For μ: $E = z_c \dfrac{\sigma}{\sqrt{n}}$ when σ is known;

$E = t_c \dfrac{s}{\sqrt{n}}$ with $d.f. = n - 1$ when σ is unknown

For p: $E = z_c \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$ when $n\hat{p} > 5$ and $n\hat{q} > 5$

For $\mu_1 - \mu_2$: $E = z_c \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$ when σ_1 and σ_2 are known

$E = t_c \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$ when σ_1 or σ_2 is unknown with $d.f.$ = smaller of $n_1 - 1$ or $n_2 - 1$

Software uses Satterthwaite's approximation for $d.f.$

For $p_1 - p_2$: $E = z_c \sqrt{\dfrac{\hat{p}_1\hat{q}_1}{n_1} + \dfrac{\hat{p}_2\hat{q}_2}{n_2}}$ for sufficiently large n

- Confidence intervals have an associated probability c called the confidence level. For a given sample size, the proportion of all corresponding confidence intervals that contain the parameter in question is c.

IMPORTANT WORDS & SYMBOLS

Section 8.1
Maximal margin of error E
Confidence level c
Critical values z_c
Point estimate for μ
Confidence interval for μ
c confidence interval
Sample size for specified E

Section 8.2
Student's t variable
Degrees of freedom ($d.f.$)
Critical values t_c

Section 8.3
Point estimate for p, $\hat{p}$
Confidence interval for p
Margin of error for polls
Sample size for specified E

Section 8.4
Independent samples
Dependent samples
Confidence interval for $\mu_1 - \mu_2$ (σ_1 and σ_2 known)
Confidence interval for $\mu_1 - \mu_2$ (σ_1 and σ_2 unknown)
Confidence interval for $p_1 - p_2$

VIEWPOINT All Systems Go?

On January 28, 1986, the Space Shuttle Challenger *caught fire and blew up only seconds after launch. A great deal of good engineering went into the design of the* Challenger. *However, when a system has several confidence levels operating at once, it can happen, in rare cases, that risks will increase rather than cancel out. (See Chapter Review Problem 19.) Diane Vaughn is a professor of sociology at Boston College and author of the book* The Challenger Launch Decision *(University of Chicago Press). Her book contains an excellent discussion of risks, the normalization of deviants, and cost/safety tradeoffs. Vaughn's book is described as "a remarkable and important analysis of how social structures can induce consequential errors in a decision process" (Robert K. Merton, Columbia University).*

CHAPTER REVIEW PROBLEMS

1. See text.
2. $\bar{x}$ = mean of the confidence interval endpoints = \$3.30; E = \$0.15.

1. *Statistical Literacy* In your own words, carefully explain the meanings of the following terms: point estimate, critical value, maximal margin of error, confidence level, and confidence interval.

2. ***Critical Thinking*** Suppose you are told that a 95% confidence interval for the average price of a gallon of regular gasoline in your state is from \$3.15 to \$3.45. Use the fact that the confidence interval for the mean is in the form $\bar{x} - E$ to $\bar{x} + E$ to compute the sample mean and the maximal margin of error E.

3. (a) No, the probability that μ is in the interval is either 0 or 1.
   (b) Yes, 99% confidence intervals are constructed in such a way that 99% of all such confidence intervals based on random samples of the designated size will contain μ.

3. *Critical Thinking* If you have a 99% confidence interval for μ based on a simple random sample,
   (a) is it correct to say that the *probability* that μ is in the specified interval is 99%? Explain.
   (b) is it correct to say that in the long run, if you computed many, many confidence intervals using the prescribed method, about 99% of such intervals would contain μ? Explain.

For Problems 4–19, categorize each problem according to parameter being estimated, proportion p, mean μ, difference of means $\mu_1 - \mu_2$, or difference of proportions $p_1 - p_2$. Then solve the problem.

4. Interval for a mean; \$1549 to \$1592; \$1536 to \$1604.

4. *Auto Insurance: Claims* Anystate Auto Insurance Company took a random sample of 370 insurance claims paid out during a 1-year period. The average claim paid was \$1570. Assume $\sigma = \$250$. Find 0.90 and 0.99 confidence intervals for the mean claim payment.

5. Interval for a mean; 176.91 to 180.49.

5. *Psychology: Closure* Three experiments investigating the relation between need for cognitive closure and persuasion were reported in "Motivated Resistance and Openness to Persuasion in the Presence or Absence of Prior Information," by A. W. Kruglanski (*Journal of Personality and Social Psychology*, Vol. 65, No. 5, pp. 861–874). Part of the study involved administering a "need for closure scale" to a group of students enrolled in an introductory psychology course. The "need for closure scale" has scores ranging from 101 to 201. For the 73 students in the highest quartile of the distribution, the mean score was $\bar{x} = 178.70$. Assume a population standard deviation of $\sigma = 7.81$. These students were all classified as high on their need for closure. Assume that the 73 students represent a random sample of all students who are classified as high on their need for closure. Find a 95% confidence interval for the population mean score μ on the "need for closure scale" for all students with a high need for closure.

6. Sample size for a mean; 102.

6. *Psychology: Closure* How large a sample is needed in Problem 5 if we wish to be 99% confident that the sample mean score is within 2 points of the population mean score for students who are high on the need for closure?

7. Interval for a mean.
   (a) Use a calculator.
   (b) 64.1 to 84.3.

7. *Archaeology: Excavations* The Wind Mountain archaeological site is located in southwestern New Mexico. Wind Mountain was home to an ancient culture of prehistoric Native Americans called Anasazi. A random sample of excavations at Wind Mountain gave the following depths (in centimeters) from present-day surface grade to the location of significant archaeological artifacts (Source: *Mimbres Mogollon Archaeology*, by A. Woosley and A. McIntyre, University of New Mexico Press).

85	45	120	80	75	55	65	60
65	95	90	70	75	65	68	

   (a) Use a calculator with mean and sample standard deviation keys to verify that $\bar{x} \approx 74.2$ cm and $s \approx 18.3$ cm.
   (b) Compute a 95% confidence interval for the mean depth μ at which archaeological artifacts from the Wind Mountain excavation site can be found.

8. Interval for a mean.
   (a) Use a calculator.
   (b) 14.27 to 17.33.

8. *Archaeology: Pottery* Sherds of clay vessels were put together to reconstruct rim diameters of the original ceramic vessels at the Wind Mountain archaeological site (see source in Problem 7). A random sample of ceramic vessels gave the following rim diameters (in centimeters):

15.9	13.4	22.1	12.7	13.1	19.6	11.7	13.5	17.7	18.1

   (a) Use a calculator with mean and sample standard deviation keys to verify that $\bar{x} \approx 15.8$ cm and $s \approx 3.5$ cm.
   (b) Compute an 80% confidence interval for the population mean μ of rim diameters for such ceramic vessels found at the Wind Mountain archaeological site.

9. | *Telephone Interviews: Survey* The National Study of the Changing Work Force conducted an extensive survey of 2958 wage and salaried workers on issues ranging from relationships with their bosses to household chores. The data were gathered through hour-long telephone interviews with a nationally representative sample (*The Wall Street Journal*). In response to the question, "What does success mean to you?" 1538 responded, "Personal satisfaction from doing a good job." Let p be the population proportion of all wage and salaried workers who would respond the same way to the stated question. Find a 90% confidence interval for p.

9. Interval for a proportion; 0.50 to 0.54.

10. | *Telephone Interviews: Survey* How large a sample is needed in Problem 9 if we wish to be 95% confident that the sample percentage of those equating success with personal satisfaction is within 1% of the population percentage? (*Hint:* Use $p \approx 0.52$ as a preliminary estimate.)

10. Sample size for a proportion; 9589.

11. | *Archaeology: Pottery* Three-circle, red-on-white is one distinctive pattern painted on ceramic vessels of the Anasazi period found at the Wind Mountain archaeological site (see source for Problem 7). At one excavation, a sample of 167 potsherds indicated that 68 were of the three-circle, red-on-white pattern.
(a) Find a point estimate $\hat{p}$ for the proportion of all ceramic potsherds at this site that are of the three-circle, red-on-white pattern.
(b) Compute a 95% confidence interval for the population proportion p of all ceramic potsherds with this distinctive pattern found at the site.

11. Interval for a proportion.
(a) $\hat{p} = 0.4072$.
(b) 0.333 to 0.482.

12. | *Archaeology: Pottery* Consider the three-circle, red-on-white pattern discussed in Problem 11. How many ceramic potsherds must be found and identified if we are to be 95% confident that the sample proportion $\hat{p}$ of such potsherds is within 6% of the population proportion of three-circle, red-on-white patterns found at this excavation site? (*Hint:* Use the results of Problem 11 as a preliminary estimate.)

12. Sample size for a proportion; $n = 258$ total, or 91 more.

13. | *Agriculture: Bell Peppers* The following data represent soil water content (percent water by volume) for independent random samples of soil taken from two experimental fields growing bell peppers (Reference: *Journal of Agricultural, Biological, and Environmental Statistics*). *Note:* These data are also available for download on-line in HM StatSPACE™

Soil water content from field I: x_1; $n_1 = 72$

15.1	11.2	10.3	10.8	16.6	8.3	9.1	12.3	9.1	14.3
10.7	16.1	10.2	15.2	8.9	9.5	9.6	11.3	14.0	11.3
15.6	11.2	13.8	9.0	8.4	8.2	12.0	13.9	11.6	16.0
9.6	11.4	8.4	8.0	14.1	10.9	13.2	13.8	14.6	10.2
11.5	13.1	14.7	12.5	10.2	11.8	11.0	12.7	10.3	10.8
11.0	12.6	10.8	9.6	11.5	10.6	11.7	10.1	9.7	9.7
11.2	9.8	10.3	11.9	9.7	11.3	10.4	12.0	11.0	10.7
8.8	11.1								

Soil water content from field II: x_2; $n_2 = 80$

12.1	10.2	13.6	8.1	13.5	7.8	11.8	7.7	8.1	9.2
14.1	8.9	13.9	7.5	12.6	7.3	14.9	12.2	7.6	8.9
13.9	8.4	13.4	7.1	12.4	7.6	9.9	26.0	7.3	7.4
14.3	8.4	13.2	7.3	11.3	7.5	9.7	12.3	6.9	7.6
13.8	7.5	13.3	8.0	11.3	6.8	7.4	11.7	11.8	7.7
12.6	7.7	13.2	13.9	10.4	12.8	7.6	10.7	10.7	10.9
12.5	11.3	10.7	13.2	8.9	12.9	7.7	9.7	9.7	11.4
11.9	13.4	9.2	13.4	8.8	11.9	7.1	8.5	14.0	14.2

13. Difference of means.
(a) Use a calculator.
(b) $d.f. \approx 71$; round down to $d.f. \approx 70$; $E \approx 0.83$; interval from -0.06 to 1.6.
(c) Because the interval contains both positive and negative values, we cannot conclude at the 95% confidence level that there is any difference in soil water content between the two fields.
(d) Student's t distribution because σ_1 and σ_2 are unknown. Both samples are large, so no assumptions about the original distributions are needed.

Now, what is the probability that *both* intervals hold together? Use methods of Section 4.2 to show that

$$P(A_1 < \mu_1 < B_1 \quad and \quad A_2 < \mu_2 < B_2) = 0.64$$

Hint: We are combining independent events. If the confidence is 64% that both intervals hold together, explain why the risk that at least one interval does not hold (i.e., fails) must be 36%.

(b) Suppose we want *both* intervals to hold with 90% confidence (i.e., only 10% risk level). How much confidence c should each interval have to achieve this combined level of confidence? (Assume that each interval has the same confidence level c.)

Hint: $P(A_1 < \mu_1 < B_1 \quad and \quad A_2 < \mu_2 < B_2) = 0.90$

$$P(A_1 < \mu_1 < B_1) \times P(A_2 < \mu_2 < B_2) = 0.90$$

$$c \times c = 0.90$$

Now solve for c.

(c) If we want *both* intervals to hold at the 90% level of confidence, then the individual intervals must hold at a *higher* level of confidence. Write a brief but detailed explanation of how this could be of importance in a large, complex engineering design such as a rocket booster or a spacecraft.

DATA HIGHLIGHTS: GROUP PROJECTS

Digging clams

Break into small groups and discuss the following topics. Organize a brief outline in which you summarize the main points of your group discussion.

1. Garrison Bay is a small bay in Washington state. A popular recreational activity in the bay is clam digging. For several years, this harvest has been monitored and the size distribution of clams recorded. Data for lengths and widths of little neck clams (*Protothaca staminea*) were recorded by a method of systematic sampling in a study done by S. Scherba and V. F. Gallucci ("The Application of Systematic Sampling to a Study of Infaunal Variation in a Soft Substrate Intertidal Environment," *Fishery Bulletin* 74:937–948). The data in Tables 8-4 and 8-5 give lengths and widths for 35 little neck clams.
   (a) Use a calculator to compute the sample mean and sample standard deviation for the lengths and widths. Compute the coefficient of variation for each.
   (b) Compute a 95% confidence interval for the population mean length of all Garrison Bay little neck clams.
   (c) How many more little neck clams would be needed in a sample if you wanted to be 95% sure that the sample mean length is within a maximal margin of error of 10 mm of the population mean length?

TABLE 8-4 Lengths of Little Neck Clams (mm)

530	517	505	512	487	481	485	479	452	468
459	449	472	471	455	394	475	335	508	486
474	465	420	402	410	393	389	330	305	169
91	537	519	509	511					

TABLE 8-5 Widths of Little Neck Clams (mm)

494	477	471	413	407	427	408	430	395	417
394	397	402	401	385	338	422	288	464	436
414	402	383	340	349	333	356	268	264	141
77	498	456	433	447					

(d) Compute a 95% confidence interval for the population mean width of all Garrison Bay little neck clams.

(e) How many more little neck clams would be needed in a sample if you wanted to be 95% sure that the sample mean width is within a maximal margin of error of 10 mm of the population mean width?

(f) The *same* 35 clams were used for measures of length and width. Are the sample measurements length and width independent or dependent? Why?

2. Examine Figure 8-8, "Fall Back."

   (a) Of the 1024 adults surveyed, 66% were reported to favor daylight saving time. How many people in the sample preferred daylight saving time? Using the statistic $\hat{p} = 0.66$ and sample size $n = 1024$, find a 95% confidence interval for the proportion of people p who favor daylight saving time. How could you report this information in terms of a margin of error?

   (b) Look at Figure 8-8 to find the sample statistic $\hat{p}$ for the proportion of people preferring standard time. Find a 95% confidence interval for the population proportion p of people who favor standard time. Report the same information in terms of a margin of error.

3. Examine Figure 8-9, "Coupons: Limited Use."

   (a) Use Figure 8-9 to estimate the percentage of merchandise coupons that were redeemed. Also estimate the percentage dollar value of the coupons that were redeemed. Are these numbers approximately equal?

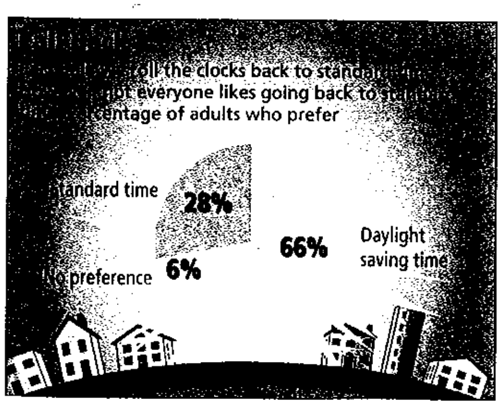
   (b) Suppose you are a marketing executive working for a national chain of toy stores. You wish to estimate the percentage of coupons that will be redeemed for the toy stores. How many coupons should you check to be 95% sure that the percentage of coupons redeemed is within 1% of the population proportion of all coupons redeemed for the toy store?

   (c) Use the results of part (a) as a preliminary estimate for p, the percentage of coupons that are redeemed, and redo part (b).

   (d) Suppose you sent out 937 coupons and found that 27 were redeemed. Explain why you could be 95% confident that the proportion of such coupons redeemed in the future would be between 1.9% and 3.9%.

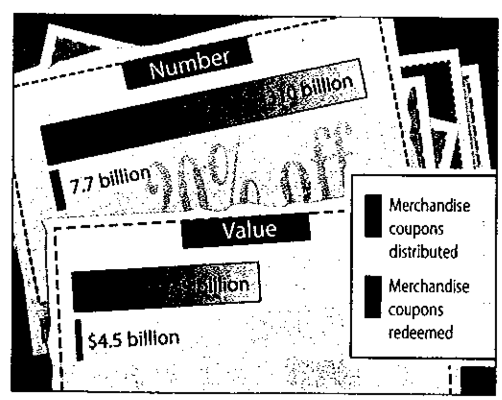
   (e) Suppose the dollar value of a collection of coupons was \$10,000. Use the data in Figure 8-9 to find the expected value and standard deviation of the dollar value of the redeemed coupons. What is the probability that between \$225 and \$275 (out of the \$10,000) is redeemed?

FIGURE 8-8



Source: Hilton Time Survey of 1024 adults

FIGURE 8-9 Coupons: Limited Use



Source: NCH Promotional Services

LINKING CONCEPTS: WRITING PROJECTS

Discuss each of the following topics in class or review the topics on your own. Then write a brief but complete essay in which you summarize the main points. Please include formulas and graphs as appropriate.

1. In this chapter, we have studied confidence intervals. Carefully read the following statements about confidence intervals:
   (a) Once the endpoints of the confidence interval are numerically fixed, then the parameter in question (either μ or p) does or does not fall inside the "fixed" interval.
   (b) A given fixed interval either does or does not contain the parameter μ or p, therefore, the probability is 1 or 0 that the parameter is in the interval.

   Next, read the following statements. Then discuss all four statements in the context of what we actually mean by a confidence interval.
   (c) Nontrivial probability statements can be made only about variables, not constants.
   (d) The confidence level c represents the proportion of all (fixed) intervals that would contain the parameter if we repeated the process many, many times.
2. Throughout Chapter 8, we have used the normal distribution, the central limit theorem, or the Student's t distribution.
   (a) Give a brief outline describing how confidence intervals for means use the normal distribution or Student's t distribution in their basic construction.
   (b) Give a brief outline describing how the normal approximation to the binomial distribution is used in the construction of confidence intervals for a proportion p.
   (c) Give a brief outline describing how the sample size for a predetermined error tolerance and level of confidence is determined from the normal distribution or the central limit theorem.
3. When the results of a survey or a poll are published, the sample size is usually given, as well as the margin of error. For example, suppose the *Honolulu Star Bulletin* reported that it surveyed 385 Honolulu residents and 78% said they favor mandatory jail sentences for people convicted of driving under the influence of drugs or alcohol (with margin of error of 3 percentage points in either direction). Usually the confidence level of the interval is not given, but it is standard practice to use the margin of error for a 95% confidence interval when no other confidence level is given.
   (a) The paper reported a point estimate of 78%, with margin of error of $\pm 3\%$. Write this information in the form of a confidence interval for p, the population proportion of residents favoring mandatory jail sentences for people convicted of driving under the influence. What is the assumed confidence level?
   (b) The margin of error is simply the error due to using a sample instead of the entire population. It does not take into account the bias that might be introduced by the wording of the question, by the truthfulness of the respondents, or by other factors. Suppose the question was asked in this fashion: "Considering the devastating injuries suffered by innocent victims in auto accidents caused by drunken or drugged drivers, do you favor a mandatory jail sentence for those convicted of driving under the influence of drugs or alcohol?" Do you think the wording of the question would influence the respondents? Do you think the population proportion of those favoring mandatory jail sentences is accurately represented by a confidence interval based on responses to such a question? Explain your answer.

       If the question had been: "Considering existing overcrowding of our prisons, do you favor a mandatory jail sentence for people convicted of driving under the influence of drugs or alcohol?" Do you think the population proportion of those favoring mandatory sentences is accurately represented by a confidence interval based on responses to such a question? Explain.