15. (a) \( \alpha = 0.01; H_0: \) The distributions are the same; \( H_1: \) The distributions are different.
(b) Sample \( \chi^2 = 3.559; df = 8. \)
(c) 0.100 < \( P \)-value < 0.900.
(d) Do not reject \( H_0. \)
(e) At the 1% level of significance, the evidence is insufficient to conclude that the distribution of first nonzero digits in the accounting file does not follow Benford’s Law.

16. (a) \( \alpha = 0.01; H_0: \) The distributions are the same; \( H_1: \) The distributions are different.
(b) Sample \( \chi^2 = 13.44; df = 5. \)
(c) 0.010 < \( P \)-value < 0.025.
(d) Do not reject \( H_0. \)
(e) At the 1% level of significance, the evidence is insufficient to conclude that the distribution of observed outcomes for the die is different from the expected distribution of a fair die. In other words, at the 1% level of significance, we cannot conclude that the die is not balanced.

12. (a) \( P(0) = 0.179; P(1) = 0.308; \)
\( P(2) = 0.265; P(3) = 0.152; \)
\( P(4) = 0.049. \)
(b) For \( r = 0, E = 16.11; \) for \( r = 1, E = 27.72; \) for \( r = 2, E = 23.85; \)
\( r = 3, E = 13.58; \) for \( r = 4, E = 8.64. \)
(c) \( \chi^2 = 12.55 \) with \( df = 4. \)
(d) \( \alpha = 0.01; H_0: \) The Poisson distribution fits; \( H_1: \) The Poisson distribution does not fit.
0.01 < \( P \)-value < 0.025; do not reject \( H_0. \) At the 1% level of significance, we cannot say that the Poisson distribution does not fit the sample data.

17. **Fair Dice: Uniform Distribution** A gambler complained about the dice. They seemed to be loaded! The dice were taken off the table and tested one at a time. One die was rolled 300 times and the following frequencies were recorded.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed frequency ( O )</td>
<td>62</td>
<td>45</td>
<td>63</td>
<td>32</td>
<td>18</td>
<td>13</td>
</tr>
</tbody>
</table>

Do these data indicate that the die is unbalanced? Use a 1% level of significance.

**Hint:** If the die is balanced, all outcomes should have the same expected frequency.

18. **Bacteria Colonies: Poisson Distribution** A pathologist has been studying the frequency of bacterial colonies within the field of a microscope using samples of throat cultures from healthy adults. Long-term history indicates that there is an average of 2.80 bacteria colonies per field. Let \( r \) be a random variable that represents the number of bacteria colonies per field. Let \( O \) represent the number of observed bacteria colonies per field for throat cultures from healthy adults. A random sample of 100 healthy adults gave the following information.

<table>
<thead>
<tr>
<th>( r )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O )</td>
<td>12</td>
<td>15</td>
<td>29</td>
<td>18</td>
<td>19</td>
<td>7</td>
</tr>
</tbody>
</table>
Test of Variance

**Hypothesis Testing:** To test the hypothesis that the variance is less than a specified value, we use the chi-square distribution as our test statistic.

**Procedure:**
1. Set the significance level.
2. Compute the sample variance.
3. Compute the test statistic:
   
   $$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

   where $n$ is the sample size, $s^2$ is the sample variance, and $\sigma^2$ is the hypothesized population variance.
4. Compare the test statistic to the chi-square distribution with $n-1$ degrees of freedom.
5. Make a decision based on the comparison.

**Example:** If we have a sample of 20 observations with a sample variance of 5, and we want to test if this variance is significantly less than 4 at a 0.05 significance level.

- **Step 1:** Set the significance level at 0.05.
- **Step 2:** Compute the sample variance: $s^2 = 5$.
- **Step 3:** Compute the test statistic:
   
   $$\chi^2 = \frac{(20-1)5}{4} = \frac{95}{4} = 23.75$$

- **Step 4:** Compare the test statistic to the chi-square distribution with 19 degrees of freedom.
   - The critical value for a right-tailed test at 0.05 significance level is approximately 30.14.
   - Since 23.75 < 30.14, we fail to reject the null hypothesis.

**Conclusion:** There is not enough evidence to conclude that the variance is significantly less than 4 at the 0.05 significance level.