ferior (material) activity. In the "Gospel according to Thomas" discovered in 1945,* Jesus says ironically:

> If the flesh came for the sake of the spirit, it
> is a miracle. But if the spirit for the sake
> of the flesh—it is a miracle of miracles.

All the history of mathematics is a convincing proof that such a "miracle of miracles" is impossible. If we look upon the decisive moment in the development of mathematics, the moment when it took its first step and when the ground on which it is based came into being—I have in mind *logical proof*—we shall see that this was done with material that actually excluded the very possibility of practical applications. The first theorems of Thales of Miletus proved statements evident to every sensible man—for instance that a diameter divides the circle into two equal parts. Genius was needed not to be convinced of the justice of these statements, but to understand that they need proofs. Obviously the practical value of such discoveries is nought.

In ending, I want to express a hope that . . . mathematics may serve now as a model for the solution of the main problem of our epoch: to reveal a supreme religious goal and to fathom the meaning of the spiritual activity of mankind."

Thus, Shafarevitch—a surprising statement to come from the lips of any contemporary mathematician in or out of Russia. But it is hardly a new statement. The Greek philosophers thought of mathematics as a bridge between theology and the perceptible, physical world, and this view was stressed and developed by the Neoplatonists. The quadrivium: arithmetic, music, geometry, astronomy, already known to Protagoras (d. 411 B.C.), was thought to lead the mind upward through mathematics to the heavenly sphere where the eternal movements were the perceptible form of the world soul.

Further Readings. See Bibliography

P. Merlan; I. R. Shafarevitch

* (Footnote added by P.J.D.) The Gospel of Thomas is probably the most significant of the books discovered in the 1940s at Nag Hammadi in Egypt. It is a compilation of the "sayings of Jesus," placed in a Gnostic context. Gnosticism asserts that there is a secret knowledge (gnosis) through which salvation can be achieved and that this knowledge is superior to ordinary faith. (See R. M. Grant, "Gnosticism, Marcion, Origen" in "The Crucible of Christianity," A. Toynbee, ed., London: Thames and Hudson, 1969.)

Unorthodoxies

MOST MATHEMATICIANS have had the following experience and those whose activities are somewhat more public have had it often: an unsolicited letter arrives from an unknown individual and contained in the letter is a piece of mathematics of a very sensational nature. The writer claims that he has solved one of the great unsolved mathematical problems or that he has refuted one of the standard mathematical assertions. In times gone by, circle squaring was a favorite activity; in fact, this activity is so old that Aristophanes parodies the circle squarers of the world. In more recent times, proofs of Fermat's "Last Theorem" have been very popular. The writer of such a letter is usually an amateur, with very little training in mathematics. Very often he has a poor understanding of the nature of the problem he is dealing with, and an imperfect notion of just what a mathematical proof is and how it operates. The writer is usually male, frequently a retired person with leisure to pursue his mathematics, often he has achieved considerable professional status in the larger community and he exhibits his status symbols within the mathematical work itself.

Very often the correspondent not only "succeeds" in solving one of the great mathematical unsolvables, but has also found a way to construct an antigravity shield, to interpret the mysteries of the Great Pyramid and of Stonehenge, and is well on his way to producing the Philosophers' Stone. This is no exaggeration.

If the recipient of such a letter answers it, he will generally find himself entangled with a person with whom he cannot communicate scientifically and who exhibits many symptoms of paranoia. One gets to recognize such correspondents on sight, and to leave their letters unanswered, thus unfortunately increasing the paranoia.

I have on my desk as I write a paper of just this sort which was passed on to me by the editor of one of the leading mathematical journals in the United States. For self-protection I shall change the personal details, retaining the flavor as best I can. The paper is nicely and expensively printed on glossy stock and comes from the Philippines. It is written in Spanish and purports to be a demonstration of Fermat's Last Theorem. There is a photograph of the author, a fine-looking gentleman in his eighties, who had been a general in the Philippine army. Along with the mathematics there is a lengthy autobiography of the author. It would appear that the author's ancestors were French aristocrats, that after the French Revolution the cadet branch was sent to the East, whence the family made its way to the Philippines, etc. There are also included in this paper on Fermat's Last Theorem, nice engravings of the last three reigning Louis of France and a long plea for the restoration of the Bourbon dynasty. After page one, the mathematics rapidly wanders into incomprehensibility. I spent ten minutes with this paper; your average editor would spend less. Why? The Fermat "Last Theorem" is at the time of this writing a great unsolved problem. Perhaps the man from the Philippines has solved it. Why did I not examine his work carefully?

There are many types of anomalous or idiosyncratic writing in mathematics. How does the community strain out what it wants? How does one recognize brilliance, genius, crankiness, madness? Anyone can make an honest error. Shortly after World War II, Professor Hans Rademacher of the University of Pennsylvania, one of the leading number theoreticians in the world, thought he had proved the famous Riemann Hypothesis. (See page 405 for

a statement of this conjecture.) The media got wind of this news and an account was published in *Time* magazine. It is not often that a mathematical discovery makes the popular press. But shortly thereafter, an error was found in Rademacher's work. The problem is still open as these words are being written.

This is an example of incorrect mathematics produced within the bounds of mathematical orthodoxy—and detected there as well. This happens to the best of us every day of the week. When the error is pointed out, one recognizes it as an error and acknowledges it. This kind of situation is dealt with routinely.

At the opposite pole, there is the type whose psychopathology has just been described above. This type of writing is usually dismissed at sight. The probability that it contains something of interest is extremely small and it is a risk that the mathematical community is willing to take. But it is not always easy to draw the line between the crank and the genius.

An obscure and poor young man from a little-known place in India writes a letter around 1913 to G. H. Hardy, the leading English mathematician of the day. The letter betrays signs of inadequate training, it is intuitive and disorganized, but Hardy recognizes in it brilliant pearls of mathematics. The Indian's name was Srinivasa Ramanujan. If Hardy had not arranged for a fellowship for Ramanujan, some very interesting mathematics might have been lost forever.

Srinivasa Ramanujan 1887–1920

Then there was the case of Hermann Grassman (1809–1877). In 1844 Grassman published a book called *Die lineale Ausdehnungslehre.* This work is today recognized as a work of genius. It was an anticipation of what would be subsequently worked out as vector and tensor analysis and associative algebras (quaternions). But because Grassman's exposition was obscure, mystical, and unusually abstract for its period, this work repelled the mathematical community and was ignored for many years.

Less known than either Grassman or Ramanujan is the story of Jozef Maria Wronski (1776–1853), whose personality and work combined elements from pretentious naiveté to genius near madness. Today Wronski is chiefly remembered for a certain determinant $W[u_1, u_2, \ldots, u_n] =$

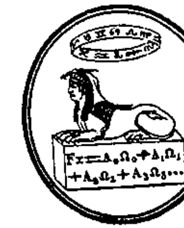
$$\begin{vmatrix} u_1 & u_2 & \cdots & u_n \\ u_1' & u_2' & \cdots & u_n' \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ u_1^{(n-1)} & u_2^{(n-1)} & \cdots & u_n^{(n-1)} \end{vmatrix}$$ formed from n functions $u_1, \ldots, u_n$.

This determinant is related to theories of linear independence and is of importance in the theory of linear differential equations. Every student of differential equations has heard of the Wronskian.

Wronski was a Pole who fought with Kosciuszko for Polish independence, yet, dedicated his book "Introduction à la Philosophie des Mathématiques et Technie de l'Algorithmie" to His Majesty, Alexander I, Autocrat of all the Russias. A political realist, one would think.

On the 15th of August 1803, Wronski experienced a revelation which enabled him to conceive of "the absolute." His subsequent mathematical and philosophical work was motivated by a drive to expound the absolute and its laws of unification. In addition to his mathematics and philosophy, Wronski pursued theosophy, political and cultural messianism (he wrote five books on this topic), promoted the ideas of arithmosophism, mathematical vitalism, and something which he called "séchelianisme" (from the Hebrew; sechel: reason). This latter purported to change Christianity from a revealed religion to a proved religion. Wronski distinguished three forces which control history: providence, fatality (destiny), and reason. He constructed almost all of his system around the negation of the principle of inertia. Inasmuch as the material has no inertia it does not compete with the spiritual. The scientific ideal would be a kind of panmathematism which unites the knowledge of the formation of mathematical systems with the laws of living beings.

Wronski's philosophy is, apparently, not uninteresting and ties in with the later writings of Bergson.



Wronski's key to the universe; placed in a cartouche, sanctified by the zodiac, guarded by a sphinx, and printed on all his works.

What do we find, mathematically speaking, when we open up the first volume of his *Oeuvres Mathématiques?*

It appears, at a quick glance, to be mixture of the theory of infinite series, difference equations, differential equations, and complex variables. It is long, rambling, polemical, tedious, obscure, egocentric, and full of philosophical interpolations giving unifying schemata. The "Grand Law of the Generation of Quantities," which contains the Key to the Universe, appears as equation (7). Wronski sold it to a wealthy banker. The banker did not pay up and Wronski aired his complaints publicly. Here is the Grand Law:

"$Fx = A_0\Omega_0 + A_1\Omega_1 + A_2\Omega_2 + A_3\Omega_3 + A_4\Omega_4$ + etc. à l'infini."

What does it mean? It appears to be a general scheme for the expansion of functions as linear combinations of other functions; a kind of generalized Taylor expansion which contains all expansions of the past and all future expansions.

It is not possible for me to grasp the essential spirit of Wronski's work; and it would take a profound student of eighteenth century mathematics to tell what, if anything, is new or useful in the four volumes. I am only too willing to accept the judgment of history that Wronski deserves to be remembered only for the Wronskian. The doors of the mathematical past are often rusted. If an inner chamber is difficult of access, it does not necessarily mean that treasure is to be found therein.

There is work, then, which is wrong, is acknowledged to be wrong and which, at some later date may be set to rights. There is work which is dismissed without examination. There is work which is so obscure that it is difficult to interpret and is perforce ignored. Some of it may emerge later. There is work which may be of great importance—such as Cantor's set theory—which is heterodox, and as a result, is ignored or boycotted. There is also work, perhaps the bulk of the mathematical output, which is admittedly correct, but which in the long run is ignored, for lack of interest, or because the main streams of mathematics did not choose to pass that way. In the final analysis, there can be no formalization of what is right and how we know it right, what is accepted, and what the mechanism for acceptance is. As Hermann Weyl has written, "Mathematizing may well be a creative activity of man . . . whose historical decisions defy complete objective rationalization."

Further Readings. See Bibliography

J. M. Wronski

The Individual and the Culture

THE RELATIONSHIP BETWEEN the individual and society has never been of greater concern than it is today. The opposing tendencies of amalgamation versus fragmentation, of nationalism versus regionalism, of the freedom of the individual as opposed to the security within a larger group are acting out a drama on history's stage which may settle a direction for civilization for the next several centuries. Running perpendicularly to these struggles is the conflict between the "Two Cultures": the humanistic and the technological.

Mathematics, being a human activity, possesses all four components. It profits greatly from individual genius, but thrives only with the tacit approval of the wider community. As a great art form, it is humanistic; it is scientific-technological in its applications.

To understand just where and how mathematics fits into the human condition, it is important that we pay heed to all four of these components.

There are two extreme positions on the history of discovery. The first position holds that individual genius is the wellspring of discovery. The second position is that social and economic forces bring forth discovery. Most people do not hold with the one or with the other in a pure form, but try to find a mixture which is compatible with their own experiences.

The doctrine of the individual is the more familiar of the two, the easier of the two, and we are rather more comfortable with it. As teachers, we try our best to concentrate on the individual student; we do not attempt to teach people in their multitude. Methods of teaching en masse, through media of some sort, all postulate an individual at the receiving end. On the contrary, the word "indoctrination," which implies a kind of group phenomenon, worries us.

We study mathematical didactics and strategies of discovery as in Pólya's books (See Chapter 6) and try to transfer some of the insights of a great mathematician to our students. We read biographies of great geniuses and study their works carefully.

One of the most striking statements of the doctrine of the individual in mathematics was put forward in an article by Alfred Adler. The author is a professional mathematician and his article is as eloquent as it is dramatic. The article is also a very personal statement; its views are romanticized, manic-depressive, and apocalyptic.

Adler begins by putting the case for an extreme form of élitism:

> Each generation has its few great mathematicians, and mathematics would not even notice the absence of the others. They are useful as teachers, and their research harms no one, but it is of no importance at all. A mathematician is great or he is nothing.

This is accompanied by the statement of "The Happy Few."

> But there is never any doubt about who is and who is not a creative mathematician, so all that is required is to keep track of the activities of these few men.

"The Few"—or at least five of them—are then identified (as of 1972).

It is noted that the creation of mathematics appears to be a young man's business:

> The mathematical life of a mathematician is short. Work rarely improves after the age of twenty-five or thirty. If little has been accomplished by then, little will ever be ac-