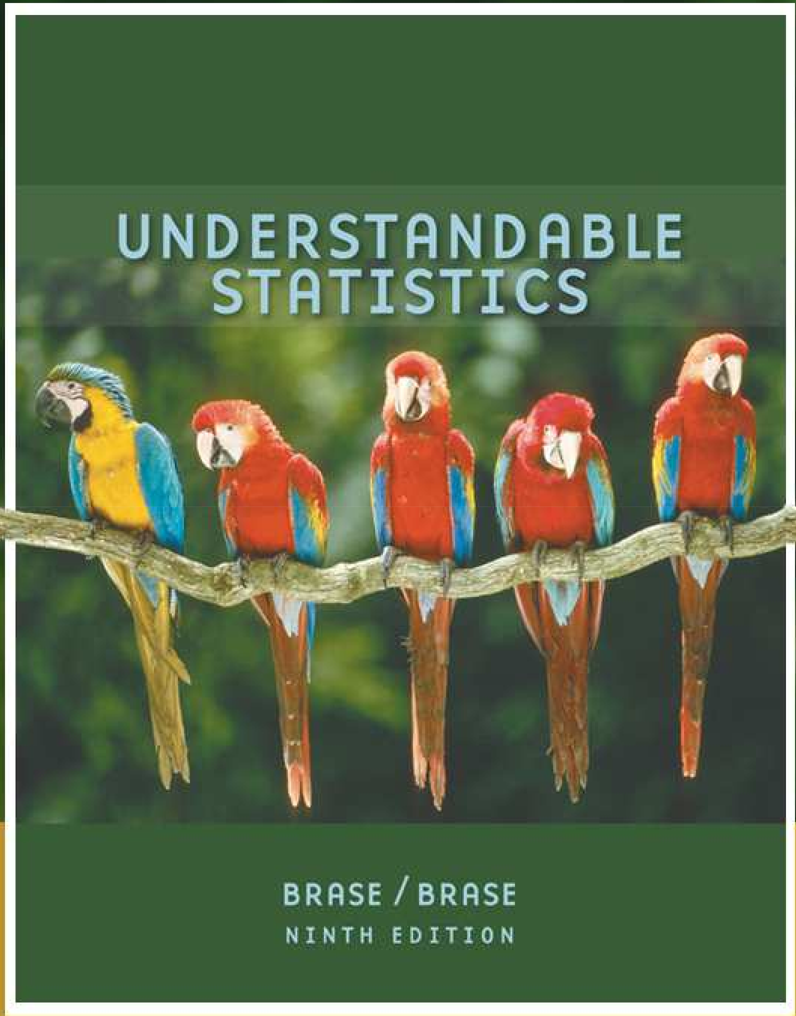

Chapter 4
Elementary Probability Theory

Understandable Statistics
Ninth Edition
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Probability

- Probability is a numerical measure that indicates the likelihood of an event.
- All probabilities are between 0 and 1, inclusive.
- A probability of 0 means the event is impossible.
- A probability of 1 means the event is certain to occur.
- Events with probabilities near 1 are likely to occur.

Probability

- Events can be named with capital letters: A, B, C…

- $P(A)$ means the probability of A occurring.
  - $P(A)$ is read “P of A”
  - $0 \leq P(A) \leq 1$

Probability Assignment

- Assignment by intuition – based on intuition, experience, or judgment.
- Assignment by relative frequency –

  $P(A)$ = Relative Frequency = $\frac{f}{n}$
- Assignment for equally likely outcomes

$$P(A) = \frac{Number\ of\ Outcomes\ Favorable\ to\ Event\ A}{Total\ Number\ of\ Outcomes}$$

Law of Large Numbers

In the long run, as the sample size increases, the relative frequency will get closer and closer to the theoretical probability.

- Example: We repeat the penny experiment, and the relative frequency gets closer and closer to $P(\text{head}) = 0.50$

Relative Frequency	**0.52**	**0.518**	**0.495**	**0.503**	**0.4996**
f = number of flips	**104**	**259**	**495**	**1006**	**2498**
n = number of heads	**200**	**500**	**1000**	**2000**	**5000**

Probability Definitions

- Statistical Experiment: Any random activity that results in a definite outcome.
- Event: A collection of one or more outcomes in a statistical experiment.
- Simple Event: An event that consists of exactly one outcome in a statistical experiment.
- Sample Space: The set of all simple events.

- Review for the video, Probability Concepts (math.uprag.edu/ProbabilityConcepts.avi)

- Probability deal with population
- Statistic deal with samples
- Simulation
- Subjective Probability
- Relative Probability   P(A)= #(A)/#(S)
- “Hot Hands”

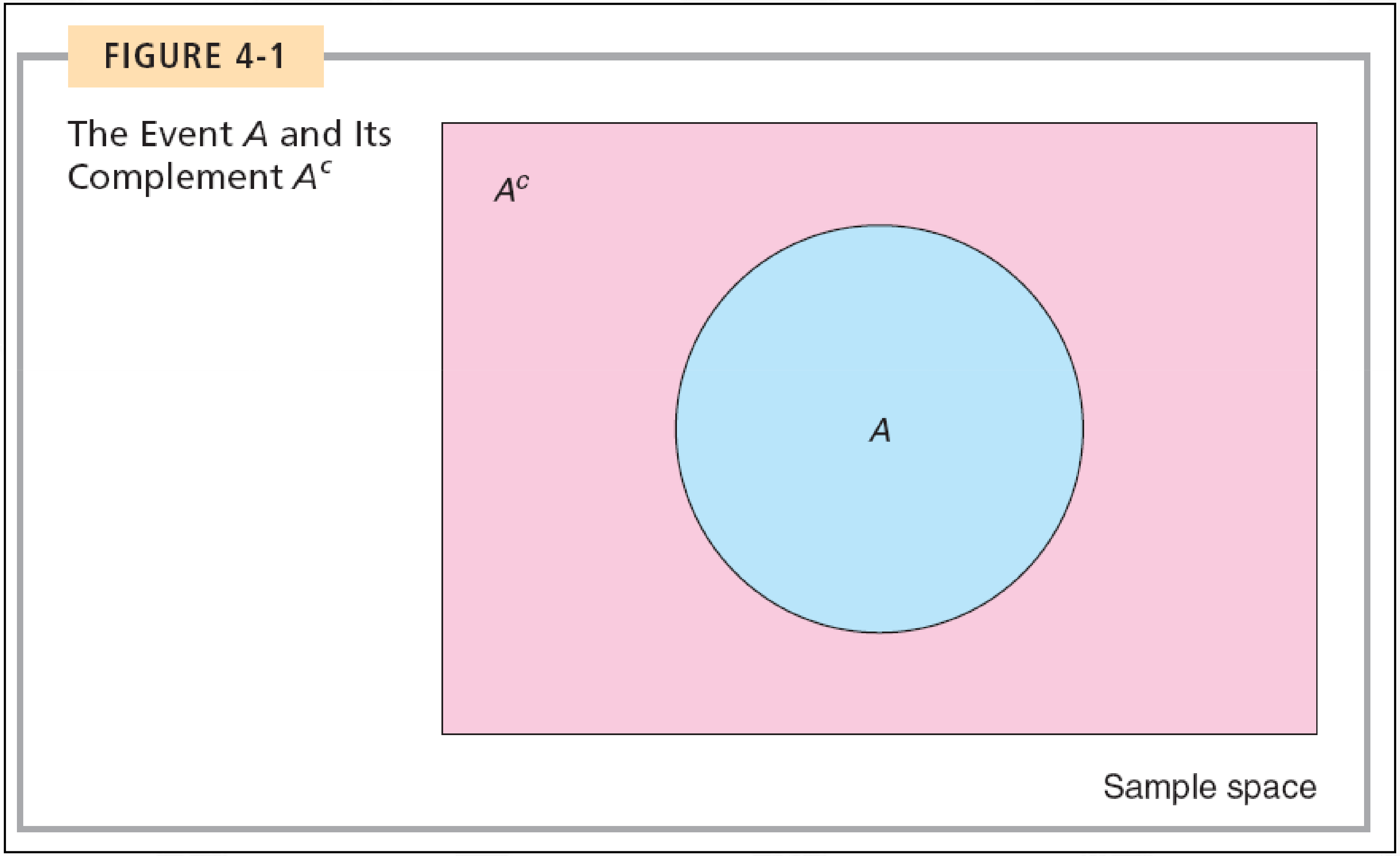
The Sum Rule and The Complement Rule

- The sum of the probabilities of all the simple events in the sample space must equal 1.

- The complement of event *A* is the event that *A* *does not occur,* denoted by A^c

- $P(A^c) = 1 - P(A)$

The Complement Rule



Probability versus Statistics

Probability is the field of study that makes statements about what will occur when a sample is drawn from a known population.

Statistics is the field of study that describes how samples are to be obtained and how inferences are to be made about unknown populations.

Independent Events

- Two events are independent if the occurrence or nonoccurrence of one event does *not* change the probability of the other event.

Multiplication Rule for Independent Events

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

General Multiplication Rule – For all events (independent or not):

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

$$P(A \text{ and } B) = P(B) \cdot P(A \mid B)$$

Conditional Probability (when $P(B) \neq 0$):

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

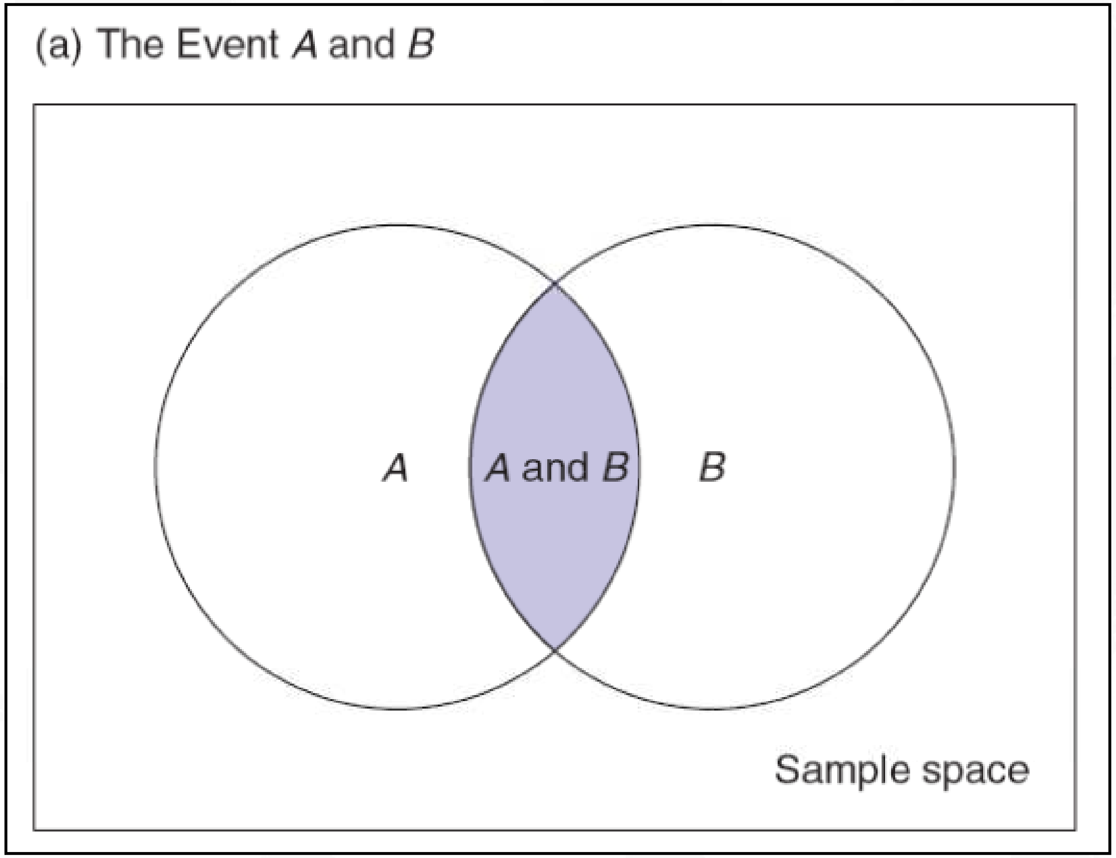
Suppose you are going to throw two fair die. What is the probability of getting a 5 on each die?

Solution:
Sample Space

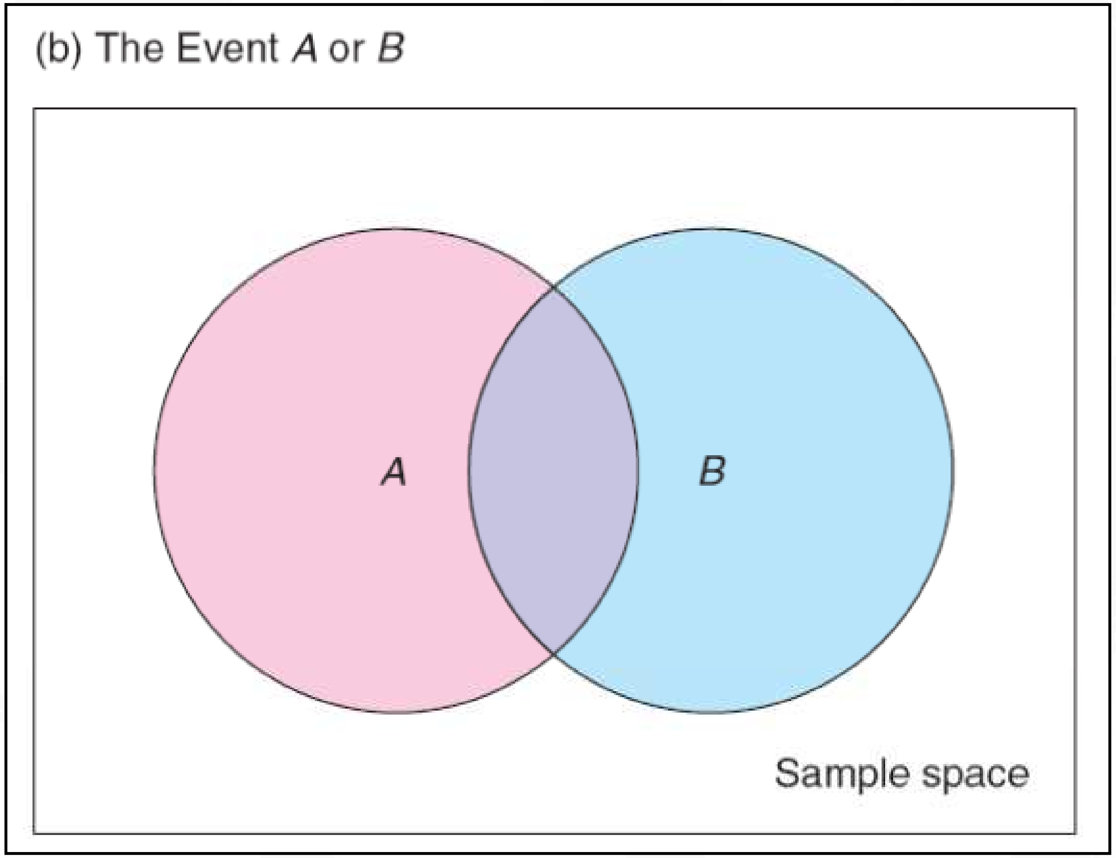
Multiplication Rule

- Compute the probability of drawing two aces from a well-deck of 52 cards if the first card is not replaced before the second is draw.

Two Events Occurring Together



Either or Both of Two Events Occurring



Mutually Exclusive Events

Two events are mutually exclusive if they cannot occur at the same time.

Mutually Exclusive = Disjoint

If A and B are mutually exclusive, then

$$P(A \text{ and } B) = 0$$

Addition Rules

- If A and B are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$.

- If A and B are not mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B)$.

- Rosa, Carmen and Alberto studying together for an exam. The probability of passing Rosa is 0.65, Carmen pass is 0.75 and Alberto pass is 0.50. The probability that Rosa and Carmen pass is 0.55, that Carmen and Alberto pass is 0.35 and that Rosa and Alberto pass is 0.25. The probability that the three pass is 0.20. What is the probability that:
- a) At least one of them pass the test?
- b) Only one of them pass the test?
- c) Carmen and Alberto pass the exam but not Rose?
- d) Alberto not pass the test but at least one of the women?
- e) None pass the test?

Critical Thinking

- Pay attention to translating events described by common English phrases into events described using *and, or, complement*, or *given.*

- Rules and definitions of probabilities have extensive applications in everyday lives.

Bayes's Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$$

- At Litchfield College of Nursing, 85% of incoming freshmen nursing students are female and 15% are male. Recent records indicate that 69% of the entering female students will graduate with a BSN degree, while 95% of the male students will obtain a BSN degree. If an incoming freshman nursing student is selected at random, find the following probabilities. (Use 2 decimal places.)
- (a) *P*(student will graduate | student is female).

  (b) *P*(student will graduate *and* student is female).

  (c) *P*(student will graduate | student is male).

  (d) *P*(student will graduate *and* student is male).

  (e) *P*(student will graduate). Note that those who will graduate are either males who will graduate or females who will graduate.

- 70% of hospital patients are women and 20% are smokers. On the other hand 40% of male patients are smokers. Randomly selected a patient from hospital. What is the probability that is a smoker?

- In one hospital, 98% of babies born alive. On the other hand, 40% of all births are by cesarean and 96% of them survive childbirth. If randomly chooses a woman who will not perform caesarean section. What is the probability that the child live?

Multiplication Rule for Counting

Multiplication rule of counting

If there are n possible outcomes for event E_1 and m possible outcomes for event E_2, then there are a total of $n \times m$ or nm possible outcomes for the series of events E_1 followed by E_2.

This rule extends to outcomes involving three, four, or more series of events.

- A young man has 4 different pants and 6 different shirts. The boy is dressed differently each day. How many days he is dressed different?

- A password to access a computer consists of 36 characters can be letters (26) or numbers (10).

a) How many different passwords can be formed?
b) How many different passwords can be formed containing only numbers?
c) How many different passwords can be formed if they should have at least one letter?

Tree Diagrams

Displays the outcomes of an experiment consisting of a sequence of activities.

- The total number of branches equals the total number of outcomes.
- Each unique outcome is represented by following a branch from start to finish.

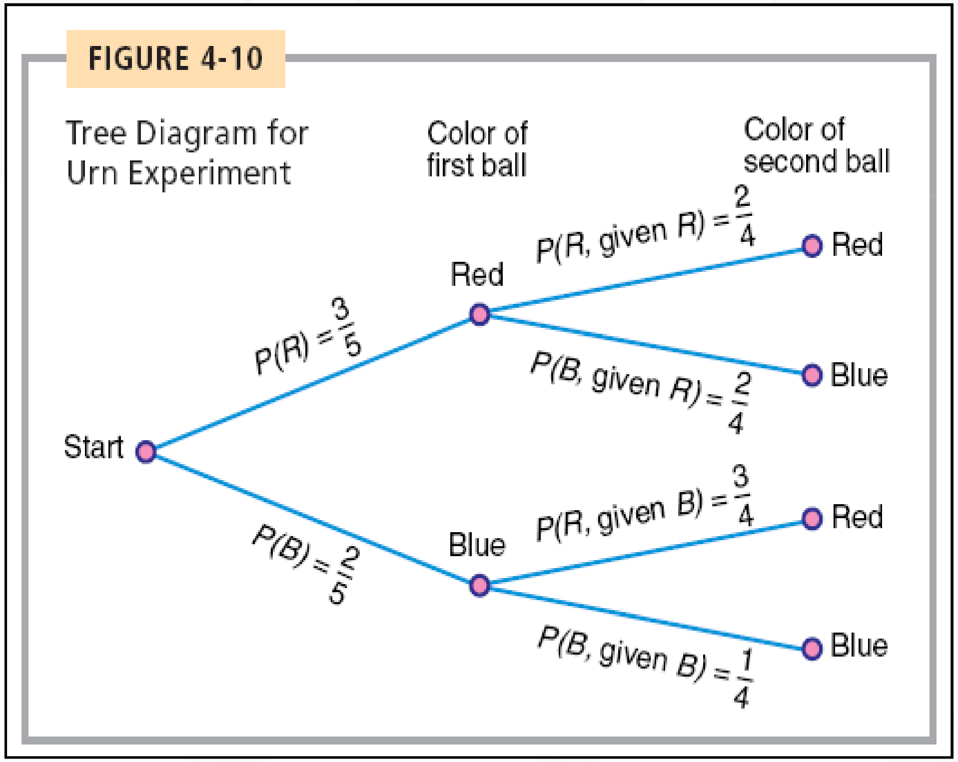
Tree Diagrams with Probability

We can also label each branch of the tree with its respective probability.

To obtain the probability of the events, we can multiply the probabilities as we work down a particular branch.

Urn Example

- Suppose there are five balls in an urn. Three are red and two are blue. We will select a ball, note the color, and, without replacing the first ball, select a second ball.



There are four possible outcomes:
Red, Red
Red, Blue
Blue, Red
Blue, Blue

We can find the probabilities of the outcomes by using the multiplication rule for dependent events.

An urn contains 5 red and 3 black balls. If you drawn two balls, one by one and without replacement. Draw the tree diagram for the sample space.

An urn contains 5 red and 3 black balls. If you drawn two balls, one by one and with replacement. Draw the tree diagram for the sample space.

Factorials

- For counting numbers 1, 2, 3, …
- ! is read "factorial"
  - So for example, 5! is read "five factorial"
- $n! = n * (n-1) * (n-2) * \ldots * 3 * 2 * 1$
  - So for example, 5! = 5 * 4 * 3 * 2 * 1 = 120
- 1! = 1
- 0! = 1

Permutations

- Permutation: ordered grouping of objects.
- Counting Rule for Permutations

Counting rule for permutations

The number of ways to *arrange in order* n distinct objects, taking them r at a time, is

$$P_{n,r} = \frac{n!}{(n-r)!} \qquad (9)$$

where n and r are whole numbers and $n \geq r$. Another commonly used notation for permutations is nPr.

Example. Eight athletes compete in the Olympic final of the 110-meter hurdles. Assuming that they cross the line in different moments. How many different ways there to deliver the gold, silver and bronze?

Example. Ten people of different heights in a row pose for a photo.

a) How many different pictures can be taken?

b) How many different photographs can be taken if the tallest person and the Shoter should not go together in the picture?

1) Calculator Exercises. Compute the following. To check, answers are provided.

a) $7! =$ b) $3P3 =$ c) $48C3 =$

2) Determine which method to use to solve, and solve.

a) In how many ways can 5 different cars be parked in a row in a parking lot?

b) In how many different ways can 4 horses be lined up for a race?

c) Suppose 40 cars start at the Indianapolis 500. In how many ways can the top three cars finish the race?

Michigan Lotto. The state of Michigan runs a 6-out-of-44-number lotto twice a week that pays at least $1.5 million. You purchase a card for $1 and pick any 6 numbers from 1 to 44. If your 6 numbers match those that the state draws, you win.

i) How many possible 6-number combinations are there for drawing?

ii) What is the probability of winning the lotto?

iii) Suppose it takes 10 minutes to pick your numbers and buy a ticket. How many tickets can you buy in 4 days.

iv) How many people would you have to hire to buy all the tickets and ensure that you win?

Combinations

- A combination is a grouping

Counting rule for combinations

The number of *combinations* of n objects taken r at a time is

$$C_{n,r} = \frac{n!}{r!(n-r)!} \qquad (10)$$

where n and r are whole numbers and $n \geq r$. Other commonly used notations for combinations include nCr and $\binom{n}{r}$.

ions

- Example. From a group of 4 women and 6 men a committee of 5 members will be choose.

  a) How many committees can be chosen?
  b) How many committees can choose if we want 3 men?
  c) How many committees can choose if there should be at least one woman?
  Solution:

- Example . A woman has 8 friends and you want to invite 5 of them to a party. What how many ways can you do if two of them are angry with each other and can not be invited together?

- Example. A group of scientists of 5 Argentine, 3 Chileans, 2 Colombians and 2 Peruvians will randomly select 6, to represent South America in a world congress.
  What is the probability that
  a) 2 Argentines and 2 Chileans is elected ?
  b) Take at least one elected Peruvian?

- A teacher assigned a week before the test a set of 10 problems. The exam will consist of 5 problems selected at random from among the 10 allotted.
  A student could only resolve 7 of those problems. What is the probability that student
  a) Answer 3 out of 5 good questions?
  b) Have at least 4 good questions?

