(c) Find or estimate the P-value of the sample test statistic.

(d) Based on your answers in parts (a) to (c), will you reject or fail to reject the null hypothesis of independence?

(e) *Interpret* your conclusion in the context of the application.

Use the expected values E to the hundredths place.

9. *Psychology: Myers–Briggs* The following table shows the Myers–Briggs personality preferences for a random sample of 406 people in the listed professions (*Atlas of Type Tables* by Macdaid, McCaulley, and Kainz). E refers to extroverted and I refers to introverted.

	Personality Preference Type		
Occupation	**E**	**I**	**Row Total**
Clergy (all denominations)	62	45	107
M.D.	68	94	162
Lawyer	56	81	137
Column Total	186	220	406

Use the chi-square test to determine if the listed occupations and personality preferences are independent at the 0.05 level of significance.

10. *Psychology: Myers–Briggs* The following table shows the Myers–Briggs personality preferences for a random sample of 519 people in the listed professions (*Atlas of Type Tables* by Macdaid, McCaulley, and Kainz). T refers to thinking and F refers to feeling.

	Personality Preference Type		
Occupation	**T**	**F**	**Row Total**
Clergy (all denominations)	57	91	148
M.D.	77	82	159
Lawyer	118	94	212
Column Total	252	267	519

Use the chi-square test to determine if the listed occupations and personality preferences are independent at the 0.01 level of significance.

11. *Archaeology: Pottery* The following table shows site type and type of pottery for a random sample of 628 sherds at a location in Sand Canyon Archaeological Project, Colorado (*The Sand Canyon Archaeological Project*, edited by Lipe).

	Pottery Type			
Site Type	**Mesa Verde Black-on-White**	**McElmo Black-on-White**	**Mancos Black-on-White**	**Row Total Black-on-White**
Mesa Top	75	61	53	189
Cliff-Talus	81	70	62	213
Canyon Bench	92	68	66	226
Column Total	248	199	181	628

Use a chi-square test to determine if site type and pottery type are independent at the 0.01 level of significance.

12. *Archaeology: Pottery* The following table shows ceremonial ranking and type of pottery sherd for a random sample of 434 sherds at a location in the Sand Canyon Archaeological Project, Colorado (*The Architecture of Social Integration in Prehistoric Pueblos*, edited by Lipe and Hegmon).

Use a chi-square test to determine if ceremonial ranking and pottery type are independent at the 0.05 level of significance.

No. All she can say is that the two variables, source of fraud/identity theft and gender, are dependent. She cannot tell which of the sources are dependent with respect to gender.

$\alpha = 0.05$; H_0: Myers–Briggs preference and profession are independent; H_1: Myers–Briggs preference and profession are not independent.
$\chi^2 = 8.649$; $d.f. = 2$.
$0.010 < P\text{-value} < 0.025$. From TI-84, P-value ≈ 0.0132.
Reject H_0.
At the 5% level of significance, there is sufficient evidence to conclude that Myers–Briggs preference and profession are not independent.

(a) $\alpha = 0.01$; H_0: Myers–Briggs preference and profession are independent; H_1: Myers–Briggs preference and profession are not independent.
(b) $\chi^2 = 10.26$; $d.f. = 2$.
(c) $0.005 < P\text{-value} < 0.010$. From TI-84, P-value ≈ 0.0059.
(d) Reject H_0.
(e) At the 1% level of significance, there is sufficient evidence to conclude that Myers–Briggs preference and profession are not independent.

(a) $\alpha = 0.01$; H_0: Site type and pottery type are independent; H_1: Site type and pottery type are not independent.
(b) $\chi^2 = 0.5552$; $d.f. = 4$.
(c) $0.950 < P\text{-value} < 0.975$. From TI-84, P-value ≈ 0.9679.
(d) Do not reject H_0.
(e) At the 1% level of significance, there is insufficient evidence to conclude that site type and pottery type are not independent.

(a) $\alpha = 0.05$; H_0: Ceremonial ranking and pottery type are independent; H_1: Ceremonial ranking and pottery type are not independent.
(b) $\chi^2 = 6.198$; $d.f. = 2$.
(c) $0.025 < P\text{-value} < 0.050$. From TI-84, P-value ≈ 0.0451.
(d) Reject H_0.
(e) At the 5% level of significance, there is sufficient evidence to conclude that ceremonial ranking and pottery type are not independent.

(c) Find or estimate the P-value of the sample test statistic.

(d) Based on your answers in parts (a) to (c), will you reject or fail to reject the null hypothesis of independence?

(e) *Interpret* your conclusion in the context of the application.

Use the expected values E to the hundredths place.

9. *Psychology: Myers–Briggs* The following table shows the Myers–Briggs personality preferences for a random sample of 406 people in the listed professions (*Atlas of Type Tables* by Macdaid, McCaulley, and Kainz). E refers to extroverted and I refers to introverted.

	Personality Preference Type		
Occupation	**E**	**I**	**Row Total**
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Lawyer	56	81	137
Column Total	186	220	406

Use the chi-square test to determine if the listed occupations and personality preferences are independent at the 0.05 level of significance.

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	Personality Preference Type		
Occupation	**T**	**F**	**Row Total**
Clergy (all denominations)	57	91	148
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Lawyer	118	94	212
Column Total	252	267	519

Use the chi-square test to determine if the listed occupations and personality preferences are independent at the 0.01 level of significance.

11. *Archaeology: Pottery* The following table shows site type and type of pottery for a random sample of 628 sherds at a location in Sand Canyon Archaeological Project, Colorado (*The Sand Canyon Archaeological Project,* edited by Lipe).

	Pottery Type			
Site Type	**Mesa Verde Black-on-White**	**McElmo Black-on-White**	**Mancos Black-on-White**	**Row Total Black-on-White**
Mesa Top	75	61	53	189
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Use a chi-square test to determine if site type and pottery type are independent at the 0.01 level of significance.

12. *Archaeology: Pottery* The following table shows ceremonial ranking and type of pottery sherd for a random sample of 434 sherds at a location in the Sand Canyon Archaeological Project, Colorado (*The Architecture of Social Integration in Prehistoric Pueblos,* edited by Lipe and Hegmon).

Use a chi-square test to determine if ceremonial ranking and pottery type are independent at the 0.05 level of significance.

No. All she can say is that the two variables, source of fraud/identity theft and gender, are dependent. She cannot tell which of the sources are dependent with respect to gender.

$\alpha = 0.05$; H_0: Myers–Briggs preference and profession are independent; H_1: Myers–Briggs preference and profession are not independent.
$\chi^2 = 8.649$; $d.f. = 2$.
$0.010 < P\text{-value} < 0.025$. From TI-84, $P\text{-value} \approx 0.0132$.
Reject H_0.
At the 5% level of significance, there is sufficient evidence to conclude that Myers–Briggs preference and profession are not independent.

$\alpha = 0.01$; H_0: Myers–Briggs preference and profession are independent; H_1: Myers–Briggs preference and profession are not independent.
$\chi^2 = 10.26$; $d.f. = 2$.
$0.005 < P\text{-value} < 0.010$. From TI-84, $P\text{-value} \approx 0.0059$.
Reject H_0.
At the 1% level of significance, there is sufficient evidence to conclude that Myers–Briggs preference and profession are not independent.

$\alpha = 0.01$; H_0: Site type and pottery type are independent; H_1: Site type and pottery type are not independent.
$\chi^2 = 0.5552$; $d.f. = 4$.
$0.950 < P\text{-value} < 0.975$. From TI-84, $P\text{-value} \approx 0.9679$.
Do not reject H_0.
At the 1% level of significance, there is insufficient evidence to conclude that site type and pottery type are not independent.

$\alpha = 0.05$; H_0: Ceremonial ranking and pottery type are independent; H_1: Ceremonial ranking and pottery type are not independent.
$\chi^2 = 6.198$; $d.f. = 2$.
$0.025 < P\text{-value} < 0.050$. From TI-84, $P\text{-value} \approx 0.0451$.
Reject H_0.
At the 5% level of significance, there is sufficient evidence to conclude that ceremonial ranking and pottery type are not independent.

Ceremonial Ranking	Cooking Jar Sherds	Decorated Jar Sherds (Noncooking)	Row Total
A	86	49	135
B	92	53	145
C	79	75	154
Column Total	257	177	434

13. (a) $\alpha = 0.05$; H_0: Age distribution and location are independent; H_1: Age distribution and location are not independent.
(b) $\chi^2 = 0.6704$; *d.f.* = 4.
(c) $0.950 < P\text{-value} < 0.975$. From TI-84, *P*-value ≈ 0.9549.
(d) Do not reject H_0.
(e) At the 5% level of significance, there is insufficient evidence to conclude that age distribution and location are not independent.

13. *Ecology: Buffalo* The following table shows age distribution and location of a random sample of 166 buffalo in Yellowstone National Park (based on information from *The Bison of Yellowstone National Park,* National Park Service Scientific Monograph Series).

Age	Lamar District	Nez Perce District	Firehole District	Row Total
Calf	13	13	15	41
Yearling	10	11	12	33
Adult	34	28	30	92
Column Total	57	52	57	166

Use a chi-square test to determine if age distribution and location are independent at the 0.05 level of significance.

14. (a) $\alpha = 0.05$; H_0: Myers–Briggs type and area of study are independent; H_1: Myers–Briggs type and area of study are not independent.
(b) $\chi^2 = 15.6017$; *d.f.* = 6.
(c) $0.010 < P\text{-value} < 0.025$. From TI-84, *P*-value ≈ 0.0161.
(d) Reject H_0.
(e) At the 5% level of significance, there is sufficient evidence to conclude that Myers–Briggs type and area of study are not independent.

14. *Psychology: Myers–Briggs* The following table shows the Myers–Briggs personality preference and area of study for a random sample of 519 college students (*Applications of the Myers–Briggs Type Indicator in Higher Education,* edited by Provost and Anchors). In the table, IN refers to introvert, intuitive; EN refers to extrovert, intuitive; IS refers to introvert, sensing; and ES refers to extrovert, sensing.

Myers–Briggs Preference	Arts & Science	Business	Allied Health	Row Total
IN	64	15	17	96
EN	82	42	30	154
IS	68	35	12	115
ES	75	42	37	154
Column Total	289	134	96	519

Use a chi-square test to determine if Myers–Briggs preference type is independent of area of study at the 0.05 level of significance.

15. (a) $\alpha = 0.05$; H_0: Age of young adult and movie preference are independent; H_1: Age of young adult and movie preference are not independent.
(b) $\chi^2 = 3.6230$; *d.f.* = 4.
(c) $0.100 < P\text{-value} < 0.900$. From TI-84, *P*-value ≈ 0.4594.
(d) Do not reject H_0.
(e) At the 5% level of significance, there is insufficient evidence to conclude that age of young adult and movie preference are not independent.

15. *Sociology: Movie Preference* Mr. Acosta, a sociologist, is doing a study to see if there is a relationship between the age of a young adult (18 to 35 years old) and the type of movie preferred. A random sample of 93 young adults revealed the following data. Test whether age and type of movie preferred are independent at the 0.05 level.

	Person's Age			
Movie	18–23 yr	24–29 yr	30–35 yr	Row Total
Drama	8	15	11	34
Science fiction	12	10	8	30
Comedy	9	8	12	29
Column Total	29	33	31	93

16. (a) $\alpha = 0.01$; H_0: Contribution level and ethnic group are independent; H_1: Contribution level and ethnic group are not independent.
(b) $\chi^2 = 13.35$; *d.f.* = 12.

16. *Sociology: Ethnic Groups* After a large fund drive to help the Boston City Library, the following information was obtained from a random sample of contributors to the library fund. Using a 1% level of significance, test the claim that the amount contributed to the library fund is independent of ethnic group.

(c) $0.100 < P\text{-value} < 0.900$. From TI-84, $P\text{-value} \approx 0.3444$.
(d) Do not reject H_0.
(e) At the 1% level of significance, there is insufficient evidence to conclude that contribution level and ethnic group are not independent.

Ethnic Group	Number of People Making Contribution					Row Total
	$1–50	$51–100	$101–150	$151–200	Over $200	
A	83	62	53	35	18	251
B	94	77	48	25	20	264
C	78	65	51	40	32	266
D	105	89	63	54	29	340
Column Total	360	293	215	154	99	1121

17. (a) $\alpha = 0.05$; H_0: Stone tool construction material and site are independent; H_1: Stone tool construction material and site are not independent.
(b) $\chi^2 = 11.15$; $d.f. = 3$.
(c) $0.010 < P\text{-value} < 0.025$. From TI-84, $P\text{-value} \approx 0.0110$.
(d) Reject H_0.
(e) At the 5% level of significance, there is sufficient evidence to conclude that stone tool construction material and site are not independent.

17. *Focus Problem: Archaeology* The Focus Problem at the beginning of the chapter refers to excavations at Burnt Mesa Pueblo in Bandelier National Monument. One question the archaeologists asked was: Is raw material used by prehistoric Indians for stone tool manufacture independent of the archaeological excavation site? Two different excavation sites at Burnt Mesa Pueblo gave the information in the following table. Use a chi-square test with 5% level of significance to test the claim that raw material used for construction of stone tools and excavation site are independent.

Stone Tool Construction Material, Burnt Mesa Pueblo			
Material	Site A	Site B	Row Total
Basalt	731	584	1315
Obsidian	102	93	195
Pedernal chert	510	525	1035
Other	85	94	179
Column Total	1428	1296	2724

18. (ii) (a) $\alpha = 0.01$; H_0: The proportions of Democratic and Republican congress members spending specific amounts on home district projects are the same. H_1: The proportions of Democratic and Republican congress members spending specific amounts on home district projects are not the same.
(b) $\chi^2 = 2.17$; $d.f. = 2$.
(c) $0.100 < P\text{-value} < 0.900$. From TI-84, $P\text{-value} \approx 0.3370$.
(d) Do not reject H_0.
(e) At the 1% level of significance, there is insufficient evidence to conclude that party affiliation and dollars spent are not independent.

18. *Political Affiliation: Spending* Two random samples were drawn from members of the U.S. Congress. One sample was taken from members who are Democrats and the other from members who are Republicans. For each sample, the number of dollars spent on federal projects in each congressperson's home district was recorded.
(i) Make a cluster bar graph showing the percentages of Congress members from each party who spent each designated amount in their respective home districts.
(ii) Use a 1% level of significance to test whether congressional members of each political party spent designated amounts in the same proportions.

Party	Dollars Spent on Federal Projects in Home Districts			Row Total
	Less than 5 Billion	5 to 10 Billion	More than 10 Billion	
Democratic	8	15	22	45
Republican	12	19	16	47
Column Total	20	34	38	92

19. (ii) (a) H_0: The proportions of the different age groups having each communication preference are the same.

19. *Sociology: Methods of Communication* Random samples of people ages 15–24 and 25–34 were asked about their preferred method of (remote) communication with friends. The respondents were asked to select one of the methods from the following list: cell phone, instant message, e-mail, other.

H_1: The proportions of the different age groups having each communication preference are not the same.
(b) $\chi^2 = 9.312$; $d.f. = 3$.
(c) $0.025 < P\text{-value} < 0.050$. From TI-84, $P\text{-value} \approx 0.0254$.
(d) Reject H_0.
(e) At the 5% level of significance, there is sufficient evidence to conclude that the two age groups do not have the same proportions of communications preferences.

(i) Make a cluster bar graph showing the percentages in each age group who selected each method.

(ii) Test whether the two populations share the same proportions of preferences for each type of communication method. Use $\alpha = 0.05$.

	Preferred Communication Method				
Age	Cell Phone	Instant Message	E-mail	Other	Row Total
15–24	48	40	5	7	100
25–34	41	30	15	14	100
Column Total	89	70	20	21	200

SECTION 10.2 Chi-Square: Goodness of Fit

FOCUS POINTS

- Set up a test to investigate how well a sample distribution fits a given distribution.
- Use observed and expected frequencies to compute the sample χ^2 statistic.
- Find or estimate the P-value and complete the test.

You can present this section first or by itself if you use the overview of the χ^2 distribution at the beginning of Part I.

Last year, the labor union bargaining agents listed five categories and asked each employee to mark the *one* most important to her or him. The categories and corresponding percentages of favorable responses are shown in Table 10-8. The bargaining agents need to determine if the *current* distribution of responses "fits" last year's distribution or if it is different.

In questions of this type, we are asking whether a population follows a specified distribution. In other words, we are testing the hypotheses

Hypotheses

H_0: The population fits the given distribution.
H_1: The population has a different distribution.

Computing sample χ^2

We use the chi-square distribution to test "goodness-of-fit" hypotheses. Just as with tests of independence, we compute the sample statistic:

$$\chi^2 = \Sigma \frac{(O - E)^2}{E} \text{ with degrees of freedom } = k - 1$$

where E = expected frequency
O = observed frequency
$\frac{(O - E)^2}{E}$ is summed for each category in the distribution
k = number of categories in the distribution

Next we use the chi-square distribution table (Table 7, Appendix II) to estimate the P-value of the sample χ^2 statistic. Finally, we compare the P-value to the level of significance α and conclude the test.

VIEWPOINT Run! Run! Run!

What description would you use for marathon runners? How about age distribution? Body weight? Length of stride? Heart rate? Blood pressure? What countries do these runners come from? What are their best running times? Make your own estimated distribution for these variables, and then consider a goodness-of-fit test for your distribution compared with available data. For more information on marathon runners, visit the Brase/Brase statistics site at **http://www.cengage.com/statistics/brase** *and find links to the Honolulu marathon site and to the* Runners World *site.*

SECTION 10.2 PROBLEMS

1. *Statistical Literacy* For a chi-square goodness-of-fit test, how are the degrees of freedom computed?

2. *Statistical Literacy* How are expected frequencies computed for goodness-of-fit tests?

3. *Statistical Literacy* Explain why goodness-of-fit tests are always right-tailed tests.

4. *Critical Thinking* When the sample evidence is sufficient to justify rejecting the null hypothesis in a goodness-of-fit test, can we tell exactly how the distribution of observed values over the specified categories differs from the expected distribution? Explain.

For Problems 5–16, please provide the following information.
(a) What is the level of significance? State the null and alternate hypotheses.
(b) Find the value of the chi-square statistic for the sample. Are all the expected frequencies greater than 5? What sampling distribution will you use? What are the degrees of freedom?
(c) Find or estimate the P-value of the sample test statistic.
(d) Based on your answers in parts (a) to (c), will you reject or fail to reject the null hypothesis that the population fits the specified distribution of categories?
(e) *Interpret* your conclusion in the context of the application.

5. *Census: Age* The age distribution of the Canadian population and the age distribution of a random sample of 455 residents in the Indian community of Red Lake Village (Northwest Territories) are shown below (based on *U.S. Bureau of the Census, International Data Base*).

Age (years)	Percent of Canadian Population	Observed Number in Red Lake Village
Under 5	7.2%	47
5 to 14	13.6%	75
15 to 64	67.1%	288
65 and older	12.1%	45

Use a 5% level of significance to test the claim that the age distribution of the general Canadian population fits the age distribution of the residents of Red Lake Village.

6. *Census: Type of Household* The type of household for the U.S. population and for a random sample of 411 households from the community of Dove Creek, Montana, are shown (based on *Statistical Abstract of the United States*).

1. $d.f.$ = number of categories − 1.
2. Consider the sample size n of all the observed frequencies. The expected frequency for a category is computed by taking the proportion of n designated by the proposed distribution.
3. The greater the differences between the observed frequencies and the expected frequencies, the higher the sample χ^2 value. Greater χ^2 values lead to the conclusion that the differences between expected and observed frequencies are too large to be explained by chance alone.
4. No. When we reject the null hypothesis in a goodness-of-fit test, we say that the observed distribution is simply different from the expected distribution. The test does not tell us how the distributions differ in each category.
5. (a) $\alpha = 0.05$; H_0: The distributions are the same; H_1: The distributions are different.
(b) Sample $\chi^2 = 11.788$; $d.f. = 3$.
(c) $0.005 < P\text{-value} < 0.010$.
(d) Reject H_0.
(e) At the 5% level of significance, the evidence is sufficient to conclude that the age distribution of the Red Lake Village population does not fit that of the general Canadian population.

6. (a) $\alpha = 0.05$; H_0: The distributions are the same; H_1: The distributions are different.
(b) Sample $\chi^2 = 13.017$; $d.f. = 4$.
(c) $0.010 < P\text{-value} < 0.025$.
(d) Reject H_0.
(e) At the 5% level of significance, the evidence is sufficient to conclude that the Dove Creek household distribution does not fit the general U.S. household distribution.

Type of Household	Percent of U.S. Households	Observed Number of Households in Dove Creek
Married with children	26%	102
Married, no children	29%	112
Single parent	9%	33
One person	25%	96
Other (e.g., roommates, siblings)	11%	68

Use a 5% level of significance to test the claim that the distribution of U.S. households fits the Dove Creek distribution.

7. (a) $\alpha = 0.01$; H_0: The distributions are the same; H_1: The distributions are different.
(b) Sample $\chi^2 = 0.1984$; $d.f. = 4$.
(c) $P\text{-value} > 0.995$. (Note that as the χ^2 values decrease, the area in the right tail increases, so $\chi^2 < 0.207$ means that the corresponding $P\text{-value} > 0.995$.)
(d) Do not reject H_0.
(e) At the 1% level of significance, the evidence is insufficient to conclude that the regional distribution of raw materials does not fit the distribution at the current excavation site.

7. *Archaeology: Stone Tools* The types of raw materials used to construct stone tools found at the archaeological site Casa del Rito are shown below (Bandelier *Archaeological Excavation Project*, edited by Kohler and Root). A random sample of 1486 stone tools was obtained from a current excavation site.

Raw Material	Regional Percent of Stone Tools	Observed Number of Tools at Current Excavation Site
Basalt	61.3%	906
Obsidian	10.6%	162
Welded tuff	11.4%	168
Pedernal chert	13.1%	197
Other	3.6%	53

Use a 1% level of significance to test the claim that the regional distribution of raw materials fits the distribution at the current excavation site.

8. (a) $\alpha = 0.05$; H_0: The distributions are the same; H_1: The distributions are different.
(b) Sample $\chi^2 = 1.084$; $d.f. = 4$.
(c) $0.100 < P\text{-value} < 0.900$.
(d) Do not reject H_0.
(e) At the 5% level of significance, the evidence is insufficient to conclude that the natural distribution of browse does not fit the feeding pattern.

8. *Ecology: Deer* The types of browse favored by deer are shown in the following table (*The Mule Deer of Mesa Verde National Park*, edited by Mierau and Schmidt). Using binoculars, volunteers observed the feeding habits of a random sample of 320 deer.

Type of Browse	Plant Composition in Study Area	Observed Number of Deer Feeding on This Plant
Sage brush	32%	102
Rabbit brush	38.7%	125
Salt brush	12%	43
Service berry	9.3%	27
Other	8%	23

Use a 5% level of significance to test the claim that the natural distribution of browse fits the deer feeding pattern.

Students sometimes ask how to determine if a distribution is normal. There are several techniques to test whether a distribution is normal. One technique, shown in Problems 9 and 10, utilizes the empirical rule to see if the observed distribution "fits" a normal distribution.

9. *Meteorology: Normal Distribution* The following problem is based on information from the *National Oceanic and Atmospheric Administration* (NOAA) *Environmental Data Service*. Let x be a random variable that represents the average daily temperature (in degrees Fahrenheit) in July in the town of Kit Carson, Colorado. The x distribution has a mean μ of approximately 75°F and standard deviation σ of approximately 8°F. A 20-year study (620 July days) gave the entries in the rightmost column of the following table.

6. (a) $\alpha = 0.05$; H_0: The distributions are the same; H_1: The distributions are different.
   (b) Sample $\chi^2 = 13.017$; *d.f.* $= 4$.
   (c) $0.010 < P\text{-value} < 0.025$.
   (d) Reject H_0.
   (e) At the 5% level of significance, the evidence is sufficient to conclude that the Dove Creek household distribution does not fit the general U.S. household distribution.

Type of Household	Percent of U.S. Households	Observed Number of Households in Dove Creek
Married with children	26%	102
Married, no children	29%	112
Single parent	9%	33
One person	25%	96
Other (e.g., roommates, siblings)	11%	68

Use a 5% level of significance to test the claim that the distribution of U.S. households fits the Dove Creek distribution.

7. (a) $\alpha = 0.01$; H_0: The distributions are the same; H_1: The distributions are different.
   (b) Sample $\chi^2 = 0.1984$; *d.f.* $= 4$.
   (c) *P*-value > 0.995. (Note that as the χ^2 values decrease, the area in the right tail increases, so $\chi^2 < 0.207$ means that the corresponding *P*-value > 0.995.)
   (d) Do not reject H_0.
   (e) At the 1% level of significance, the evidence is insufficient to conclude that the regional distribution of raw materials does not fit the distribution at the current excavation site.

7. *Archaeology: Stone Tools* The types of raw materials used to construct stone tools found at the archaeological site Casa del Rito are shown below (*Bandelier Archaeological Excavation Project*, edited by Kohler and Root). A random sample of 1486 stone tools was obtained from a current excavation site.

Raw Material	Regional Percent of Stone Tools	Observed Number of Tools at Current Excavation Site
Basalt	61.3%	906
Obsidian	10.6%	162
Welded tuff	11.4%	168
Pedernal chert	13.1%	197
Other	3.6%	53

Use a 1% level of significance to test the claim that the regional distribution of raw materials fits the distribution at the current excavation site.

8. (a) $\alpha = 0.05$; H_0: The distributions are the same; H_1: The distributions are different.
   (b) Sample $\chi^2 = 1.084$; *d.f.* $= 4$.
   (c) $0.100 < P\text{-value} < 0.900$.
   (d) Do not reject H_0.
   (e) At the 5% level of significance, the evidence is insufficient to conclude that the natural distribution of browse does not fit the feeding pattern.

8. *Ecology: Deer* The types of browse favored by deer are shown in the following table (*The Mule Deer of Mesa Verde National Park*, edited by Mierau and Schmidt). Using binoculars, volunteers observed the feeding habits of a random sample of 320 deer.

Type of Browse	Plant Composition in Study Area	Observed Number of Deer Feeding on This Plant
Sage brush	32%	102
Rabbit brush	38.7%	125
Salt brush	12%	43
Service berry	9.3%	27
Other	8%	23

Use a 5% level of significance to test the claim that the natural distribution of browse fits the deer feeding pattern.

Students sometimes ask how to determine if a distribution is normal. There are several techniques to test whether a distribution is normal. One technique, shown in Problems 9 and 10, utilizes the empirical rule to see if the observed distribution "fits" a normal distribution.

9. *Meteorology: Normal Distribution* The following problem is based on information from the *National Oceanic and Atmospheric Administration* (NOAA) *Environmental Data Service*. Let x be a random variable that represents the average daily temperature (in degrees Fahrenheit) in July in the town of Kit Carson, Colorado. The x distribution has a mean μ of approximately 75°F and standard deviation σ of approximately 8°F. A 20-year study (620 July days) gave the entries in the rightmost column of the following table.

(i) Answers vary.
(ii) (a) $\alpha = 0.01$; H_0: The distributions are the same; H_1: The distributions are different.
(b) Sample $\chi^2 = 1.5693$; $d.f. = 5$.
(c) $0.900 < P\text{-value} < 0.950$.
(d) Do not reject H_0.
(e) At the 1% level of significance, the evidence is insufficient to conclude that the average daily July temperature does not follow a normal distribution.

I	II	III	IV
Region under Normal Curve	x°F	Expected % from Normal Curve	Observed Number of Days in 20 Years
$\mu - 3\sigma \le x < \mu - 2\sigma$	$51 \le x < 59$	2.35%	16
$\mu - 2\sigma \le x < \mu - \sigma$	$59 \le x < 67$	13.5%	78
$\mu - \sigma \le x < \mu$	$67 \le x < 75$	34%	212
$\mu \le x < \mu + \sigma$	$75 \le x < 83$	34%	221
$\mu + \sigma \le x < \mu + 2\sigma$	$83 \le x < 91$	13.5%	81
$\mu + 2\sigma \le x < \mu + 3\sigma$	$91 \le x < 99$	2.35%	12

(i) Remember that $\mu = 75$ and $\sigma = 8$. Examine Figure 6-5 in Chapter 6. Write a brief explanation for Columns I, II, and III in the context of this problem.
(ii) Use a 1% level of significance to test the claim that the average daily July temperature follows a normal distribution with $\mu = 75$ and $\sigma = 8$.

(i) Answers vary.
(ii) (a) $\alpha = 0.01$; H_0: The distributions are the same; H_1: The distributions are different.
(b) Sample $\chi^2 = 0.2562$; $d.f. = 5$.
(c) $P\text{-value} > 0.995$.
(d) Do not reject H_0.
(e) At the 1% level of significance, the evidence is insufficient to conclude that the average daily January temperature does not follow a normal distribution.

10. *Meteorology: Normal Distribution* Let x be a random variable that represents the average daily temperature (in degrees Fahrenheit) in January for the town of Hana, Maui. The x variable has a mean μ of approximately 68°F and standard deviation σ of approximately 4°F (see reference in Problem 9). A 20-year study (620 January days) gave the entries in the rightmost column of the following table.

I	II	III	IV
Region Under Normal Curve	x°F	Expected % from Normal Curve	Observed Number of Days in 20 Years
$\mu - 3\sigma \le x < \mu - 2\sigma$	$56 \le x < 60$	2.35%	14
$\mu - 2\sigma \le x < \mu - \sigma$	$60 \le x < 64$	13.5%	86
$\mu - \sigma \le x < \mu$	$64 \le x < 68$	34%	207
$\mu \le x < \mu + \sigma$	$68 \le x < 76$	34%	215
$\mu + \sigma \le x < \mu + 2\sigma$	$72 \le x < 76$	13.5%	83
$\mu + 2\sigma \le x < \mu + 3\sigma$	$76 \le x < 80$	2.35%	15

(i) Remember that $\mu = 68$ and $\sigma = 4$. Examine Figure 6-5 in Chapter 6. Write a brief explanation for Columns I, II, and III in the context of this problem.
(ii) Use a 1% level of significance to test the claim that the average daily January temperature follows a normal distribution with $\mu = 68$ and $\sigma = 4$.

(a) $\alpha = 0.05$; H_0: The distributions are the same; H_1: The distributions are different.
(b) Sample $\chi^2 = 9.333$; $d.f. = 3$.
(c) $0.025 < P\text{-value} < 0.050$.
(d) Reject H_0.
(e) At the 5% level of significance, the evidence is sufficient to conclude that the current fish distribution is different than it was 5 years ago.

11. *Ecology: Fish* The Fish and Game Department stocked Lake Lulu with fish in the following proportions: 30% catfish, 15% bass, 40% bluegill, and 15% pike. Five years later it sampled the lake to see if the distribution of fish had changed. It found that the 500 fish in the sample were distributed as follows.

Catfish	Bass	Bluegill	Pike
120	85	220	75

In the 5-year interval, did the distribution of fish change at the 0.05 level?

12. *Library: Book Circulation* The director of library services at Fairmont College did a survey of types of books (by subject) in the circulation library. Then she used library records to take a random sample of 888 books checked out last

12. (a) $\alpha = 0.05$; H_0: The distributions are the same; H_1: The distributions are different.
(b) Sample $\chi^2 = 11.92$; *d.f.* $= 4$.
(c) $0.010 < P\text{-value} < 0.025$.
(d) Reject H_0.
(e) At the 5% level of significance, the evidence is sufficient to conclude that the subject distribution of books in the library is different from that of books checked out by students.

13. (a) $\alpha = 0.01$; H_0: The distributions are the same; H_1: The distributions are different.
(b) Sample $\chi^2 = 13.70$; *d.f.* $= 5$.
(c) $0.010 < P\text{-value} < 0.025$.
(d) Do not reject H_0.
(e) At the 1% level of significance, the evidence is insufficient to conclude that the census ethnic origin distribution and the ethnic origin distribution of city residents are different.

14. (a) $\alpha = 0.01$; H_0: The distributions are the same; H_1: The distributions are different.
(b) Sample $\chi^2 = 15.65$; *d.f.* $= 5$.
(c) $0.005 < P\text{-value} < 0.010$.
(d) Reject H_0.
(e) At the 1% level of significance, the evidence is sufficient to conclude that the distribution of customer ages and the age distribution shown in the Snoop report are different.

term and classified the books in the sample by subject. The results are shown below.

Subject Area	Percent of Books in Circulation Library on This Subject	Number of Books in Sample on This Subject
Business	32%	268
Humanities	25%	214
Natural science	20%	215
Social science	15%	115
All other subjects	8%	76

Using a 5% level of significance, test the claim that the subject distribution of books in the library fits the distribution of books checked out by students.

13. *Census: California* The accuracy of a census report on a city in southern California was questioned by some government officials. A random sample of 1215 people living in the city was used to check the report, and the results are shown here:

Ethnic Origin	Census Percent	Sample Result
Black	10%	127
Asian	3%	40
Anglo	38%	480
Latino/Latina	41%	502
Native American	6%	56
All others	2%	10

Using a 1% level of significance, test the claim that the census distribution and the sample distribution agree.

14. *Marketing: Compact Discs* Snoop Incorporated is a firm that does market surveys. The Rollum Sound Company hired Snoop to study the age distribution of people who buy compact discs. To check the Snoop report, Rollum used a random sample of 519 customers and obtained the following data:

Customer Age (years)	Percent of Customers from Snoop Report	Number of Customers from Sample
Younger than 14	12%	88
14–18	29%	135
19–23	11%	52
24–28	10%	40
29–33	14%	76
Older than 33	24%	128

Using a 1% level of significance, test the claim that the distribution of customer ages in the Snoop report agrees with that of the sample report.

15. *Accounting Records: Benford's Law* Benford's law states that the first nonzero digits of numbers drawn at random from a large complex data file have the following probability distribution (Reference: American Statistical Association, *Chance*, Vol. 12, No. 3, pp. 27–31; see also the Focus Problem of Chapter 9).

First nonzero digit	1	2	3	4	5	6	7	8	9
Probability	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046