rule. He did not even employ an algebraic equation; he used Newton's old-fashioned geometric notation to calculate and add areas. Nor did he develop his theorem into a powerful mathematical method. Above all, unlike Price, he did not mention Hume, religion, or God.

Instead, he cautiously confined himself to the probability of events and did not mention hypothesizing, predicting, deciding, or taking action. He did not suggest possible uses for his work, whether in theology, science, or social science. Future generations would extend Bayes' discovery to do all these things and to solve a myriad of practical problems. Bayes did not even name his breakthrough. It would be called the probability of causes or inverse probability for the next 200 years. It would not be named Bayesian until the 1950s.

In short, Bayes took the first steps. He composed the prelude for what was to come.

For the next two centuries few read the Bayes-Price article. In the end, this is the story of two friends, Dissenting clergymen and amateur mathematicians, whose labor had almost no impact. Almost, that is, except on the one person capable of doing something about it, the great French mathematician Pierre Simon Laplace.

2.

the man who did everything

Just across the English Channel from Tunbridge Wells, about the time that Thomas Bayes was imagining his perfectly smooth table, the mayor of a tiny village in Normandy was celebrating the birth of a son, Pierre Simon Laplace, the future Einstein of his age.

Pierre Simon, born on March 23, 1749, and baptized two days later, came from several generations of literate and respected dignitaries. His mother's relatives were well-to-do farmers, but she died when he was young, and he never referred to her. His father kept the stagecoach inn in picturesque Beaumont-en-Auge, was a leader of the community's 472 inhabitants, and served 30 years as mayor. By the time Pierre Simon was a teenager his father seems to have been his only close relative. In years to come Pierre Simon's decision to become a mathematician would shatter their relationship almost irretrievably.[1]

Fortunately for the boy there was never any question about his getting an education. Attending school was becoming the norm in France in the 1700s, an enormous revolution fueled by the Catholic Church's fight against Protestant heresy and by parents convinced that education would enrich their children spiritually, intellectually, and financially. The question was, what kind of schooling?

Decades of religious warfare between Protestants and Catholics and several horrendous famines caused by cold weather had made France a determinedly secular country intent on developing its resources. Pierre Simon could have studied modern science and geometry in one of the country's many new secular schools. Instead, the elder Laplace enrolled his son in a local primary and secondary school where Benedictine monks produced clergy for the church and soldiers, lawyers, and bureaucrats for the crown. Thanks to the patronage of the Duke of Orleans, local day students like Pierre Simon attended free. The curriculum was conservative and Latin-based, heavy on copying, memorization, and philosophy. But it left Laplace with a fabulous memory and almost unbelievable perseverance.

Although the monks probably did not know it, they were competing with the French Enlightenment for the child's attention. Contemporaries called it the Century of Lights and the Age of Science and Reason, and the popularization of science was its most important intellectual phenomenon. Given the almost dizzying curiosity of the times, it is not surprising that, shortly after his tenth birthday, Pierre Simon was profoundly affected by a spectacular scientific prediction.[2]

Decades before, the English astronomer Edmond Halley had predicted the reappearance of the long-tailed comet that now bears his name. A trio of French astronomers, Alexis Claude Clairaut, Joseph Lalande, and Nicole-Reine Lepaute, the wife of a celebrated clockmaker, solved a difficult three-body problem and discovered that the gravitational pull of Jupiter and Saturn would delay the arrival of Halley's comet. The French astronomers accurately pinpointed the date—mid-April 1759 plus or minus a month—when Europeans would be able to see the comet returning from its orbit around the sun. The comet's appearance on schedule and on course electrified Europeans. Years later Laplace said it was the event that made his generation realize that extraordinary events like comets, eclipses, and severe droughts were caused not by divine anger but by natural laws that mathematics could reveal.

Laplace's extraordinary mathematical ability may not yet have been apparent when he turned 17 in 1766, because he did not go to the University of Paris, which had a strong science faculty. Instead he went to the University of Caen, which was closer to home and had a solid theological program suitable for a future cleric.

Yet even Caen had mathematical firebrands offering advanced lectures on differential and integral calculus. While English mathematicians were getting mired in Newton's awkward geometric version of calculus, their rivals on the Continent were using Gottfried Leibniz's more supple algebraic calculus. With it, they were forming equations and discovering a fabulous wealth of enticing new information about planets, their masses and details of their orbits. Laplace emerged from Caen a swashbuckling mathematical virtuoso eager to take on the scientific world. He had also become, no doubt to his father's horror, a religious skeptic.

At graduation Laplace faced an anguishing dilemma. His master's degree permitted him to take either the priestly vows of celibacy or the title of abbé, signifying a low-ranking clergyman who could marry and inherit property. Abbés did not have good reputations; Voltaire called them "that indefinable being which is neither ecclesiastic nor secular . . . young men, who are known for their debauchery."[3] An engraving of the period, "What Does the Abbé Think of It?" shows the clergyman peering appreciatively down a lady's bosom as she dresses.[4] Still, the elder Laplace wanted his son to become a clergyman.

If Laplace had been willing to become an abbé, his father might have helped him financially, and Laplace could have combined church and science. A number of abbés supported themselves in science, the most famous being Jean Antoine Nollet, who demonstrated spectacular physics experiments to the paying public. For the edification of the king and queen of France, Nollet sent a charge of static electricity through a line of 180 soldiers to make them leap comically into the air. Two abbés were even elected to the prestigious Royal Academy of Sciences. Still, the lot of most abbé-scientists was neither lucrative nor intellectually challenging. The majority found low-level jobs tutoring the sons of rich nobles or teaching elementary mathematics and science in secondary schools. University-level opportunities were limited because during the 1700s professors transmitted knowledge from the past instead of doing original research.

But Caen had convinced Laplace that he wanted to do something quite new. He wanted to be a full-time, professional, secular, mathematical researcher. And he wanted to explore the new algebra-generated, data-rich world of science. To his father, an ambitious man in bucolic France, a career in mathematics must have seemed preposterous.

Young Laplace made his move in the summer of 1769, shortly after completing his studies at Caen. He left Normandy and traveled to Paris, clutching a letter of recommendation to Jean Le Rond d'Alembert, the most powerful mathematician of the age, one of Europe's most notorious anticlerics, and the object of almost incessant Jesuit attacks. D'Alembert was a star of the Enlightenment and the chief spokesman for the *Encyclopédie*, which was making an enormous body of empirical knowledge universally available, scientific, and free of religious dogma. By throwing in his lot with d'Alembert, Laplace effectively cut his ties to the Catholic Church. We can only imagine his father's reaction, but we know that Laplace did not return home for 20 years and did not attend the old man's funeral.

Once in Paris, Laplace immediately approached the great d'Alembert and showed him a four-page student essay on inertia. Years later Laplace could still recite passages from it. Although besieged by applicants, d'Alembert was so impressed that within days he had arranged a paying job for Laplace as an instructor of mathematics at the new secular, mathematics-based Royal Military School for the younger sons of minor nobles. The school, located behind Les Invalides in Paris, provided Laplace with a salary, housing, meals, and money for wood to heat his room in winter. It was precisely the kind of job he had hoped to avoid.

Laplace could have tried to find work applying mathematics to practical problems in one of the monarchy's numerous research establishments or manufacturing plants. Many mathematically talented young men from modest families were employed in such institutions. But Laplace and his mentor were aiming far higher. Laplace wanted the challenge of doing basic research full time. And to do that, as d'Alembert must have told him, he had to get elected to the Royal Academy of Sciences.

In striking contrast to the amateurism of the Royal Society of London, the French Royal Academy of Sciences was the most professional scientific institution in Europe. Although aristocratic amateurs could become honorary members, the organization's highest ranks were composed of working scientists chosen by merit and paid to observe, collect, and investigate facts free of dogma; to publish their findings after peer review; and to advise the government on technical issues like patents. To augment their low salaries, academicians could use their prestige to cobble together various part-time jobs.

Without financial support from the church or his father, however, Laplace had to work fast. Since most academy members were chosen on the basis of a long record of solid accomplishment, he would have to be elected over the heads of more senior men. And for that to happen, he needed to make a spectacular impact.

D'Alembert, who had made Newton's revolution the focus of French mathematics, urged Laplace to concentrate on astronomy. D'Alembert had a clear problem in mind.

Over the previous two centuries mathematical astronomy had made great strides. Nicolaus Copernicus had moved Earth from the center of the solar system to a modest but accurate position among the planets; Johannes Kepler had connected the celestial bodies by simple laws; and Newton had introduced the concept of gravity. But Newton had described the motions of heavenly bodies roughly and without explanation. His death in 1727 left Laplace's generation an enormous challenge: showing that gravitation was not a hypothesis but a fundamental law of nature.

Astronomy was the era's most quantified and respected science, and only it could test Newton's theories by explaining precisely how gravitation affects the movements of tides, interacting planets and comets, our moon, and the shape of Earth and other planets. Forty years of empirical data had been collected, but, as d'Alembert warned, a single exception could bring the entire edifice tumbling down.

The burning scientific question of the day was whether the universe was stable. If Newton's gravitational force operates throughout the universe, why don't the planets collide with each other and cause the cosmic Armageddon described in the biblical book of Revelation? Was the end of the world at hand?

Astronomers had long been aware of alarming evidence suggesting that the solar system was inherently unstable. Comparing the actual positions of the most remote known planets with centuries-old astronomical observations, they could see that Jupiter was slowly accelerating in its orbit around the sun while Saturn was slowing down. Eventually, they thought, Jupiter would smash into the sun, and Saturn would spin off into space. The problem of predicting the motions of many interacting bodies over long periods of time is complex even today, and Newton concluded that God's miraculous intervention kept the heavens in equilibrium. Responding to the challenge, Laplace decided to make the stability of the universe his lifework. He said his tool would be mathematics and it would be like a telescope in the hands of an astronomer.

For a short time Laplace actually considered modifying Newton's theory by making gravity vary with a body's velocity as well as with its mass and distance. He also wondered fleetingly whether comets might be disturbing the orbits of Jupiter and Saturn. But he changed his mind almost immediately. The problem was not Newton's theory. The problem was the data astronomers used.

Newton's system of gravitation could be accepted as true only if it agreed with precise measurements, but observational astronomy was awash with information, some of it uncertain and inadequate. Working on the problem of Jupiter and Saturn, for example, Laplace would use observations made by Chinese astronomers in 1100 BC, Chaldeans in 600 BC, Greeks in 200 BC, Romans in AD 100, and Arabs in AD 1000. Obviously, not all data were equally valuable. How to resolve errors, known delicately as discrepancies, was anybody's guess.

The French academy was tackling the problem by encouraging the development of more precise telescopes and graduated arcs. And as algebra improved instrumentation, experimentalists were producing more quantitative results. In a veritable information explosion, the sheer collection and systemization of data accelerated through the Western world. Just as the number of known plant and animal species expanded enormously during the 1700s, so did knowledge about the physical universe. Even as Laplace arrived in Paris, the French and British academies were sending trained observers with state-of-the-art instrumentation to 120 carefully selected locations around the globe to time Venus crossing the face of the sun; this was a critical part of Capt. James Cook's original mission to the South Seas. By comparing all the measurements, French mathematicians would determine the approximate distance between the sun and Earth, a fundamental natural constant that would tell them the size of the solar system. But sometimes even up-to-date expeditions provided contradictory data about whether, for instance, Earth was shaped like an American football or a pumpkin.

Dealing with large amounts of complex data was emerging as a major scientific problem. Given a wealth of observations, how could scientists evaluate the facts at their disposal and choose the most valid? Observational astronomers typically averaged their three best observations of a particular phenomenon, but the practice was as straightforward as it was ad hoc; no one had ever tried to prove its validity empirically or theoretically. The mathematical theory of errors was in its infancy.

Problems were ripe for the picking and, with his eye on membership in the Royal Academy, Laplace bombarded the society with 13 papers in five years. He submitted hundreds of pages of powerful and original mathematics needed in astronomy, celestial mechanics, and important related issues. Astutely, he timed his reports to appear when openings occurred in the academy's membership. The secretary of the academy, the Marquis de Condorcet, wrote that never before had the society seen "anyone so young, present to it in so little time, so many important Mémoires, and on such diverse and such difficult matters."[5]

Academy members considered Laplace for membership six times but rejected him repeatedly in favor of more senior scientists. D'Alembert complained furiously that the organization refused to recognize talent. Laplace considered emigrating to Prussia or Russia to work in their academies.

During this frustrating period Laplace spent his free afternoons digging in the mathematical literature in the Royal Military School's 4,000-volume library. Analyzing large amounts of data was a formidable problem, and Laplace was already beginning to think it would require a fundamentally new way of thinking. He was beginning to see probability as a way to deal with the uncertainties pervading many events and their causes. Browsing in the library's stacks, he discovered an old book on gambling probability, *The Doctrine of Chances*, by Abraham de Moivre. The book had appeared in three editions between 1718 and 1756, and Laplace may have read the 1756 version. Thomas Bayes had studied an earlier edition.

Reading de Moivre, Laplace became more and more convinced that probability might help him deal with uncertainties in the solar system. Probability barely existed as a mathematical term, much less as a theory. Outside of gambling, it was applied in rudimentary form to philosophical questions like the existence of God and to commercial risk, including contracts, marine and life insurance, annuities, and money lending.

Laplace's growing interest in probability created a diplomatic problem of some delicacy because d'Alembert believed probability was too subjective for science. Young as he was, Laplace was confident enough in his mathematical judgment to disagree with his powerful patron. To Laplace, the movements of celestial bodies seemed so complex that he could not hope for precise solutions. Probability would not give him absolute answers, but it might show him which data were more likely to be correct. He began thinking about a method for deducing the probable causes of divergent, error-filled observations in astronomy. He was feeling his way toward a broad general theory for moving mathematically from known events back to their most probable causes. Continental mathematicians did not know yet about Bayes' discovery, so Laplace called his idea "the probability of causes" and "the probability of causes and future events, derived from past events."[6]

Wrestling with the mathematics of probability in 1773, he reflected on its philosophical counterpoint. In a paper submitted and read to the academy in March, the former abbé compared ignorant mankind, not with God but with an imaginary intelligence capable of knowing All. Because humans can never know everything with certainty, probability is the mathematical expression of our ignorance: "We owe to the frailty of the human mind one of the most delicate and ingenious of mathematical theories, namely the science of chance or probabilities."[7]

The essay was a grand combination of mathematics, metaphysics, and the heavens that Laplace held to his entire life. His search for a probability of causes and his view of the deity were deeply congenial. Laplace was all of one piece and for that reason all the more formidable. He often said he did not believe in God, and not even his biographer could decide whether he was an atheist or a deist. But his probability of causes was a mathematical expression of the universe, and for the rest of his days he updated his theories about God and the probability of causes as new evidence became available.

Laplace was struggling with probability when one day, ten years after the publication of Bayes' essay, he picked up an astronomy journal and was shocked to read that others might be hot on the same trail. They were not, but the threat of competition galvanized him. Dusting off one of his discarded manuscripts, Laplace transformed it into a broad method for determining the most likely causes of events and phenomena. He called it "Mémoire on the Probability of the Causes Given Events."

It provided the first version of what today we call Bayes' rule, Bayesian probability, or Bayesian statistical inference. Not yet recognizable as the modern Bayes' rule, it was a one-step process for moving backward, or inversely, from an effect to its most likely cause. As a mathematician in a gambling-addicted culture, Laplace knew how to work out the gambler's future odds of an event knowing its cause (the dice). But he wanted to solve scientific problems, and in real life he did not always know the gambler's odds and often had doubts about what numbers to put into his calculations. In a giant and intellectually nimble leap, he realized he could inject these uncertainties into his thinking by considering *all* possible causes and then choosing among them.

Laplace did not state his idea as an equation. He intuited it as a principle and described it only in words: the probability of a cause (given an event) is proportional to the probability of the event (given its cause). Laplace did not translate his theory into algebra at this point, but modern readers might find it helpful to see what his statement would look like today:

$$P(C|E) = \frac{P(E|C)}{\Sigma P(E|C')}$$

where $P(C|E)$ is the probability of a particular cause (given the data), and $P(E|C)$ represents the probability of an event or datum (given that cause). The sign in the denominator represented with Newton's sigma sign makes the total probability of all possible causes add up to one.

Armed with his principle, Laplace could do everything Thomas Bayes could have done—as long as he accepted the restrictive assumption that all his possible causes or hypotheses were equally likely. Laplace's goal, however, was far more ambitious. As a scientist, he needed to study the various possible causes of a phenomenon and then determine the best one. He did not yet know how to do that mathematically. He would need to make two more major breakthroughs and spend decades in thought.

Laplace's principle, the proportionality between probable events and their probable causes, seems simple today. But he was the first mathematician to work with large data sets, and the proportionality of cause and effect would make it feasible to make complex numerical calculations using only goose quills and ink pots.

In a mémoire read aloud to the academy, Laplace first applied his new probability of causes to two gambling problems. In each case he understood intuitively what should happen but got bogged down trying to prove it mathematically. First, he imagined an urn filled with an unknown ratio of black and white tickets (his cause). He drew a number of tickets from the urn and, based on that experience, asked for the probability that his next ticket would be white. Then in a frustrating battle to prove the answer he wrote no fewer than 45 equations covering four quarto-sized pages.

His second gambling problem involved piquet, a game requiring both luck and skill. Two people start playing but stop midway through the game and have to figure out how to divide the kitty by estimating their relative skill levels (the cause). Again, Laplace understood instinctively how to solve the problem but could not yet do so mathematically.

After dealing with gambling, which he loathed, Laplace moved happily on to the critical scientific problem faced by working astronomers. How should they deal with different observations of the same phenomenon? Three of the era's biggest scientific problems involved gravitational attraction on the motions of our moon, the motions of the planets Jupiter and Saturn, and the shape of the Earth. Even if observers repeated their measurements at the same time and place with the same instrument, their results could be slightly different each time. Trying to calculate a midvalue for such discrepant observations, Laplace limited himself to three observations but still needed seven pages of equations to formulate the problem. Scientifically, he understood the right answer—average the three data points—but he would have no mathematical justification for doing so until 1810, when, without using the probability of causes, he invented the central limit theorem.

Although Bayes originated the probability of causes, Laplace clearly discovered his version on his own. Laplace was 15 when the Bayes-Price essay was published; it appeared in an English-language journal for the English gentry and was apparently never mentioned again. Even French scientists who kept up with foreign journals thought Laplace was first and congratulated him wholeheartedly on his originality.

Mathematics confirms that Laplace discovered the principle independently. Bayes solved a special problem about a flat table using a two-step process that involved a prior guess and new data. Laplace did not yet know about the initial guess but dealt with the problem generally, making it useful for a variety of problems. Bayes laboriously explained and illustrated why uniform probabilities were permissible; Laplace assumed them instinctively. The Englishman wanted to know the range of probabilities that something will happen in light of previous experience. Laplace wanted more: as a working scientist, he wanted to know the probability that certain measurements and numerical values associated with a phenomenon were realistic. If Bayes and Price searched for the probability that, on the basis of today's puddles, it had rained yesterday and would rain tomorrow, Laplace asked for the probability that a particular amount of rain would fall and then refined his opinion over and over with new information to get a better value. Laplace's method was immensely influential; scientists did not pay Bayes serious heed until the twentieth century.

Most strikingly of all, Laplace at 25 was already steadfastly determined to develop his new method and make it useful. For the next 40 years he would work to clarify, simplify, expand, generalize, prove, and apply his new rule. Yet while Laplace became the indisputable intellectual giant of Bayes' rule, it represented only a small portion of his career. He also made important advances in celestial mechanics, mathematics, physics, biology, Earth science, and statistics. He juggled projects, moving from one to another and then back to the first. Happily blazing trails through every field of science known to his age, he transformed and mathematized everything he touched. He never stopped being thrilled by examples of Newton's theory.

Although he was fast becoming the leading scientist of his era, the academy waited five years before electing him a member on March 31, 1773. A few weeks later he was formally inducted into the world's leading scientific organization. His mémoire on the probability of causes was published a year later, in 1774. At the age of 24, Laplace was a professional researcher. The academy's annual stipend, together with his teaching salary, would help support him while he refined his research on celestial mechanics and the probability of causes.

Laplace was still grappling with probability in 1781, when Richard Price visited Paris and told Condorcet about Bayes' discovery. Laplace immediately latched onto the Englishman's ingenious invention, the starting guess, and incorporated it into his own, earlier version of the probability of causes. Strictly speaking, he did not produce a new formula but rather a statement about the first formula assuming equal probabilities for the causes. The statement gave him confidence that he was on the right track and told him that as long as all his prior hypotheses were equally probable, his earlier principle of 1774 was correct.[8]

Laplace could now confidently marry his intuitive grasp of a scientific situation with the eighteenth century's passion for new and precise scientific discoveries. Every time he got new information he could use the answer from his last solution as the starting point for another calculation. And by assuming that all his initial hypotheses were equally probable he could even derive his theorem.

As Academy secretary, Condorcet wrote an introduction to Laplace's essay and explained Bayes' contribution. Laplace later publicly credited Bayes with being first when he wrote, "The theory whose principles I explained some years after, . . . he accomplished in an acute and very ingenious, though slightly awkward, manner."[9]

Over the next decade, however, Laplace would realize with increasing clarity and frustration that his mathematics had shortcomings. It limited him to assigning equal probabilities to each of his initial hypotheses. As a scientist, he disapproved. If his method was ever going to reflect the actual state of affairs, he needed to be able to differentiate dubious data from more valid observations. Calling all events or observations equally probable could be true only theoretically. Many dice, for example, that appeared perfectly cubed were actually skewed. In one case he started by assigning players equal probabilities of winning, but with each round of play their respective skills emerged and their probabilities changed. "The science of chances must be used with care and must be modified when we pass from the mathematical case to the physical," he counseled.[10]

Moreover, as a pragmatist, he realized he had to confront a serious technical difficulty. Probability problems require multiplying numbers over and over, whether tossing coin after coin or measuring and remeasuring an observation. The process generated huge numbers—nothing as large as those common today but definitely cumbersome for a man working alone without mechanical or electronic aids. (He did not even get an assistant to help with calculations until about 1785.)

Laplace was never one to shrink from difficult computations, but, as he complained, probability problems were often impossible because they presented great difficulties and numbers raised to "very high powers."[11] He could use logarithms and an early generating function that he considered inadequate. But to illustrate how tedious calculations with big numbers could be, he described multiplying 20,000 × 19,999 × 19,998 × 19,997, etc. and then dividing by 1 × 2 × 3 × 4 up to 10,000. In another case he bet in a lottery only to realize he could not calculate its formula numerically; the French monarchy's winning number had 90 digits, drawn five at a time.

Such big-number problems were new. Newton had calculated with geometry, not numbers. Many mathematicians, like Bayes, used thought experiments to separate real problems from abstract and methodological issues. But Laplace wanted to use mathematics to illuminate natural phenomena, and he insisted that theories had to be based on actual fact. Probability was propelling him into an unmanageable world.

Armed with the Bayes–Price starting point, Laplace broke partway through the logjam that had stymied him for seven years. So far he had concentrated primarily on probability as a way to resolve error-prone astronomical observations. Now he switched gears to concentrate on finding the most probable causes of known events. To do so, he needed to practice with a big database of real and reliable values. But astronomy seldom provided extensive or controlled data, and the social sciences often involved so many possible causes that algebraic equations were useless.

Only one large amalgamation of truly trustworthy numbers existed in the 1700s: parish records of births, christenings, marriages, and deaths. In 1771 the French government ordered all provincial officials to report birth and death figures regularly to Paris; and three years later, the Royal Academy published 60 years of data for the Paris region. The figures confirmed what the Englishman John Graunt had discovered in 1662: slightly more boys than girls were born, in a ratio that remained constant over many years. Scientists had long assumed that the ratio, like other newly discovered regularities in nature, must be the result of "Divine Providence." Laplace disagreed.

Soon he was assessing not gambling or astronomical statistics but infants. For anyone interested in large numbers, babies were ideal. First, they came in binomials, either boys or girls, and eighteenth-century mathematicians already knew how to treat binomials. Second, infants arrived in abundance and, as Laplace emphasized, "It is necessary in this delicate research to employ sufficiently large numbers in view of the small difference that exists between . . . the births of boys and girls."[12] When the great naturalist Comte de Buffon discovered a small village in Burgundy where, for five years running, more girls had been born than boys, he asked whether this village invalidated Laplace's hypotheses. Absolutely not, Laplace replied firmly. A study based on a few facts cannot overrule a much larger one.

The calculations would be formidable. For example, if he had started with a 52:48 ratio of newborn boys to girls and a sample of 58,000 boys, Laplace would have had to multiply .52 by itself 57,999 times—and then do a similar calculation for girls. This was definitely not something anyone, not even the indomitable Laplace, wanted to do by hand.

He started out, however, as Bayes had suggested, by pragmatically assigning equal probabilities to all his initial hunches, whether 50–50, 33–33–33, or 25–25–25–25. Because their sums equal one, multiplication would be easier. He employed equal probabilities only provisionally, as a starting point, and his final hypotheses would depend on all the observational data he could add.

Next, he tried to confirm that Graunt was correct about the probability of a boy's birth being larger than 50%. He was building the foundation of the modern theory of testing statistical hypotheses. Poring over records of christenings in Paris and births in London, he was soon willing to bet that boys would outnumber girls for the next 179 years in Paris and for the next 8,605 years in London. "It would be extraordinary if it was the effect of chance," he wrote, tut-tutting that people really should make sure of their facts before theorizing about them.[13]

To transform probability's large numbers into smaller, more manageable terms Laplace invented a multitude of mathematical shortcuts and clever approximations. Among them were new generating functions, transforms, and asymptotic expansions. Computers have made many of his shortcuts unnecessary, but generating functions remain deeply embedded in mathematical analyses used for practical applications. Laplace used generating functions as a form of mathematical wizardry to trick a function he could deal with into providing him with the function he really wanted.

To Laplace, these mathematical pyrotechnics seemed as obvious as common sense. To students' frustration, he sprinkled his reports with phrases like, "It is easy to see, it is easy to extend, it is easy to apply, it is obvious that. . . ."[14] When a confused student once asked how he had jumped intuitively from one equation to another, Laplace had to work hard to reconstruct his thought process.

He was soon asking whether boys were more apt to be born in certain geographic regions. Perhaps "climate, food or customs . . . facilitates the birth of boys" in London.[15] Over the next 30-odd years Laplace collected birth ratios from Naples in the south, St. Petersburg in the north, and French provinces in between. He concluded that climate could not explain the disparity in births. But would more boys than girls always be born? As each additional piece of evidence appeared, Laplace found his probabilities approaching certainty "at a dramatically increasing rate."

He was refining hunches with objective data. In building a mathematical model of scientific thinking, where a reasonable person could develop a hypothesis and then evaluate it relentlessly in light of new knowledge, he became the first modern Bayesian. His system was enormously sensitive to new information. Just as each throw of a coin increases the probability of its being fair or rigged, so each additional birth record narrowed the range of uncertainties. Eventually, Laplace decided that the probability of boys exceeding girls was as "certain as any other moral truth" with an extremely tiny margin of being wrong.[16]

Generalizing from babies, he found a way to determine not just the probability of simple events, like the birth of one boy, but also the probability of future composite events like an entire year of births—even when the probability of simple events (whether the next newborn will be male) was uncertain. By 1786 he was determining the influence of past events on the probability of future events and wondering how big his sample of newborns had to be. By then Laplace saw probability as the primary way to overcome uncertainty. Pounding the point home in one short paragraph, he wrote, "Probability is relative in part to this *ignorance*, in part to our knowledge . . . a state of indecision, . . . it's impossible to announce with certainty."[17]

Persevering for years, he used insights gained in one science to shed light on others, researching a puzzle and inventing a mathematical technique to resolve it, integrating, approximating, and generalizing broadly when there was no other way to proceed. Like a modern researcher, he competed and collaborated with others and published reports on his interim progress as he went. Above all, he was tenacious. Twenty-five years later he was still eagerly testing his probability of causes with new information. He combed 65 years' worth of orphanage registries, asked friends in Egypt and Alexander von Humboldt in Central America about birth ratios there, and called on naturalists to check the animal kingdom. Finally, in 1812, after decades of work, he cautiously concluded that the birth of more boys than girls seemed to be "a general law for the human race."[18]

To test his rule on a larger sample Laplace decided in 1781 to determine the size of the French population, the thermometer of its health and prosperity. A conscientious administrator in eastern France had carefully counted heads in several parishes; to estimate the population of the entire nation, he recommended multiplying the annual number of births in France by 26. His proposal produced what was thought to be France's population, approximately 25.3 million. But no one knew how accurate his estimate was. Today's demographers believe that France's population had actually grown rapidly, to almost 28 million, because of fewer famines and because a government-trained midwife was touring the countryside promoting the use of soap and boiling water during childbirth.

Using his probability of causes, Laplace combined his prior information from parish records about births and deaths throughout France with his new information about headcounts in eastern France. He was adjusting estimates of the nation's population with more precise information from particular regions. In 1786 he reached a figure closer to modern estimates and calculated odds of 1,000 to 1 that his estimate was off by less than half a million. In 1802 he was able to advise Napoleon Bonaparte that a new census should be augmented with detailed samples of about a million residents in 30 representative departments scattered equally around France.

As he worked on his birth and census studies during the monarchy's last years, Laplace became involved in an inflammatory debate about France's judicial system. Condorcet believed the social sciences should be as quantifiable as the physical sciences. To help transform absolutist France into an English-style constitutional monarchy, he wanted Laplace to use mathematics to explore a variety of issues. How confident can we be in a sentence handed down by judge or jury? How probable is it that voting by an assembly or judicial tribunal will establish the truth? Laplace agreed to apply his new theory of probability to questions about electoral procedures, the credibility of witnesses, decision making by judicial panels and juries, and procedures of representative bodies and judicial panels.

Laplace took a dim view of most court judgments in France. Forensic science did not exist, so judicial systems everywhere relied on witness testimony. Taking a witness's statement for an event, Laplace asked the probability that the witness or the judge might be truthful, misled, or simply mistaken. He estimated the prior odds of an accused person's guilt at 50–50 and the probability that a juror was being truthful somewhat higher. Even at that, if a jury of eight voted by simple majority, the chance that they judged the accused's guilt wrong would be 65/256, or more than one in four. Thus for both mathematical and religious reasons Laplace sided with the Enlightenment's most radical demand, the abolition of capital punishment: "The possibility of atoning for these errors is the strongest argument of philosophers who have wanted to abolish the death penalty."[19] Laplace also used his rule for more complicated cases where a court must decide among contradictory witnesses or where the reliability of testimony decreases with each telling. For Laplace, these questions demonstrated that ancient biblical accounts by the Apostles lacked credibility.

While still counting babies, Laplace returned to study the seeming instability of Saturn and Jupiter's orbits, the problem that had helped sensitize him early in his career to uncertain data. He did not, however, use his new knowledge of probability to solve this important problem. He used other methods between 1785 and 1788 to determine that Jupiter and Saturn oscillate gently in an 877-year cycle around the sun and that the moon orbits Earth in a cycle millions of years long. The orbits of Jupiter, Saturn, and the moon were not exceptions to Newton's gravitation but thrilling examples of it. The solar system was in equilibrium, and the world would not end. This discovery was the biggest advance in physical astronomy since Newton's law of gravity.

Despite Laplace's astounding productivity, his life as a professional scientist was financially precarious. Fortunately, Paris in the 1700s had more educational institutions and scientific opportunities than anywhere else on Earth, and academy members could patch jobs together to make a respectable living. Laplace tripled his income by examining artillery and naval engineering students three or four months a year and serving as a scientist in the Duke of Orleans' entourage. His increasingly secure position also gave him access to the government statistics he needed to develop and test his probability of causes.

At the age of 39, with a bright future ahead of him, Laplace married 18-year-old Marie Anne Charlotte Courty de Romange. The average age of marriage for French women was 27, but Marie Anne came from a prosperous and recently ennobled family with multiple ties to his financial and social circle. A small street off the Boulevard Saint-Germain is named Courty for her family. The Laplaces would have two children; contraception, whether coitus interruptus or pessaries, was common, and the church itself campaigned against multiple childbirths because they endangered the lives of mothers. Some 16 months after the wedding a Parisian mob stormed the Bastille, and the French Revolution began.

After the revolutionary government was attacked by foreign monarchies, France spent a decade at war. Few scientists or engineers emigrated, even during the Reign of Terror. Mobilized for the national defense, they organized the conscription of soldiers, collected raw materials for gunpowder, supervised munitions factories, drew military maps, and invented a secret weapon, reconnaissance balloons. Laplace worked throughout the upheaval and served as the central figure in one of the Revolution's most important scientific projects, the metric reform to standardize weights and measures. It was Laplace who named the meter, centimeter, and millimeter.

Nevertheless, during the 18 months of the Terror, as almost 17,000 French were executed and half a million imprisoned, his position became increasingly precarious. Radicals attacked the elite Royal Academy of Sciences, and publications denounced him as a modern charlatan and a "Newtonian idolator." A month after the Royal Academy was abolished Laplace was arrested on suspicion of disloyalty to the Revolution but neighbors interceded and he was released the next day at 4 a.m. A few months later he was purged from the metric system commission as not "worthy of confidence as to [his] republican virtues and [his] hatred of kings."[20] His assistant, Jean-Baptiste Delambre, was arrested while measuring the meridian for the meter and then released. At one point Laplace was relieved of his part-time job examining artillery students, only to be given the same job at the École Polytechnique. Seven scientists, including several of Laplace's closest friends and supporters, died during the Terror. Unlike Laplace, who took no part in radical politics, they had identified themselves with particular political factions. The most famous was Antoine Lavoisier, guillotined because he had been a royal tax collector. Condorcet, trying to escape from Paris, died in jail.

The Revolution, however, transformed science from a popular hobby into a full-fledged profession. Laplace emerged from the chaos as a dean of French science, charged with building new secular educational institutions and training the next generation of scientists. For almost 50 years—from the

1780s until his death in 1827—France led world science as no other country has before or since. And for 30 of those years Laplace was among the most influential scientists of all time.

As the best-selling author of books about the celestial system and the law of gravity, Laplace dedicated two volumes to a rising young general, Napoleon Bonaparte. Laplace had launched Napoleon on his military career by giving him a passing exam grade in military school. The two never became personal friends, but Napoleon appointed Laplace minister of the interior for a short time and then appointed him to the largely honorary Senate with a handsome salary and generous expense account that made him quite a rich man. Mme Laplace became a lady-in-waiting to Napoleon's sister and received her own salary. With additional financing from Napoleon, Laplace and his friend the chemist Claude Berthollet turned their country homes in Arceuil, outside Paris, into the world's only center for young postdoctoral scientists.

At a reception in Josephine Bonaparte's rose garden at Malmaison in 1802, the emperor, who was trying to engineer a rapprochement with the papacy, started a celebrated argument with Laplace about God, astronomy, and the heavens.

"And who is the author of all this?" Napoleon demanded.

Laplace replied calmly that a chain of natural causes would account for the construction and preservation of the celestial system.

Napoleon complained that "Newton spoke of God in his book. I have perused yours but failed to find His name even once. Why?"

"Sire," Laplace replied magisterially, "I have no need of that hypothesis."[21]

Laplace's answer, so different from Price's idea that Bayes' rule could prove the existence of God, became a symbol of a centuries-long process that would eventually exclude religion from the scientific study of physical phenomena. Laplace had long since separated his probability of causes from religious considerations: "The true object of the physical sciences is not the search for primary causes [that is, God] but the search for laws according to which phenomena are produced."[22] Scientific explanations of natural phenomena were triumphs of civilization whereas theological debates were fruitless because they could never be resolved.

Laplace continued his research throughout France's political upheavals. In 1810 he announced the central limit theorem, one of the great scientific and statistical discoveries of all time. It asserts that, with some exceptions, any average of a large number of similar terms will have a normal, bell-shaped distribution. Suddenly, the easy-to-use bell curve was a real mathematical construct. Laplace's probability of causes had limited him to binomial problems, but his final proof of the central limit theorem let him deal with almost any kind of data.

In providing the mathematical justification for taking the mean of many data points, the central limit theorem had a profound effect on the future of Bayes' rule. At the age of 62, Laplace, its chief creator and proponent, made a remarkable about-face. He switched allegiances to an alternate, frequency-based approach he had also developed. From 1811 until his death 16 years later Laplace relied primarily on this approach, which twentieth-century theoreticians would use to almost obliterate Bayes' rule.

Laplace made the change because he realized that where large amounts of data were concerned, both approaches generally produce much the same results. The probability of causes was still useful in particularly uncertain cases because it was more powerful than frequentism. But science matured during Laplace's lifetime. By the 1800s mathematicians had much more reliable data than they had had in his youth and dealing with trustworthy data was easier with frequentism. Mathematicians did not learn until the mid-twentieth century that, even with great amounts of data, the two methods can sometimes seriously disagree.

Looking back in 1813 on his 40-year quest to develop the probability of causes, Laplace described it as the primary method for researching unknown or complicated causes of natural phenomena. He referred to it fondly as his source of large numbers and the inspiration behind his development and use of generating functions.

And finally, in the climax of one small part of his career, he proved the elegant, general version of his theorem that we now call Bayes' rule. He had intuited its principle as a young man in 1774. In 1781 he found a way to use Bayes' two-step process to derive the formula by making certain restrictive assumptions. Between 1810 and 1814 he finally realized what the general theorem had to be. It was the formula he had been dreaming about, one broad enough to allow him to distinguish highly probable hypotheses from less valid ones. With it, the entire process of learning from evidence was displayed:

$$P(C|E) = \frac{P(E|C)\, P_{prior}(C)}{\sum P(E|C')\, P_{prior}(C')}$$

In modern terms, the equation says that $P(C|E)$, the probability of a hypothesis (given information), equals $P_{prior}(C)$, our initial estimate of its probability, times $P(E|C)$, the probability of each new piece of information (under the hypothesis), divided by the sum of the probabilities of the data in all possible hypotheses.

Undergraduates today study Laplace's first version of the equation, which deals with discrete events such as coin tosses and births. Advanced and graduate students and researchers use calculus with his later equation to work with observations on a continuous range between two values, for example, all the temperatures between 32 and 33 degrees. With it, Laplace could estimate a value as being within such and such a range with a particular degree of probability.

Laplace had owned Bayes' rule in all but name since 1781. The formula, the method, and its masterful utilization all belong to Pierre Simon Laplace. He made probability-based statistics commonplace. By transforming a theory of gambling into practical mathematics, Laplace's work dominated probability and statistics for a century. "In my mind," Glenn Shafer of Rutgers University observed, "Laplace did everything, and we just read stuff back into Thomas Bayes. Laplace put it into modern terms. In a sense, everything is Laplacean."[23]

If advancing the world's knowledge is important, Bayes' rule should be called Laplace's rule or, in modern parlance, BPL for Bayes-Price-Laplace. Sadly, a half century of usage forces us to give Bayes' name to what was really Laplace's achievement.

Since discovering his first version of Bayes' rule in 1774, Laplace had used it primarily to develop new mathematical techniques and had applied it most extensively to the social sciences, that is, demography and judicial reform. Not until 1815, at the age of 66, did he apply it to his first love, astronomy. He had received some astonishingly accurate tables compiled by his assistant Alexis Bouvard, the director of the Paris Observatory. Using Laplace's probability of causes, Bouvard had calculated a large number of observations about the masses of Jupiter and Saturn, estimated the possible error for each entry, and then predicted the probable masses of the planets. Laplace was so delighted with the tables that, despite his aversion to gambling, he used Bayes' rule to place a famous bet with his readers: odds were 11,000 to 1 that Bouvard's results for Saturn were off by less than 1%. For Jupiter, the odds were a million to one. Space-age technology confirms that Laplace and Bouvard should have won both bets.

Late in his career, Laplace also applied his probability of causes to a variety of calculations in Earth science, notably to the tides and to changes in barometric pressure. He used a nonnumerical common-sense version of his probability of causes to advance his famous nebular hypothesis: that the

planets and their satellites in our solar system originated in a swirl of dust. And he compared three hypotheses about the orbits of 100 comets to confirm what he already knew: that the comets most probably originate within the sun's sphere of influence.

After the fall of Napoleon, France's new king, Louis XVIII, bestowed the hereditary title of marquis on Laplace, the son of a village innkeeper. And on March 5, 1827, at the age of 78, Laplace died, almost exactly 100 years after his idol, Isaac Newton.

Eulogies hailed Laplace as the Newton of France. He had brought modern science to students, governments, and the reading public and had developed probability into a formidable method for handling unknown and complex causes of natural phenomena. And in one small, relatively insignificant portion of his lifework he became the first to express and use what is now called Bayes' rule. With it, he updated old knowledge with new, explained phenomena that previous centuries had ascribed to chance or to God's will, and opened the way for future scientific exploration.

Yet Laplace had built his probability theory on intuition. As far as he was concerned, "essentially, the theory of probability is nothing but good common sense reduced to mathematics. It provides an exact appreciation of what sound minds feel with a kind of instinct, frequently without being able to account for it."[24] Soon, however, scientists would begin confronting situations that intuition could not easily explain. Nature would prove to be far more complicated than even Laplace had envisioned. No sooner was the old man buried than critics began complaining about Laplace's rule.