Exercises 69–72, discuss the continuity of the composite function $h(x) = f(g(x))$.

69. $f(x) = x^2$, $g(x) = x - 1$ **70.** $f(x) = \dfrac{1}{\sqrt{x}}$, $g(x) = x - 1$

71. $f(x) = \dfrac{1}{x - 6}$, $g(x) = x^2 + 5$ **72.** $f(x) = \sin x$, $g(x) = x^2$

In Exercises 73–76, use a graphing utility to graph the function. Use the graph to determine any x-values at which the function is not continuous.

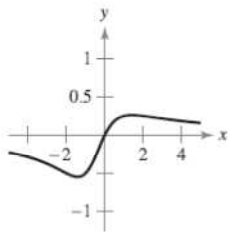
73. $f(x) = [\![x]\!] - x$ **74.** $h(x) = \dfrac{1}{x^2 - x - 2}$

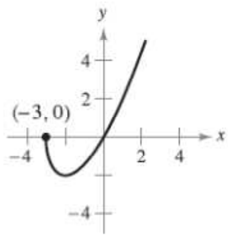
75. $g(x) = \begin{cases} x^2 - 3x, & x > 4 \\ 2x - 5, & x \le 4 \end{cases}$

76. $f(x) = \begin{cases} \dfrac{\cos x - 1}{x}, & x < 0 \\ 5x, & x \ge 0 \end{cases}$

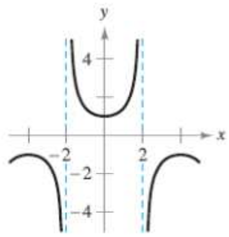
In Exercises 77–80, describe the interval(s) on which the function is continuous.

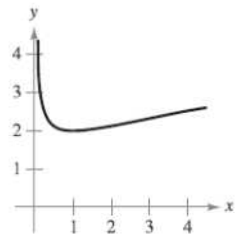
77. $f(x) = \dfrac{x}{x^2 + x + 2}$ **78.** $f(x) = x\sqrt{x + 3}$





79. $f(x) = \sec \dfrac{\pi x}{4}$ **80.** $f(x) = \dfrac{x + 1}{\sqrt{x}}$





Writing **In Exercises 81 and 82, use a graphing utility to graph the function on the interval [−4, 4]. Does the graph of the function appear to be continuous on this interval? Is the function continuous on [−4, 4]? Write a short paragraph about the importance of examining a function analytically as well as graphically.**

81. $f(x) = \dfrac{\sin x}{x}$ **82.** $f(x) = \dfrac{x^3 - 8}{x - 2}$

Writing **In Exercises 83–86, explain why the function has a zero in the given interval.**

Function	*Interval*
83. $f(x) = \frac{1}{12}x^4 - x^3 + 4$	$[1, 2]$
84. $f(x) = x^3 + 5x - 3$	$[0, 1]$
85. $f(x) = x^2 - 2 - \cos x$	$[0, \pi]$
86. $f(x) = -\dfrac{5}{x} + \tan\left(\dfrac{\pi x}{10}\right)$	$[1, 4]$

In Exercises 87–90, use the Intermediate Value Theorem and a graphing utility to approximate the zero of the function in the interval [0, 1]. Repeatedly "zoom in" on the graph of the function to approximate the zero accurate to two decimal places. Use the *zero* or *root* feature of the graphing utility to approximate the zero accurate to four decimal places.

87. $f(x) = x^3 + x - 1$

88. $f(x) = x^3 + 5x - 3$

89. $g(t) = 2 \cos t - 3t$

90. $h(\theta) = 1 + \theta - 3 \tan \theta$

In Exercises 91–94, verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem.

91. $f(x) = x^2 + x - 1, \quad [0, 5], \quad f(c) = 11$

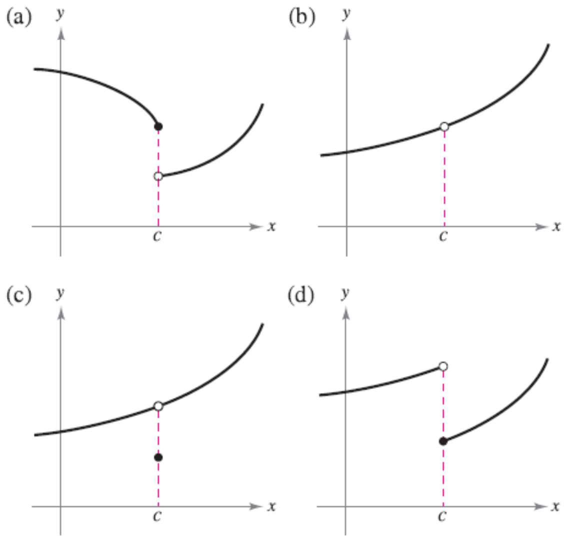
92. $f(x) = x^2 - 6x + 8, \quad [0, 3], \quad f(c) = 0$

93. $f(x) = x^3 - x^2 + x - 2, \quad [0, 3], \quad f(c) = 4$

94. $f(x) = \dfrac{x^2 + x}{x - 1}, \quad \left[\dfrac{5}{2}, 4\right], \quad f(c) = 6$

WRITING ABOUT CONCEPTS

95. State how continuity is destroyed at $x = c$ for each of the following graphs.



96. Sketch the graph of any function f such that

$$\lim_{x \to 3^+} f(x) = 1 \quad \text{and} \quad \lim_{x \to 3^-} f(x) = 0.$$

Is the function continuous at $x = 3$? Explain.

97. If the functions f and g are continuous for all real x, is $f + g$ always continuous for all real x? Is f/g always continuous for all real x? If either is not continuous, give an example to verify your conclusion.

CAPSTONE

98. Describe the difference between a discontinuity that is removable and one that is nonremovable. In your explanation, give examples of the following descriptions.

(a) A function with a nonremovable discontinuity at $x = 4$

(b) A function with a removable discontinuity at $x = -4$

(c) A function that has both of the characteristics described in parts (a) and (b)

True or False? **In Exercises 99–102, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

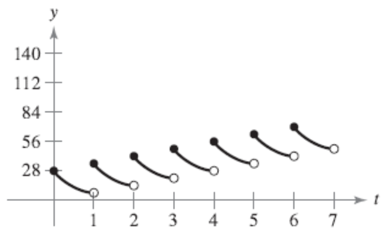
99. If $\lim_{x \to c} f(x) = L$ and $f(c) = L$, then f is continuous at c.

100. If $f(x) = g(x)$ for $x \neq c$ and $f(c) \neq g(c)$, then either f or g is not continuous at c.

101. A rational function can have infinitely many x-values at which it is not continuous.

102. The function $f(x) = |x - 1|/(x - 1)$ is continuous on $(-\infty, \infty)$.

103. ***Swimming Pool*** Every day you dissolve 28 ounces of chlorine in a swimming pool. The graph shows the amount of chlorine $f(t)$ in the pool after t days.



Estimate and interpret $\lim_{t \to 4^-} f(t)$ and $\lim_{t \to 4^+} f(t)$.

104. ***Think About It*** Describe how the functions

$$f(x) = 3 + [\![x]\!]$$

and

$$g(x) = 3 - [\![-x]\!]$$

differ.

105. ***Telephone Charges*** A long distance phone service charges \$0.40 for the first 10 minutes and \$0.05 for each additional minute or fraction thereof. Use the greatest integer function to write the cost C of a call in terms of time t (in minutes). Sketch the graph of this function and discuss its continuity.

1.5 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–4, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches 4 from the left and from the right.

1. $f(x) = \dfrac{1}{x-4}$

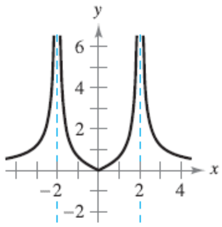
2. $f(x) = \dfrac{-1}{x-4}$

3. $f(x) = \dfrac{1}{(x-4)^2}$

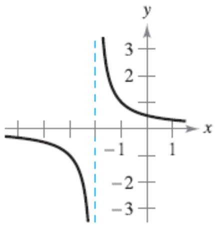
4. $f(x) = \dfrac{-1}{(x-4)^2}$

In Exercises 5–8, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -2 from the left and from the right.

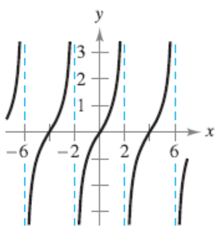
5. $f(x) = 2\left|\dfrac{x}{x^2-4}\right|$



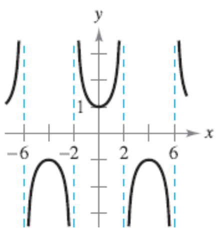
6. $f(x) = \dfrac{1}{x+2}$



7. $f(x) = \tan\dfrac{\pi x}{4}$



8. $f(x) = \sec\dfrac{\pi x}{4}$



Numerical and Graphical Analysis **In Exercises 9–12, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -3 from the left and from the right by completing the table. Use a graphing utility to graph the function to confirm your answer.**

x	-3.5	-3.1	-3.01	-3.001
$f(x)$				

x	-2.999	-2.99	-2.9	-2.5
$f(x)$				

9. $f(x) = \dfrac{1}{x^2-9}$

10. $f(x) = \dfrac{x}{x^2-9}$

11. $f(x) = \dfrac{x^2}{x^2-9}$

12. $f(x) = \sec\dfrac{\pi x}{6}$

In Exercises 13–32, find the vertical asymptotes (if any) of the graph of the function.

13. $f(x) = \dfrac{1}{x^2}$

14. $f(x) = \dfrac{4}{(x-2)^3}$

15. $f(x) = \dfrac{x^2}{x^2-4}$

16. $f(x) = \dfrac{-4x}{x^2+4}$

17. $g(t) = \dfrac{t-1}{t^2+1}$

18. $h(s) = \dfrac{2s-3}{s^2-25}$

19. $h(x) = \dfrac{x^2-2}{x^2-x-2}$

20. $g(x) = \dfrac{2+x}{x^2(1-x)}$

21. $T(t) = 1 - \dfrac{4}{t^2}$

22. $g(x) = \dfrac{\frac{1}{2}x^3 - x^2 - 4x}{3x^2 - 6x - 24}$

23. $f(x) = \dfrac{3}{x^2+x-2}$

24. $f(x) = \dfrac{4x^2+4x-24}{x^4-2x^3-9x^2+18x}$

25. $g(x) = \dfrac{x^3+1}{x+1}$

26. $h(x) = \dfrac{x^2-4}{x^3+2x^2+x+2}$

27. $f(x) = \dfrac{x^2-2x-15}{x^3-5x^2+x-5}$

28. $h(t) = \dfrac{t^2-2t}{t^4-16}$

29. $f(x) = \tan \pi x$

30. $f(x) = \sec \pi x$

31. $s(t) = \dfrac{t}{\sin t}$

32. $g(\theta) = \dfrac{\tan\theta}{\theta}$

In Exercises 33–36, determine whether the graph of the function has a vertical asymptote or a removable discontinuity at $x = -1$. Graph the function using a graphing utility to confirm your answer.

33. $f(x) = \dfrac{x^2-1}{x+1}$

34. $f(x) = \dfrac{x^2-6x-7}{x+1}$

35. $f(x) = \dfrac{x^2+1}{x+1}$

36. $f(x) = \dfrac{\sin(x+1)}{x+1}$

In Exercises 37–54, find the limit (if it exists).

37. $\displaystyle\lim_{x\to -1^+} \frac{1}{x+1}$

38. $\displaystyle\lim_{x\to 1^-} \frac{-1}{(x-1)^2}$

39. $\displaystyle\lim_{x\to 2^+} \frac{x}{x-2}$

40. $\displaystyle\lim_{x\to 1^+} \frac{2+x}{1-x}$

41. $\displaystyle\lim_{x\to 1^+} \frac{x^2}{(x-1)^2}$

42. $\displaystyle\lim_{x\to 4^-} \frac{x^2}{x^2+16}$

43. $\displaystyle\lim_{x\to -3^-} \frac{x+3}{x^2+x-6}$

44. $\displaystyle\lim_{x\to (-1/2)^+} \frac{6x^2+x-1}{4x^2-4x-3}$

45. $\displaystyle\lim_{x\to 1} \frac{x-1}{(x^2+1)(x-1)}$

46. $\displaystyle\lim_{x\to 3} \frac{x-2}{x^2}$

47. $\displaystyle\lim_{x\to 0^-} \left(1 + \frac{1}{x}\right)$

48. $\displaystyle\lim_{x\to 0^-} \left(x^2 - \frac{1}{x}\right)$

49. $\displaystyle\lim_{x\to 0^+} \frac{2}{\sin x}$

50. $\displaystyle\lim_{x\to (\pi/2)^+} \frac{-2}{\cos x}$

51. $\displaystyle\lim_{x\to \pi} \frac{\sqrt{x}}{\csc x}$

52. $\displaystyle\lim_{x\to 0} \frac{x+2}{\cot x}$

53. $\displaystyle\lim_{x\to 1/2} x \sec \pi x$

54. $\displaystyle\lim_{x\to 1/2} x^2 \tan \pi x$

In Exercises 55–58, use a graphing utility to graph the function and determine the one-sided limit.

55. $f(x) = \dfrac{x^2 + x + 1}{x^3 - 1}$

$\lim_{x \to 1^+} f(x)$

56. $f(x) = \dfrac{x^3 - 1}{x^2 + x + 1}$

$\lim_{x \to 1^-} f(x)$

57. $f(x) = \dfrac{1}{x^2 - 25}$

$\lim_{x \to 5^-} f(x)$

58. $f(x) = \sec \dfrac{\pi x}{8}$

$\lim_{x \to 4^+} f(x)$

WRITING ABOUT CONCEPTS

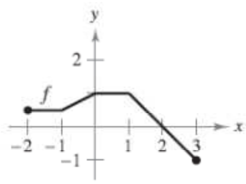
59. In your own words, describe the meaning of an infinite limit. Is ∞ a real number?

60. In your own words, describe what is meant by an asymptote of a graph.

61. Write a rational function with vertical asymptotes at $x = 6$ and $x = -2$, and with a zero at $x = 3$.

62. Does the graph of every rational function have a vertical asymptote? Explain.

63. Use the graph of the function f (see figure) to sketch the graph of $g(x) = 1/f(x)$ on the interval $[-2, 3]$. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.



CAPSTONE

64. Given a polynomial $p(x)$, is it true that the graph of the function given by $f(x) = \dfrac{p(x)}{x - 1}$ has a vertical asymptote at $x = 1$? Why or why not?

65. ***Relativity*** According to the theory of relativity, the mass m of a particle depends on its velocity v. That is,

$$m = \frac{m_0}{\sqrt{1 - (v^2/c^2)}}$$

where m_0 is the mass when the particle is at rest and c is the speed of light. Find the limit of the mass as v approaches c^-.

66. ***Boyle's Law*** For a quantity of gas at a constant temperature, the pressure P is inversely proportional to the volume V. Find the limit of P as $V \to 0^+$.

67. ***Rate of Change*** A patrol car is parked 50 feet from a long warehouse (see figure). The revolving light on top of the car turns at a rate of $\frac{1}{2}$ revolution per second. The rate at which the light beam moves along the wall is $r = 50\pi \sec^2 \theta$ ft/sec.

(a) Find the rate r when θ is $\pi/6$.

(b) Find the rate r when θ is $\pi/3$.

(c) Find the limit of r as $\theta \to (\pi/2)^-$.

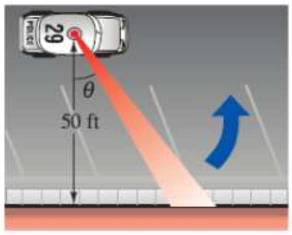


Figure for 67

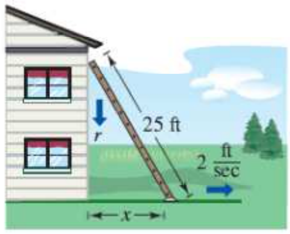


Figure for 68

68. ***Rate of Change*** A 25-foot ladder is leaning against a house (see figure). If the base of the ladder is pulled away from the house at a rate of 2 feet per second, the top will move down the wall at a rate of

$$r = \frac{2x}{\sqrt{625 - x^2}} \text{ ft/sec}$$

where x is the distance between the base of the ladder and the house.

(a) Find the rate r when x is 7 feet.

(b) Find the rate r when x is 15 feet.

(c) Find the limit of r as $x \to 25^-$.

69. ***Average Speed*** On a trip of d miles to another city, a truck driver's average speed was x miles per hour. On the return trip the average speed was y miles per hour. The average speed for the round trip was 50 miles per hour.

(a) Verify that $y = \dfrac{25x}{x - 25}$. What is the domain?

(b) Complete the table.

x	30	40	50	60
y				

Are the values of y different than you expected? Explain.

(c) Find the limit of y as $x \to 25^+$ and interpret its meaning.

70. ***Numerical and Graphical Analysis*** Use a graphing utility to complete the table for each function and graph each function to estimate the limit. What is the value of the limit when the power of x in the denominator is greater than 3?

x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$							

(a) $\lim_{x \to 0^+} \dfrac{x - \sin x}{x}$

(b) $\lim_{x \to 0^+} \dfrac{x - \sin x}{x^2}$

(c) $\lim_{x \to 0^+} \dfrac{x - \sin x}{x^3}$

(d) $\lim_{x \to 0^+} \dfrac{x - \sin x}{x^4}$