Resuelva los siguientes ejercicios

Derive las siguientes funciones

Example 1 $\frac{d}{dx}\left[\frac{\sin x}{1+\cos x}\right] = \frac{\cos x}{1+\cos x} + \frac{\sin^2 x}{(1+\cos x)^2}$

Example 2 $\frac{d}{d\mu}(\csc\mu(\mu+\cot\mu)) = -(\csc\mu(\mu+\cot\mu)\cot\mu(\mu+\cot\mu))(\mu+\cot\mu-\mu\cot^2\mu)$

Example 3 $\frac{d}{dx}\left[\frac{x-6}{x+7}\right]^3 = 3\frac{(x-6)^2}{(x+7)^3} - 3\frac{(x-6)^3}{(x+7)^4}$

Example 4 $\frac{d}{dx}(\sin(\sin(\sin x))) = \cos(\sin(\sin x))\cos(\sin x)\cos x$

nota $\frac{d}{dx}(a^x) = a^x \ln a$

Example 5 $\frac{d}{dx}(2^{\sin\pi x}) = 2^{\sin\pi x}(\cos\pi x)\pi \ln 2$

Example 6 $\frac{d}{dx}\left[\sqrt{x+\sqrt{x+\sqrt{x}}}\right] = \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)\right)$

Example 7 Encuentre $\frac{dy}{dx}$ usando diferenciacion implicita

- $\sqrt{xy} = 1 + x^2 y$ Derivative: $\frac{1}{2\sqrt{xy}}(y + xy') = 2xyx' + x^2 y'$
- $x\cos y + y\cos x = 1$ Derivative: $\cos(y) - x(\sin(y))y' + y'\cos x - y\sin x = 0$
- $\cos(x-y) = xe^x$ Derivative: $-(\sin(x-y))(1-y') = e^x + xe^x$

Example 8 Verifique la siguiente linealizacion dada alrededor de $a = 0$

- $\sqrt{1+x} \approx 1 + \frac{1}{2}x$
- $\tan x \approx x$
- $e^x \approx 1 + x$

Example 9 Explique porque la aproximación es razonable

- $\ln 1.05 \approx 0.05$
- $(1.06)^6 \approx 1.06$

Remark 1 $f(x) \approx f(a) + f'(a)(x-a)$ $f(a+dx) \approx f(a) + dy$

Sea $f(x) = \sqrt{x+3}$ $f'(x) = \frac{1}{2\sqrt{x+3}}$ $f'(1) = \frac{1}{4}$

$L(x) = f(1) + f'(1)(x-1) = 2 + \frac{1}{4}(x-1) = \frac{7}{4} + \frac{1}{4}x$ $\sqrt{x+3} \approx \frac{7}{4} + \frac{1}{4}x$ $\sqrt{3.98} \approx \frac{7}{4} + \frac{0.98}{4} = 1.995$ $\sqrt{4.05} \approx \frac{7}{4} + \frac{1.05}{4} = 2.0125$