12. Per-Pupil Expenditures The expenditures (in dollars) per pupil for states in three sections of the country are listed. Using $\alpha = 0.05$, can you conclude that there is a difference in means?

Eastern third	Middle third	Western third
4946	6149	5282
5953	7451	8605
6202	6000	6528
7243	6479	6911
6113		

Source: *New York Times Almanac.*

13. Weekly Unemployment Benefits The average weekly unemployment benefit for the entire United States is \$297. Three states are randomly selected, and a sample of weekly unemployment benefits is recorded for each. At $\alpha = 0.05$ is there sufficient evidence to conclude a difference in means? If so, perform the appropriate test to find out where the difference exists.

Florida	Pennsylvania	Maine
200	300	250
187	350	195
192	295	275
235	362	260
260	280	220
175	340	290

Source: *World Almanac.*

12–3 Two-Way Analysis of Variance

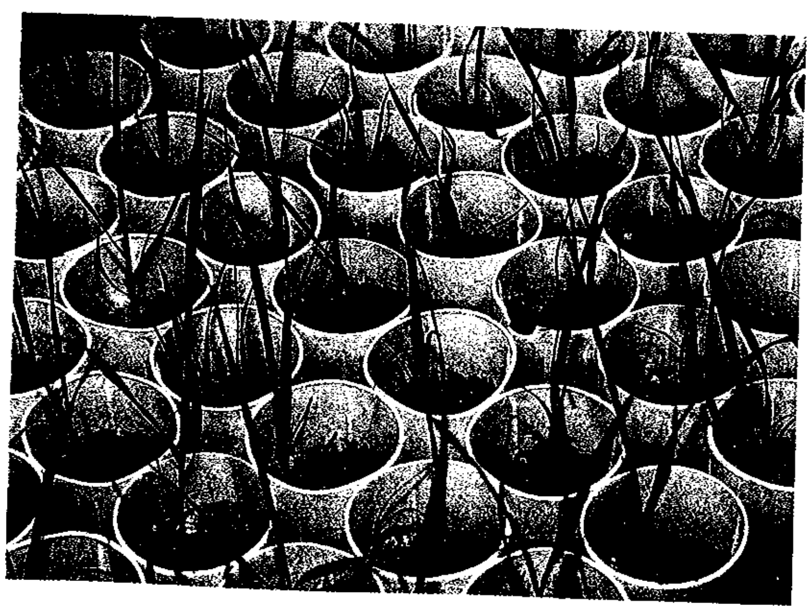
Objective 3

Use the two-way ANOVA technique to determine if there is a significant difference in the main effects or interaction.

The analysis of variance technique shown previously is called a **one-way ANOVA** since there is only *one independent variable.* The **two-way ANOVA** is an extension of the one-way analysis of variance; it involves *two independent variables.* The independent variables are also called **factors.**

The two-way analysis of variance is quite complicated, and many aspects of the subject should be considered when you are using a research design involving a two-way ANOVA. For the purposes of this textbook, only a brief introduction to the subject will be given.

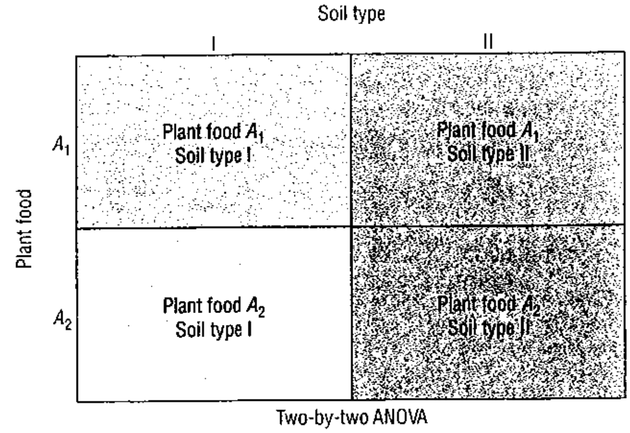
In doing a study that involves a two-way analysis of variance, the researcher is able to test the effects of two independent variables or factors on one *dependent variable.* In addition, the interaction effect of the two variables can be tested.



For example, suppose a researcher wishes to test the effects of two different types of plant food and two different types of soil on the growth of certain plants. The two independent variables are the type of plant food and the type of soil, while the dependent variable is the plant growth. Other factors, such as water, temperature, and sunlight, are held constant.

Figure 12–2

Treatment Groups for the Plant Food–Soil Type Experiment



To conduct this experiment, the researcher sets up four groups of plants. See Figure 12–2. Assume that the plant food type is designated by the letters A_1 and A_2 and the soil type by the Roman numerals I and II. The groups for such a two-way ANOVA are sometimes called **treatment groups.** The four groups are

Group 1	Plant food A_1, soil type I
Group 2	Plant food A_1, soil type II
Group 3	Plant food A_2, soil type I
Group 4	Plant food A_2, soil type II

The plants are assigned to the groups at random. This design is called a 2×2 (read "two-by-two") design, since each variable consists of two **levels,** that is, two different treatments.

The two-way ANOVA enables the researcher to test the effects of the plant food and the soil type in a single experiment rather than in separate experiments involving the plant food alone and the soil type alone. Furthermore, the researcher can test an additional hypothesis about the effect of the *interaction* of the two variables—plant food and soil type—on plant growth. For example, is there a difference between the growth of plants using plant food A_1 and soil type II and the growth of plants using plant food A_2 and soil type I? When a difference of this type occurs, the experiment is said to have a significant **interaction effect.** That is, the types of plant food affect the plant growth differently in different soil types. When the interaction effect is statistically significant the researcher should not consider the effects of the individual factors without considering the interaction effect.

Interesting Facts

As unlikely as it sounds, lightning can travel through phone wires. You should probably hold off on taking a bath or shower as well during an electrical storm. According to the *Annals of Emergency Medicine,* lightning can also travel through water pipes.

There are many different kinds of two-way ANOVA designs, depending on the number of levels of each variable. Figure 12–3 shows a few of these designs. As stated previously, the plant food–soil type experiment uses a 2×2 ANOVA.

The design in Figure 12–3(a) is called a 3×2 design, since the factor in the rows has three levels and the factor in the columns has two levels. Figure 12–3(b) is a 3×3 design, since each factor has three levels. Figure 12–3(c) is a 4×3 design.

The two-way ANOVA design has several null hypotheses. There is one for each independent variable and one for the interaction. In the plant food–soil type problem, the hypotheses are as follows:

1. H_0: There is no interaction effect between type of plant food used and type of soil used on plant growth.

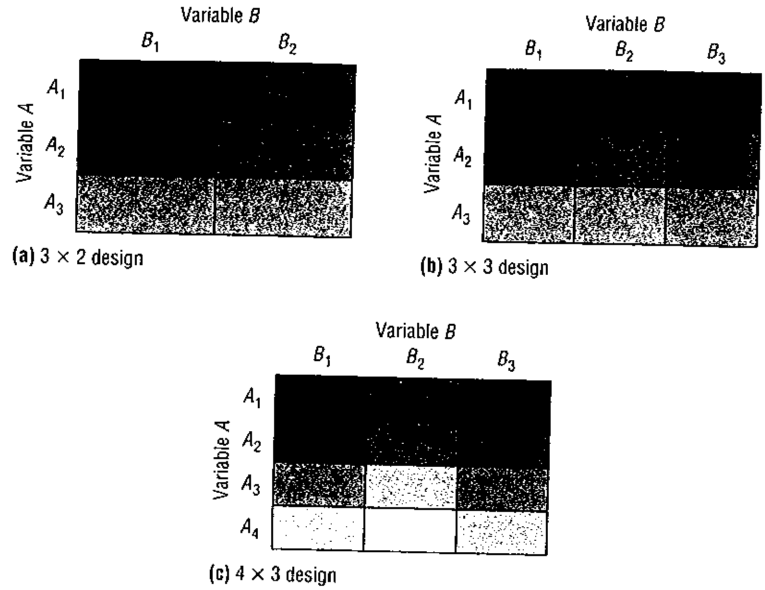
H_1: There is an interaction effect between food type and soil type on plant growth.

2. H_0: There is no difference in means of heights of plants grown using different foods.

H_1: There is a difference in means of heights of plants grown using different foods.

Figure 12–3

Some Types of Two-Way ANOVA Designs



(a) 3 × 2 design

(b) 3 × 3 design

(c) 4 × 3 design

3. H_0: There is no difference in means of heights of plants grown in different soil types.

H_1: There is a difference in means of heights of plants grown in different soil types.

The first set of hypotheses concerns the interaction effect; the second and third sets test the effects of the independent variables, which are sometimes called the **main effects.**

As with the one-way ANOVA, a between-group variance estimate is calculated, and a within-group variance estimate is calculated. An F test is then performed for each of the independent variables and the interaction. The results of the two-way ANOVA are summarized in a two-way table, as shown in Table 12–4 for the plant experiment.

Table 12–4 ANOVA Summary Table for Plant Food and Soil Type

Source	Sum of squares	d.f.	Mean square	F
Plant food				
Soil type				
Interaction				
Within (error)				
Total				

In general, the two-way **ANOVA summary table** is set up as shown in Table 12–5.

Table 12–5 ANOVA Summary Table

Source	Sum of squares	d.f.	Mean square	F
A	SS_A	$a - 1$	MS_A	F_A
B	SS_B	$b - 1$	MS_B	F_B
$A \times B$	$SS_{A\times B}$	$(a - 1)(b - 1)$	$MS_{A\times B}$	$F_{A\times B}$
Within (error)	SS_W	$ab(n - 1)$	MS_W	
Total				

In the table,

$$SS_A = \text{sum of squares for factor } A$$
$$SS_B = \text{sum of squares for factor } B$$
$$SS_{A\times B} = \text{sum of squares for interaction}$$
$$SS_W = \text{sum of squares for error term (within-group)}$$
$$a = \text{number of levels of factor } A$$
$$b = \text{number of levels of factor } B$$
$$n = \text{number of subjects in each group}$$
$$MS_A = \frac{SS_A}{a-1}$$
$$MS_B = \frac{SS_B}{b-1}$$
$$MS_{A\times B} = \frac{SS_{A\times B}}{(a-1)(b-1)}$$
$$MS_W = \frac{SS_W}{ab(n-1)}$$
$$F_A = \frac{MS_A}{MS_W} \quad \text{with d.f.N.} = a-1,\ \text{d.f.D.} = ab(n-1)$$
$$F_B = \frac{MS_B}{MS_W} \quad \text{with d.f.N.} = b-1,\ \text{d.f.D.} = ab(n-1)$$
$$F_{A\times B} = \frac{MS_{A\times B}}{MS_W} \quad \text{with d.f.N.} = (a-1)(b-1),\ \text{d.f.D.} = ab(n-1)$$

The assumptions for the two-way analysis of variance are basically the same as those for the one-way ANOVA, except for sample size.

Assumptions for the Two-Way ANOVA

1. The populations from which the samples were obtained must be normally or approximately normally distributed.
2. The samples must be independent.
3. The variances of the populations from which the samples were selected must be equal.
4. The groups must be equal in sample size.

The computational procedure for the two-way ANOVA is quite lengthy. For this reason, it will be omitted in Example 12–5, and only the two-way ANOVA summary table will be shown. The table used in Example 12–5 is similar to the one generated by most computer programs. You should be able to interpret the table and summarize the results.

Example 12–5

Gasoline Consumption

A researcher wishes to see whether the type of gasoline used and the type of automobile driven have any effect on gasoline consumption. Two types of gasoline, regular and high-octane, will be used, and two types of automobiles, two-wheel- and four-wheel-drive, will be used in each group. There will be two automobiles in each group, for a total of eight automobiles used. Using a two-way analysis of variance, the researcher will perform the following steps.

Step 1 State the hypotheses.

Step 2 Find the critical value for each F test, using $\alpha = 0.05$.

Of Americans born today, one-third of the women will reach age 100, compared to only 10% of the men, according to Ronald Klatz, M.D., president of the American Academy of Anti-Aging Medicine.

Step 3 Complete the summary table to get the test value.

Step 4 Make the decision.

Step 5 Summarize the results.

The data (in miles per gallon) are shown here, and the summary table is given in Table 12–6.

	Type of automobile	
Gas	**Two-wheel-drive**	**Four-wheel-drive**
Regular	26.7 25.2	28.6 29.3
High-octane	32.3 32.8	26.1 24.2

Table 12–6 ANOVA Summary Table for Example 12–5

Source	SS	d.f.	MS	F
Gasoline A	3.920			
Automobile B	9.680			
Interaction ($A \times B$)	54.080			
Within (error)	3.300			
Total	70.980			

Solution

Step 1 State the hypotheses. The hypotheses for the interaction are these:

H_0: There is no interaction effect between type of gasoline used and type of automobile a person drives on gasoline consumption.

H_1: There is an interaction effect between type of gasoline used and type of automobile a person drives on gasoline consumption.

The hypotheses for the gasoline types are

H_0: There is no difference between the means of gasoline consumption for two types of gasoline.

H_1: There is a difference between the means of gasoline consumption for two types of gasoline.

The hypotheses for the types of automobile driven are

H_0: There is no difference between the means of gasoline consumption for two-wheel-drive and four-wheel-drive automobiles.

H_1: There is a difference between the means of gasoline consumption for two-wheel-drive and four-wheel-drive automobiles.

Step 2 Find the critical values for each F test. In this case, each independent variable, or factor, has two levels. Hence, a 2×2 ANOVA table is used. Factor A is designated as the gasoline type. It has two levels, regular and high-octane; therefore, $a = 2$. Factor B is designated as the automobile type. It also has two levels; therefore, $b = 2$. The degrees of freedom for each factor are as follows:

Factor A: d.f.N. $= a - 1 = 2 - 1 = 1$

Factor B: d.f.N. $= b - 1 = 2 - 1 = 1$

Interaction ($A \times B$): d.f.N. $= (a - 1)(b - 1)$

$= (2 - 1)(2 - 1) = 1 \cdot 1 = 1$

Within (error): d.f.D. $= ab(n - 1)$

$= 2 \cdot 2(2 - 1) = 4$

where n is the number of data values in each group. In this case, $n = 2$.

The critical value for the F_A test is found by using $\alpha = 0.05$, d.f.N. = 1, and d.f.D. = 4. In this case, $F_A = 7.71$. The critical value for the F_B test is found by using $\alpha = 0.05$, d.f.N. = 1, and d.f.D. = 4; also F_B is 7.71. Finally, the critical value for the $F_{A\times B}$ test is found by using d.f.N. = 1 and d.f.D. = 4; it is also 7.71.

Note: If there are different levels of the factors, the critical values will not all be the same. For example, if factor A has three levels and factor b has four levels, and if there are two subjects in each group, then the degrees of freedom are as follows:

d.f.N. $= a - 1 = 3 - 1 = 2$ factor A

d.f.N. $= b - 1 = 4 - 1 = 3$ factor B

d.f.N. $= (a - 1)(b - 1) = (3 - 1)(4 - 1)$

$= 2 \cdot 3 = 6$ factor $A \times B$

d.f.N. $= ab(n - 1) = 3 \cdot 4(2 - 1) = 12$ within (error) factor

Step 3 Complete the ANOVA summary table to get the test values. The mean squares are computed first.

$$MS_A = \frac{SS_A}{a-1} = \frac{3.920}{2-1} = 3.920$$

$$MS_B = \frac{SS_B}{b-1} = \frac{9.680}{2-1} = 9.680$$

$$MS_{A\times B} = \frac{SS_{A\times B}}{(a-1)(b-1)} = \frac{54.080}{(2-1)(2-1)} = 54.080$$

$$MS_W = \frac{SS_W}{ab(n-1)} = \frac{3.300}{4} = 0.825$$

The F values are computed next.

$$F_A = \frac{MS_A}{MS_W} = \frac{3.920}{0.825} = 4.752 \qquad \text{d.f.N.} = a - 1 = 1 \qquad \text{d.f.D.} = ab(n-1) = 4$$

$$F_B = \frac{MS_B}{MS_W} = \frac{9.680}{0.825} = 11.733 \qquad \text{d.f.N.} = b - 1 = 1 \qquad \text{d.f.D.} = ab(n-1) = 4$$

$$F_{A\times B} = \frac{MS_{A\times B}}{MS_W} = \frac{54.080}{0.825} = 65.552 \qquad \text{d.f.N.} = (a-1)(b-1) = 1 \qquad \text{d.f.D.} = ab(n-1) = 4$$

The completed ANOVA table is shown in Table 12–7.

Table 12–7 Completed ANOVA Summary Table for Example 12–5

Source	SS	d.f.	MS	F
Gasoline *A*	3.920	1	3.920	4.752
Automobile *B*	9.680	1	9.680	11.733
Interaction ($A \times B$)	54.080	1	54.080	65.552
Within (error)	3.300	4	0.825	
Total	70.980	7		

Step 4 Make the decision. Since $F_B = 11.733$ and $F_{A\times B} = 65.552$ are greater than the critical value 7.71, the null hypotheses concerning the type of automobile driven and the interaction effect should be rejected. Since the interaction effect is statistically significant no decision should be made about the automobile type without further investigation.

Step 5 Summarize the results. Since the null hypothesis for the interaction effect was rejected, it can be concluded that the combination of type of gasoline and type of automobile does affect gasoline consumption.

Interesting Fact

Some birds can fly as high as 5 miles.

In the preceding analysis, the effect of the type of gasoline used and the effect of the type of automobile driven are called the *main effects.* If there is no significant interaction effect, the main effects can be interpreted independently. However, if there is a significant interaction effect, the main effects must be interpreted cautiously.

To interpret the results of a two-way analysis of variance, researchers suggest drawing a graph, plotting the means of each group, analyzing the graph, and interpreting the results. In Example 12–5, find the means for each group or cell by adding the data values in each cell and dividing by *n*. The means for each cell are shown in the chart here.

	Type of automobile	
Gas	**Two-wheel-drive**	**Four-wheel-drive**
Regular	$\bar{X} = \frac{26.7 + 25.2}{2} = 25.95$	$\bar{X} = \frac{28.6 + 29.3}{2} = 28.95$
High-octane	$\bar{X} = \frac{32.3 + 32.8}{2} = 32.55$	$\bar{X} = \frac{26.1 + 24.2}{2} = 25.15$

The graph of the means for each of the variables is shown in Figure 12–4. In this graph, the lines cross each other. When such an intersection occurs and the interaction is significant, the interaction is said to be a **disordinal interaction.** When there is a disordinal interaction, you should not interpret the main effects without considering the interaction effect.

The other type of interaction that can occur is an *ordinal interaction.* Figure 12–5 shows a graph of means in which an ordinal interaction occurs between two variables. The lines do not cross each other, nor are they parallel. If the *F* test value for the interaction is significant and the lines do not cross each other, then the interaction is said to be an **ordinal interaction** and the main effects can be interpreted independently of each other.

Finally, when there is no significant interaction effect, the lines in the graph will be parallel or approximately parallel. When this situation occurs, the main effects can be interpreted independently of each other because there is no significant interaction.

Figure 12–4

Graph of the Means of the Variables in Example 12–5

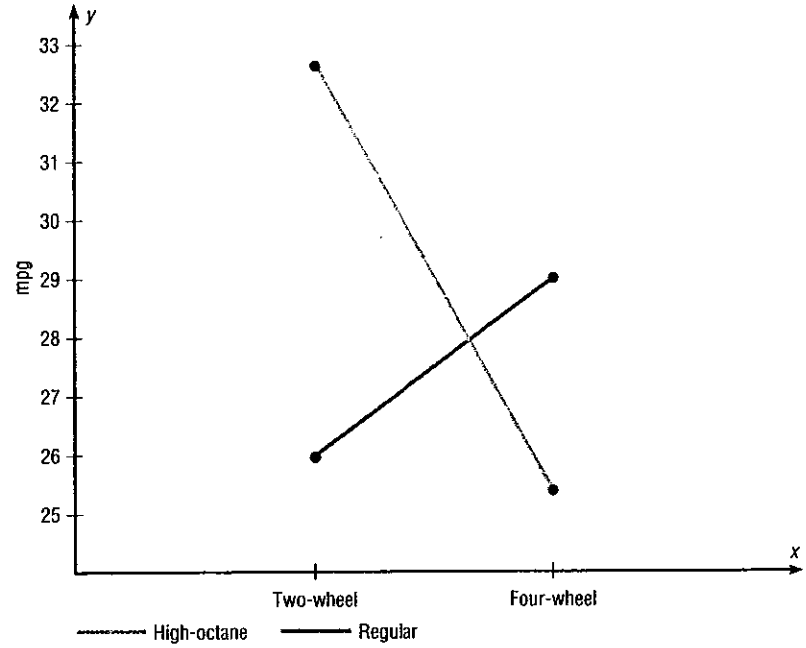


Figure 12–5

Graph of Two Variables Indicating an Ordinal Interaction

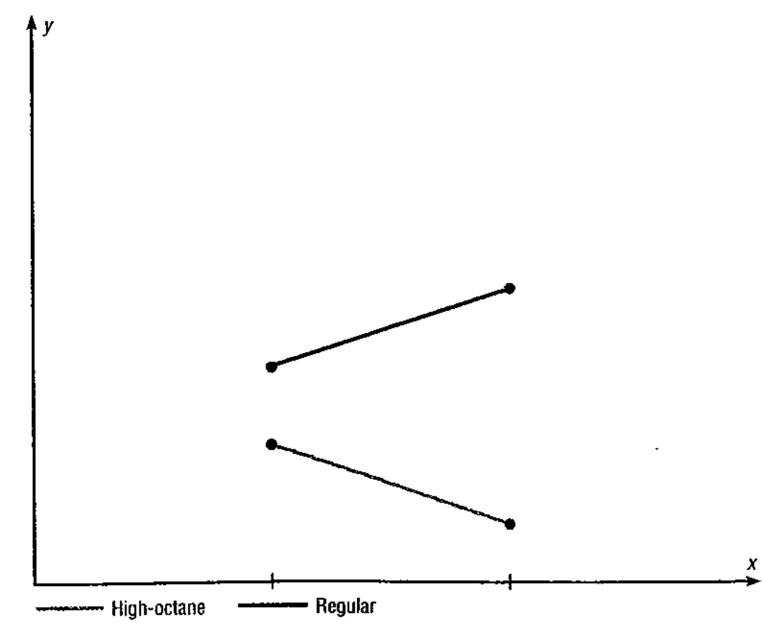
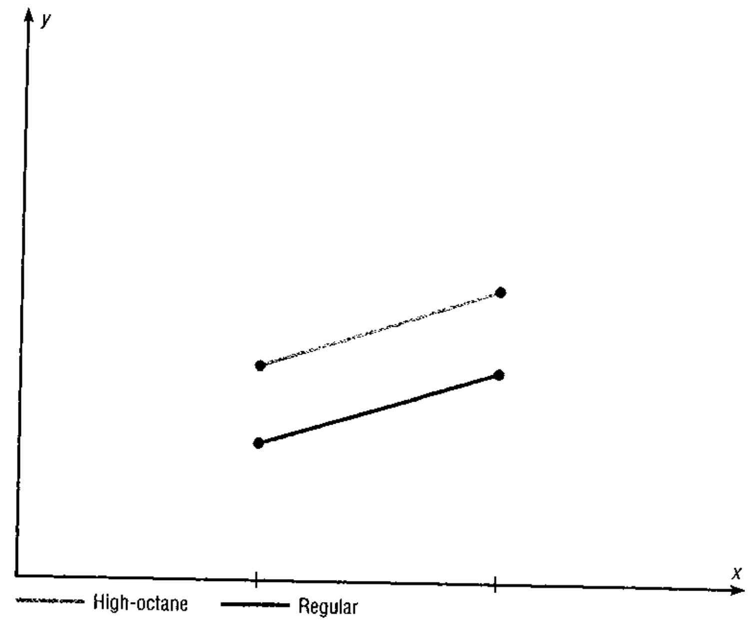


Figure 12–6 shows the graph of two variables when the interaction effect is not significant; the lines are parallel.

Example 12–5 was an example of a 2 × 2 two-way analysis of variance, since each independent variable had two levels. For other types of variance problems, such as a 3 × 2 or a 4 × 3 ANOVA, interpretation of the results can be quite complicated. Procedures using tests such as the Tukey and Scheffé tests for analyzing the cell means exist and are similar to the tests shown for the one-way ANOVA, but they are beyond the scope of this textbook. Many other designs for analysis of variance are available to researchers, such as three-factor designs and repeated-measure designs; they are also beyond the scope of this book.

Figure 12–6

Graph of Two Variables Indicating No Interaction



In summary, the two-way ANOVA is an extension of the one-way ANOVA. The former can be used to test the effects of two independent variables and a possible interaction effect on a dependent variable.

Applying the Concepts 12–3

Automobile Sales Techniques

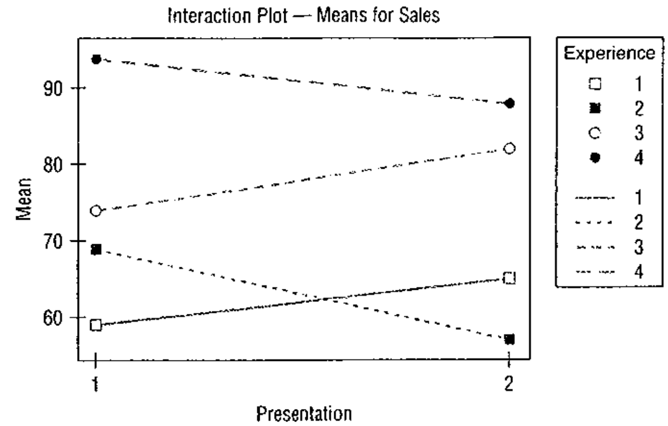
The following outputs are from the result of an analysis of how car sales are affected by the experience of the salesperson and the type of sales technique used. Experience was broken up into four levels, and two different sales techniques were used. Analyze the results and draw conclusions about level of experience with respect to the two different sales techniques and how they affect car sales.

Two-Way Analysis of Variance

```
Analysis of Variance for Sales
Source         DF        SS         MS
Experience      3    3414.0     1138.0
Presentation    1       6.0        6.0
Interaction     3     414.0      138.0
Error          16     838.0       52.4
Total          23    4672.0

                         Individual 95% CI
Experience     Mean   -----+---------+---------+---------+-----
1              62.0    (-----*-----)
2              63.0     (-----*-----)
3              78.0                    (-----*-----)
4              91.0                                 (-----*-----)
                      -----+---------+---------+---------+-----
                          60.0      70.0      80.0      90.0

                         Individual 95% CI
Presentation   Mean   ------+---------+---------+---------+-----
1              74.0      (----------------*------------------)
2              73.0   (-----------------*-----------------)
                      ------+---------+---------+---------+-----
```



See page 668 for the answers.

Exercises 12–3

1. How does the two-way ANOVA differ from the one-way ANOVA?

2. Explain what is meant by *main effects* and *interaction effect.*

3. How are the values for the mean squares computed?

4. How are the F test values computed?

5. In a two-way ANOVA, variable A has three levels and variable B has two levels. There are five data values in each cell. Find each degrees-of-freedom value.
   a. d.f.N. for factor A
   b. d.f.N. for factor B
   c. d.f.N. for factor $A \times B$
   d. d.f.D. for the within (error) factor

6. In a two-way ANOVA, variable A has six levels and variable B has five levels. There are seven data values in each cell. Find each degrees-of-freedom value.
   a. d.f.N. for factor A
   b. d.f.N. for factor B
   c. d.f.N. for factor $A \times B$
   d. d.f.D. for the within (error) factor

7. What are the two types of interactions that can occur in the two-way ANOVA?

8. When can the main effects for the two-way ANOVA be interpreted independently?

9. Describe what the graph of the variables would look like for each situation in a two-way ANOVA experiment.
   a. No interaction effect occurs.
   b. An ordinal interaction effect occurs.
   c. A disordinal interaction effect occurs.

For Exercises 10 through 15, perform these steps. Assume that all variables are normally or approximately normally distributed, that the samples are independent, and that the population variances are equal.

a. State the hypotheses.
b. Find the critical value for each F test.
c. Complete the summary table and find the test value.
d. Make the decision.
e. Summarize the results. (Draw a graph of the cell means if necessary.)

10. **Increasing Plant Growth** A gardening company is testing new ways to improve plant growth. Twelve plants are randomly selected and exposed to a combination of two factors, a "Grow-light" in two different strengths and a plant food supplement with different mineral supplements. After a number of days, the plants are measured for growth and the results (in inches) are put into the appropriate boxes.

	Grow-light 1	Grow-light 2
Plant food A	9.2, 9.4, 8.9	8.5, 9.2, 8.9
Plant food B	7.1, 7.2, 8.5	5.5, 5.8, 7.6

Can an interaction between the two factors be concluded? Is there a difference in mean growth with respect to light? With respect to plant food? Use $\alpha = 0.05$.

11. **Environmentally Friendly Air Freshener** As a new type of environmentally friendly, natural air freshener is being developed, it is tested to see whether the effects of temperature and humidity affect the length of time that the scent is effective. The numbers of days that the air freshener had a significant level of scent are listed below for two temperature and humidity levels. Can an interaction between the two factors be concluded? Is there a difference in mean length of effectiveness with respect to humidity? With respect to temperature? Use $\alpha = 0.05$.

	Temperature 1	Temperature 2
Humidity 1	35, 25, 26	35, 31, 37
Humidity 2	28, 22, 21	23, 19, 18

12. **Home-Building Times** A contractor wishes to see whether there is a difference in the time (in days) it takes two subcontractors to build three different types of homes. At $\alpha = 0.05$, analyze the data shown here, using a two-way ANOVA. See below for raw data.

Data for Exercise 12

	Home type		
Subcontractor	I	II	III
A	25, 28, 26, 30, 31	30, 32, 35, 29, 31	43, 40, 42, 49, 48
B	15, 18, 22, 21, 17	21, 27, 18, 15, 19	23, 25, 24, 17, 13

ANOVA Summary Table for Exercise 12

Source	SS	d.f.	MS	F
Subcontractor	1672.553			
Home type	444.867			
Interaction	313.267			
Within	328.800			
Total	2759.487			

13. **Durability of Paint** A pigment laboratory is testing both dry additives and solution-based additives to see their effect on the durability rating (a number from 1 to 10) of a finished paint product. The paint to be tested is divided into four equal quantities, and a different combination of the two additives is added to one-fourth of each quantity. After a prescribed number of hours, the durability rating is obtained for each of the 16 samples, and the results are recorded below in the appropriate space.

	Dry additive 1	Dry additive 2
Solution additive A	9, 8, 5, 6	4, 5, 8, 9
Solution additive B	7, 7, 6, 8	10, 8, 6, 7

Can an interaction be concluded between the dry and solution additives? Is there a difference in mean durability rating with respect to dry additive used? With respect to solution additive? Use $\alpha = 0.05$.

14. **Types of Outdoor Paint** Two types of outdoor paint, enamel and latex, were tested to see how long (in months) each lasted before it began to crack, flake, and peel. They were tested in four geographic locations in the United States to study the effects of climate on the paint. At $\alpha = 0.01$, analyze the data shown, using a two-way ANOVA shown below. Each group contained five test panels. See below for raw data.

Data for Exercise 14

	Geographic location			
Type of paint	North	East	South	West
Enamel	60, 53, 58, 62, 57	54, 63, 62, 71, 76	80, 82, 62, 88, 71	62, 76, 55, 48, 61
Latex	36, 41, 54, 65, 53	62, 61, 77, 53, 64	68, 72, 71, 82, 86	63, 65, 72, 71, 63

ANOVA Summary Table for Exercise 14

Source	SS	d.f.	MS	F
Paint type	12.1			
Location	2501.0			
Interaction	268.1			
Within	2326.8			
Total	5108.0			

15. Age and Sales A company sells three items: swimming pools, spas, and saunas. The owner decides to see whether the age of the sales representative and the type of item affect monthly sales. At $\alpha = 0.05$, analyze the data shown, using a two-way ANOVA. Sales are given in hundreds of dollars for a randomly selected month, and five salespeople were selected for each group.

ANOVA Summary Table for Exercise 15

Source	SS	d.f.	MS	F
Age	168.033			
Product	1,762.067			
Interaction	7,955.267			
Within	2,574.000			
Total	12,459.367			

Data for Exercise 15

	Product		
Age of salesperson	Pool	Spa	Sauna
Over 30	56, 23, 52, 28, 35	43, 25, 16, 27, 32	47, 43, 52, 61, 74
30 or under	16, 14, 18, 27, 31	58, 62, 68, 72, 83	15, 14, 22, 16, 27

Step by Step

MINITAB
Step by Step

Two-Way Analysis of Variance

For Example 12–5, how do gasoline type and vehicle type affect gasoline mileage?

1. Enter the data into three columns of a worksheet. The data for this analysis have to be "stacked" as shown.
   a) All the gas mileage data are entered in a single column named MPG.
   b) The second column contains codes identifying the gasoline type, a 1 for regular or a 2 for high-octane.
   c) The third column will contain codes identifying the type of automobile, 1 for two-wheel-drive or 2 for four-wheel-drive.

↓	C1	C2	C3
	MPG	GasCode	TypeCode
1	26.7	1	1
2	25.2	1	1
3	32.3	2	1
4	32.8	2	1
5	28.6	1	2
6	29.3	1	2
7	26.1	2	2
8	24.2	2	2

2. Select **Stat>ANOVA>Two-Way.**
   a) Double-click MPG in the list box.
   b) Double-click GasCode as Row factor.
   c) Double-click TypeCode as Column factor.
   d) Check the boxes for Display means, then click [OK].

The session window will contain the results.