Regresión Lineal

Demostraremos la fórmula usando los mínimos cuadrados.

Se puede demostrar que $R=\sum(y-\hat{y})=0$. Así que usaremos las desviaciones cuadradas.

$$R=\sum(y-\hat{y})^2 = \sum(y-(\hat{\alpha}+\hat{\beta}x))^2$$

De cálculo se sabe que el mínimo de una función se obtiene cuando la derivada es cero o no existe.

Derivadas parciales con respecto a α

$$\frac{dR}{d\alpha}=-2\sum(y-\hat{\alpha}-\hat{\beta}x)=0$$

$$= \sum(y-\hat{\alpha}-\hat{\beta}x)=0$$

$$\sum(y)-n\hat{\alpha}-\hat{\beta}\sum(x)=0$$

$$\sum(y)=n\hat{\alpha}+\hat{\beta}\sum(x)$$ primera ecuación normal

Derivadas parciales con respecto a β

$$\frac{dR}{d\beta}=-2\sum(y-\hat{\alpha}-\hat{\beta}x)(x)=0$$

$$\sum(y-\hat{\alpha}-\hat{\beta}x)(x)=0$$

$$\sum(yx-\hat{\alpha}x-\hat{\beta}x^2)=0$$

$$\sum(yx)-\hat{\alpha}\sum(x)-\hat{\beta}\sum(x^2)=0$$

$$\sum(yx)=\hat{\alpha}\sum(x)+\hat{\beta}\sum(x^2)$$ segunda ecuación normal

Observe que de la primera ecuación normal se obtiene.

$$\frac{(\sum(y)=n\hat{\alpha}+\hat{\beta}\sum(x))}{n}=0$$

$$\bar{y}=\hat{\alpha}+\hat{\beta}\bar{x}$$

$$\hat{\alpha}=\bar{y}+\hat{\beta}\,\bar{x}$$

Ahora despejaremos la segunda ecuación normal

$$\sum(yx)=\hat{\alpha}\sum(x)+\hat{\beta}\sum(x^2)$$

$$\sum(yx)-(\bar{y}+\hat{\beta}\,\bar{x})\sum(x)=\hat{\beta}\sum(x^2)$$ recuerde que $\hat{\alpha}=\bar{y}+\hat{\beta}\,\bar{x}$

$$\sum(yx)-(\frac{\sum y}{n}+\hat{\beta}\frac{\sum x}{n})\sum(x)=\hat{\beta}\sum(x^2)$$

$$\sum(yx)-\frac{\sum y}{n}\sum(x)+\hat{\beta}\frac{\sum x}{n}\sum(x)=\hat{\beta}\sum(x^2)$$

$$\sum(yx)-\frac{\sum y}{n}\sum(x)+\hat{\beta}\frac{(\sum x)^2}{n}=\hat{\beta}\sum(x^2)$$

$$\sum(yx)-\frac{\sum y}{n}\sum(x)=\hat{\beta}\sum(x^2)-\hat{\beta}\frac{(\sum x)^2}{n}$$

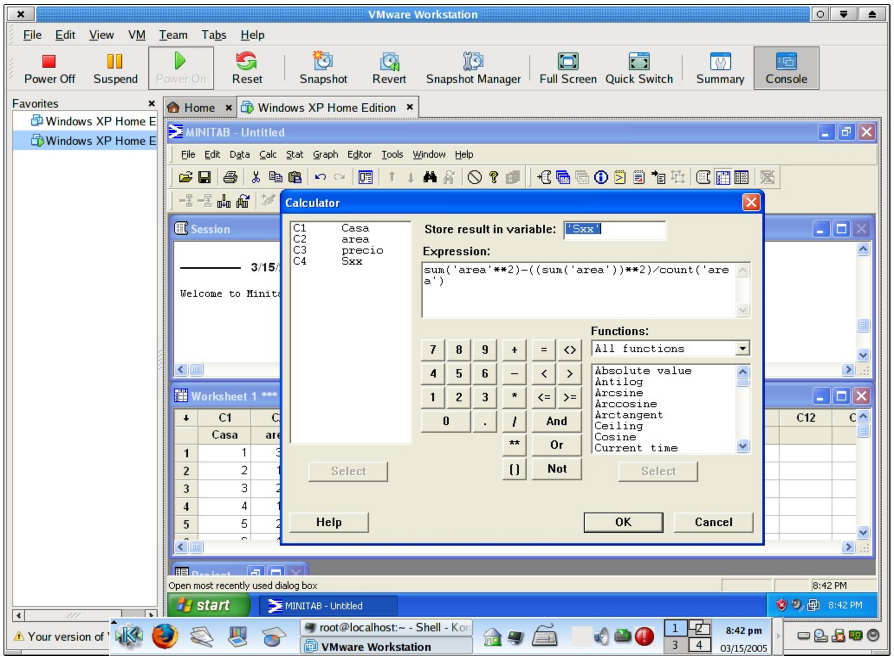
$$S_{xy}=\hat{\beta}(\sum(x^2)-\frac{(\sum x)^2}{n})$$

$$S_{xy}=\hat{\beta}\,S_{xx}$$

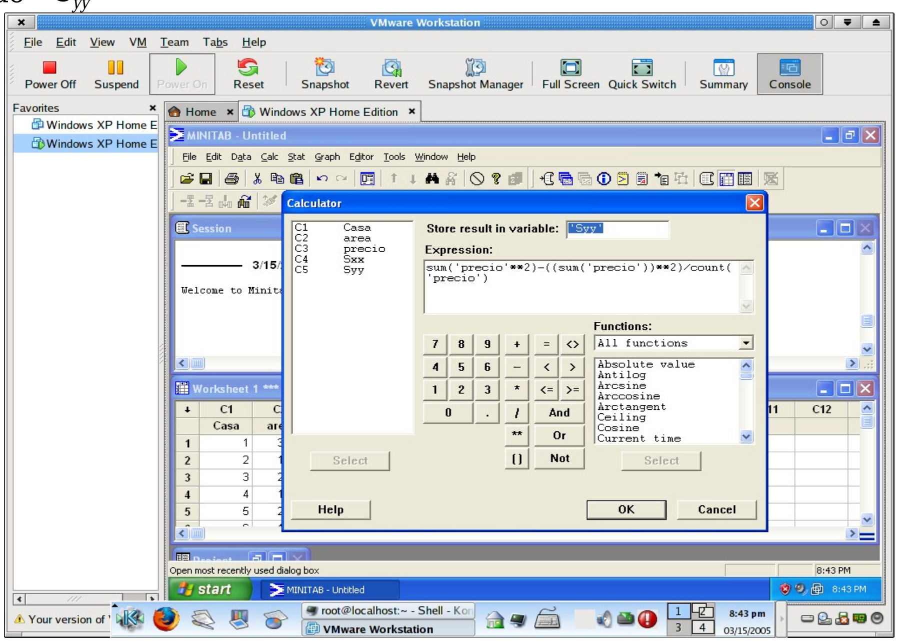
$$\hat{\beta}=\frac{S_{xy}}{S_{xx}}$$

Bueno ahora usaremos Minitab para hacer los cálculos.

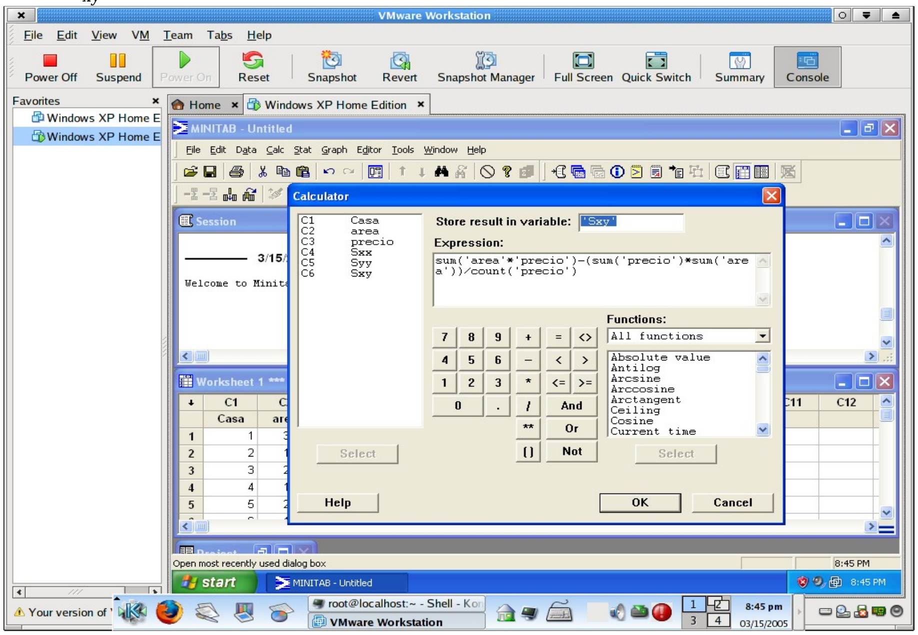
Calculando S_{xx}



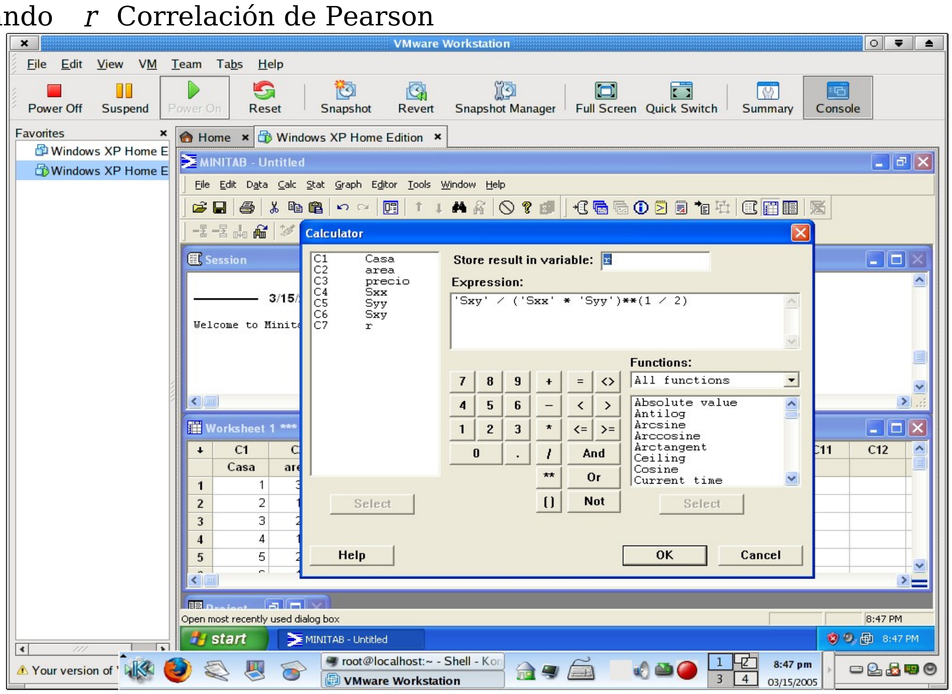
Calculando S_{yy}



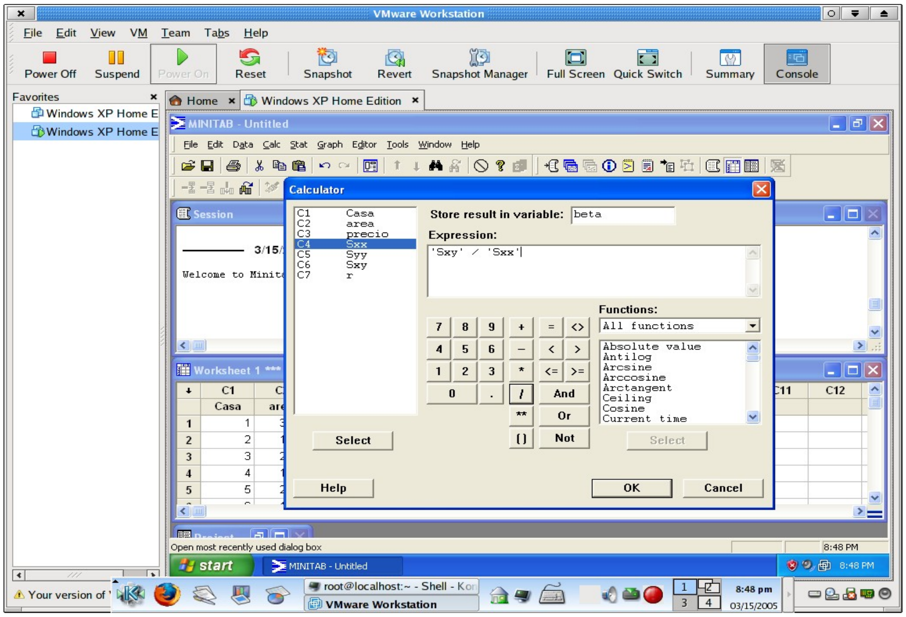
Calculando S_{xy}



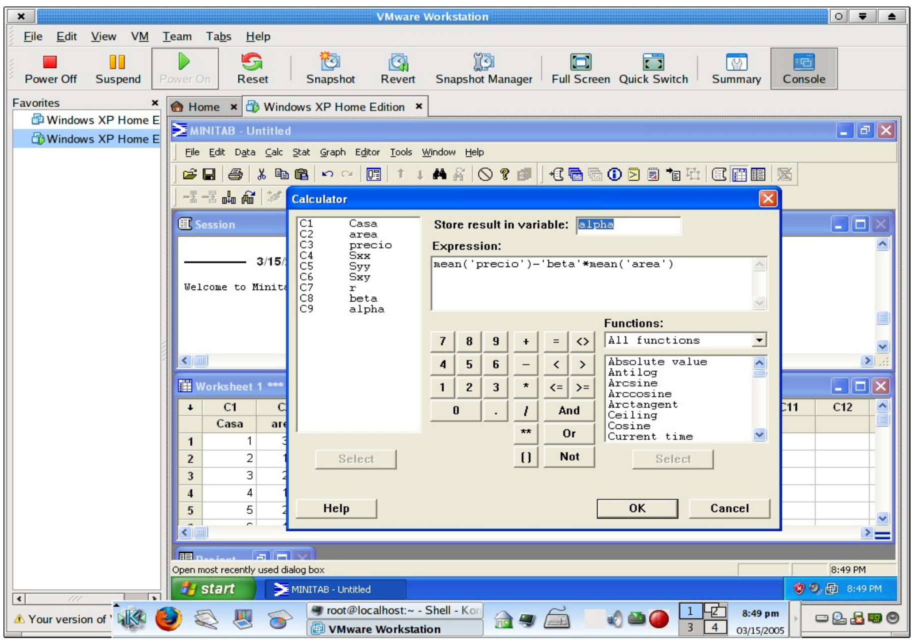
Calculando r Correlación de Pearson



Calculando β

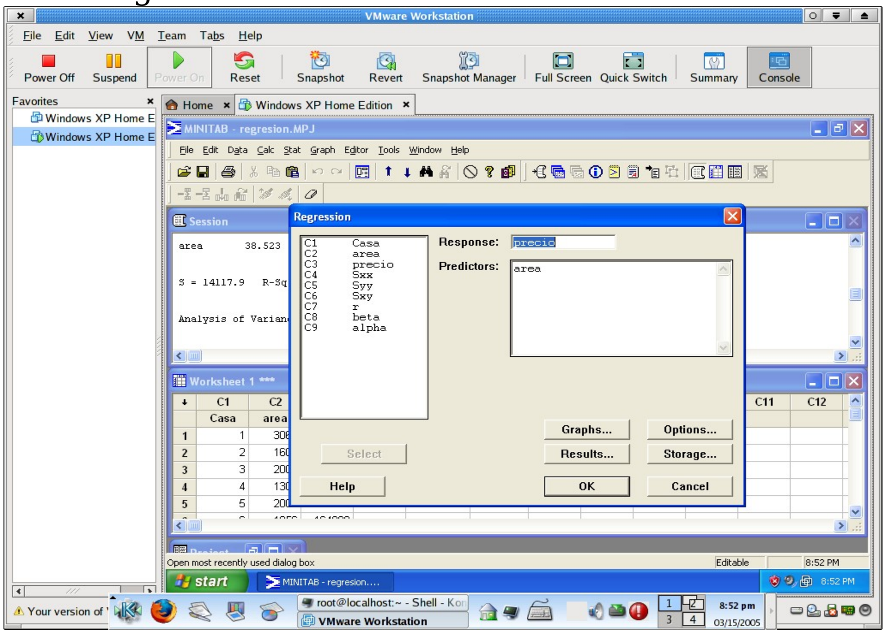


Calculando α



$$y = \alpha + \beta x$$

Calculando la regresión lineal usando el menu de Minitab



Observando la salida de Minitab